Lecture on First-principles Computations (22)
Evaluation of the Interacting Green Function

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Recall: properties of the Green function

- Defined as the expectation value of the time-ordered product of creation and annihilation operators with respect to the exact ground state.

- Once known, the Green function can be used to compute

  – the expectation value of any single-particle operator within the ground state of the system

  – the ground-state energy (through the equation of motion of the Green function)

  – the excitation spectrum of the system (through the Lehmann representation)

Evaluation of the Green function for interacting systems

How to evaluate the Green function for interacting systems ?

The usual approach is many-body perturbation theory. For this it is most convenient to work in the interaction picture !

The interaction picture

- Schrödinger picture: $$i\frac{\partial|\Psi_S(t)\rangle}{\partial t} = \hat{H}|\Psi_S(t)\rangle$$

$$|\Psi_S(t)\rangle = e^{-i\hat{H}(t-t_0)}|\Psi_S(t_0)\rangle, \qquad O(t) = \langle\Psi_S(t) \mid \hat{O}_s \mid \Psi_S(t)\rangle$$

- Heisenberger Picture: $$|\Psi_H(t)\rangle = e^{i\hat{H}t}|\Psi_S(t)\rangle = |\Psi_S(0)\rangle$$

$$i\frac{\partial|\Psi_H(t)\rangle}{\partial t} = 0, \qquad \hat{O}_H(t) = e^{i\hat{H}t}\hat{O}_S e^{-i\hat{H}t}$$

- The interaction picture: $$\hat{H} = \hat{H}_0 + \hat{H}_{int}$$

$$|\Psi_I(t)\rangle = e^{i\hat{H}_0 t}|\Psi_S(t)\rangle,$$

$$i\frac{\partial|\Psi_I(t)\rangle}{\partial t} = \hat{H}_{int}(t)|\Psi_I(t)\rangle$$

$$\hat{O}_I(t) = e^{i\hat{H}_0 t}\hat{O}_S e^{-i\hat{H}_0 t},$$

$$\hat{H}_{int}(t) = e^{i\hat{H}_0 t}\hat{H}_{int} e^{-i\hat{H}_0 t}$$

$$i\frac{\partial\hat{O}_I(t)}{\partial t} = \left[\hat{O}_I(t), \hat{H}_0\right]$$

The evolution operator in the interaction picture

The evolution operator:

$$|\Psi_I(t)\rangle = \hat{U}(t,t_0)|\Psi_I(t_0)\rangle$$

A formal solution of the evolution operator,

$$\hat{U}(t,t_0) = e^{i\hat{H}_0 t} e^{-i\hat{H}(t-t_0)} e^{-i\hat{H}_0 t_0}$$

Apparently $\hat{U}(t,t_0)$ is a unitary operator

$$\hat{U}^{\dagger}(t,t_0) = \hat{U}^{-1}(t,t_0)$$

$$\hat{U}(t_{1,}t_2)\hat{U}(t_{2,}t_3) = \hat{U}(t_{1,}t_3)$$

$$\hat{U}(t,t_0)\hat{U}(t_{0,}t) = 1$$

Integral equation of the evaluation operator

$\widehat{U}(t,t_0)$ satisfy the equation of motion:

$$i\frac{\partial}{\partial t}\widehat{U}(t,t_0) = \widehat{H}_{int}(t)\widehat{U}(t,t_0)$$

We can turn this into an integral equation:

$$\widehat{U}(t,t_0) = 1 - i\int_{t_0}^{t} \widehat{H}_{int}(t')\,\widehat{U}(t',t_0)dt'$$

Solving the integral equation iteratively:

$$\widehat{U}(t,t_0) = 1 - i\int_{t_0}^{t} \widehat{H}_{int}\,(t')dt' + (-i)^2\int_{t_0}^{t} dt\,'\int_{t_0}^{t'} d\,t''\widehat{H}_{int}(t')\widehat{H}_{int}(t") + \cdots$$

where $\widehat{H}_{int}(t) = \frac{1}{2}\int\int dx\,dx'\widehat{\psi}^{\dagger}(x,t)\widehat{\psi}^{\dagger}(x',t)V(x,x')\widehat{\psi}(x',\mathrm{t})\widehat{\psi}(x,t)$

The perturbative solution of the evaluation operator

$$\hat{U}(t,t_0) = 1 - i\int_{t_0}^{t} \hat{H}_{int}(t')dt' + (-i)^2 \int_{t_0}^{t} dt' \int_{t_0}^{t'} d\,t'' \hat{H}_{int}(t')\hat{H}_{int}(t'') + \cdots$$

$$= 1 - i\int_{t_0}^{t} \hat{H}_{int}(t')dt' + (-i)^2 \int_{t_0}^{t} dt'' \int_{t''}^{t} d\,t' \hat{H}_{int}(t')\hat{H}_{int}(t'') + \cdots$$

$$= 1 - i\int_{t_0}^{t} \hat{H}_{int}(t')dt' + \frac{(-i)^2}{2!} \int_{t_0}^{t} dt' \int_{t_0}^{t} d\,t'' \hat{T}\left[\hat{H}_{int}(t')\hat{H}_{int}(t'')\right] + \cdots$$

Changing the range of the integral

Finally

$$\hat{U}(t,t_0) = 1 + \sum_{n=1}^{\infty} \frac{(-i)^n}{n!} \int_{t_0}^{t} dt_1 \cdots \int_{t_0}^{t} d\,t_n \hat{T}\left[\hat{H}_{int}(t_1)\cdots\hat{H}_{int}(t_n)\right]$$

$$= \hat{T} e^{-i\int_{t_0}^{t} \hat{H}_{int}(t')dt'}$$

Adiabatic "switching on"

Consider a time-dependent Hamiltonian,

$$\hat{H} = \hat{H}_0 + e^{-\epsilon|t|}\hat{H}_{int}$$ (ϵ: a small positive number)

When $t \to \infty$ or $-\infty$, $\hat{H} \to \hat{H}_0$

The evolution operator can still be defined!

$$|\Psi_I(t)\rangle = \hat{U}_\epsilon(t, t_0)|\Psi_I(t_0)\rangle$$

$$i\frac{\partial}{\partial t}\hat{U}_\epsilon(t, t_0) = e^{-\epsilon|t|}\hat{H}_{int}(t)\hat{U}_\epsilon(t, t_0)$$

$$\hat{U}_\epsilon(t, t_0) = 1 + \sum_{n=1}^{\infty}\frac{(-i)^n}{n!}\int_{t_0}^{t} dt_1 \cdots \int_{t_0}^{t} d\,t_n e^{-\epsilon(|t_1|+\cdots+|t_n|)}\hat{T}\left[\hat{H}_{int}(t_1)\cdots\hat{H}_{int}(t_n)\right]$$

$\epsilon \to 0$, the interaction turned on infinitely slowly, "adiabatic".

Adiabatic "switching-on" (II)

Suppose $|\Phi_0\rangle$ is an eigenstate of $\hat{H}_0$: $\hat{H}_0|\Phi_0\rangle = E_0|\Phi_0\rangle$:

For $t_0 \to -\infty$, $|\psi_s(t_0)\rangle = e^{-iE_0 t_0}|\Phi_0\rangle$, and then

$$|\Psi_I(t_0)\rangle = e^{i\hat{H}_0 t_0}|\Psi_s(t_0)\rangle = |\Phi_0\rangle \quad (t_0 \to -\infty)$$

$$i\frac{\partial|\Psi_I(t)\rangle}{\partial t} = e^{-\epsilon|t|}\hat{H}_{int}(t)|\Psi_I(t)\rangle \to 0, \qquad t \to \pm\infty$$

$$|\Psi_{I,\epsilon}(t=0)\rangle = \hat{U}_\epsilon(0,-\infty)|\Phi_0\rangle$$ ($|\Psi_{I,\epsilon}(t=0)\rangle$ may depend on ϵ)

We would like to take $\epsilon \to 0$ (adiabatically switching on the interaction), and see what happen?

$$\lim_{\epsilon\to 0}|\Psi_{I,\epsilon}(t=0)\rangle \overset{?}{\to} |\Psi_H\rangle = |\Psi_0\rangle$$

Gell-Mann - Low theorem

$$\lim_{\epsilon\to 0}\frac{\hat{U}_\epsilon(0,-\infty)|\Phi_0\rangle}{\langle\Phi_0\mid\hat{U}_\epsilon(0,-\infty)\mid\Phi_0\rangle}=\frac{|\Psi_0\rangle}{\langle\Phi_0\mid\Psi_0\rangle}$$

The numerator and denominator don't exist separately, but the limit of the radio is well-defined.

The limit is an eigenstate of the full Hamiltonian :

$$\hat{H}\frac{|\Psi_0\rangle}{\langle\Phi_0\mid\Psi_0\rangle}=E\frac{|\Psi_0\rangle}{\langle\Phi_0\mid\Psi_0\rangle}$$

The expectation value of an operator in the interaction picture

Gell-Mann – Low theorem

$$\frac{\hat{U}_\epsilon(0,-\infty)|\Phi_0\rangle}{\langle\Phi_0 \mid \hat{U}_\epsilon(0,-\infty) \mid \Phi_0\rangle} = \frac{|\Psi_0\rangle}{\langle\Phi_0 \mid \Psi_0\rangle} \quad (\text{valid for } \epsilon \to 0)$$

$$\hat{U}_\epsilon(t,t_0) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{t_0}^{t} dt_1 \cdots \int_{t_0}^{t} d\,t_n e^{-\epsilon(|t_1|+\cdots+|t_n|)} \hat{T}\left[\hat{H}_{int}(t_1)\cdots\hat{H}_{int}(t_n)\right]$$

It is easy to prove,

$$\frac{\langle\Psi_0 \mid \hat{O}_H(t) \mid \Psi_0\rangle}{\langle\Psi_0 \mid \Psi_0\rangle} = \frac{1}{\langle\Phi_0 \mid \hat{S}(\infty,-\infty) \mid \Phi_0\rangle} \times$$

$$\langle\Phi_0 \mid \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} dt_1 \cdots \int_{-\infty}^{\infty} d\,t_n e^{-\epsilon(|t_1|+\cdots+|t_n|)} \hat{T}\left[\hat{H}_{int}(t_1)\cdots\hat{H}_{int}(t_n)\hat{O}_I(t)\right] \mid \Phi_0\rangle$$

The expectation value of the time-ordered product of two operators

$$\frac{\langle \Psi_0 \mid \hat{T}\left[\hat{O}_H(t)\hat{O}_H(t')\right] \mid \Psi_0\rangle}{\langle \Psi_0 \mid \Psi_0 \mid\rangle} = \frac{1}{\langle \Phi_0 \mid \hat{S}(-\infty,\infty) \mid \Phi_0\rangle} \times$$

$$\langle \Phi_0 \mid \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} dt_1 \cdots \int_{-\infty}^{\infty} d\, t_n e^{-\epsilon(|t_1|+\cdots+|t_n|)} \hat{T}\left[\hat{H}_{int}(t_1) \cdots \hat{H}_{int} \hat{O}_I(t)\hat{O}_I(t')\right] \mid \Phi_0\rangle$$

=1

ϵ is allowed to take the limit $\epsilon \to 0$

As an example:

$$G(x,x';t-t') = -i\frac{\langle \Psi_0 \mid \hat{T}\left[\hat{\psi}_H(x,t)\hat{\psi}_H^\dagger(x',t')\right] \mid \Psi_0\rangle}{\langle \Psi_0 \mid \Psi_0 \mid\rangle} = \frac{-i}{\langle \Phi_0 \mid \hat{S}(-\infty,\infty) \mid \Phi_0\rangle} \times$$

$$\langle \Phi_0 \mid \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} dt_1 \cdots \int_{-\infty}^{\infty} d\, t_n \hat{T}\left[\hat{H}_{int}(t_1) \cdots \hat{H}_{int}(t_n)\hat{\psi}_I(x,t)\hat{\psi}_I^\dagger(x',t')\right] \mid \Phi_0\rangle$$

Now, the interacting Green function

$$G(x,t;x',t') = -i\,\frac{\langle \Psi_0 \mid T\hat{\psi}_H(x,t)\hat{\psi}_H^\dagger(x',t') \mid \Psi_0\rangle}{\langle \Psi_0 \mid \Psi_0\rangle}$$

Switching to the interaction picture :

$$G(x,t;x',t') = -i\sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int_{t_0}^{t} dt_1 \cdots \int_{t_0}^{t} d\,t_n$$

$$\frac{\langle \Phi_0 \mid \hat{T}\left[\hat{H}_{int}(t_1)\cdots\hat{\psi}_I(x,t)\cdots\hat{\psi}_I(x',t')\cdots\hat{H}_{int}(t_n)\right] \mid \Phi_0\rangle}{\langle \Phi_0 \mid \hat{S}(\infty,-\infty) \mid \Phi_0\rangle}$$

$$\hat{S}(\infty,-\infty) = \lim_{\epsilon\to 0}\hat{U}_\epsilon(\infty,-\infty)$$

The first few terms ...

The numerator:

$$\tilde{G}(x,x';t-t') = -i\langle\Psi_0 \mid \hat{T}\left[\hat{\psi}_H(x,t)\hat{\psi}_H^\dagger(x',t')\right] \mid \Psi_0\rangle =$$

$$-i\,\langle\Phi_0 \mid \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int_{-\infty}^{\infty} dt_1 \cdots \int_{-\infty}^{\infty} d\,t_n \hat{T}\left[\hat{H}_{int}(t_1)\cdots\hat{H}_{int}(t_n)\hat{\psi}_I(x,t)\hat{\psi}_I^\dagger(x',t')\right] \mid \Phi_0\rangle$$

The denominator:

$$\langle\Phi_0 \mid \hat{S}(\infty,-\infty) \mid \Phi_0\rangle = \langle\Phi_0 \mid \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int_{-\infty}^{\infty} dt_1 \cdots \int_{-\infty}^{\infty} d\,t_n \hat{T}\left[\hat{H}_{int}(t_1)\cdots\hat{H}_{int}(t_n)\right] \mid \Phi_0\rangle$$

Look at the numerator first ...

- The zeroth-order term (non-interacting Green function):

$$\tilde{G}^{n=0}(x,x';t-t') = -i\langle\Phi_0 \mid \hat{T}\left[\hat{\psi}_I(x,t)\hat{\psi}_I^\dagger(x',t')\right] \mid \Phi_0\rangle = G_0(x,x';t-t')$$

The first few terms ...

- The first-order term :

$$\tilde{G}^{n=1}(x, x'; t-t') =$$

$$i \langle \Phi_0 \mid \int_{-\infty}^{\infty} dt_1 \, \hat{T}\left[\hat{H}_{int}(t_1) \hat{\psi}_I(x,t) \hat{\psi}_I^\dagger(x',t') \right] \mid \Phi_0 \rangle$$

$$= i \frac{1}{2} \int_{-\infty}^{\infty} dt_1 \int d\,x_1 \int d\,x_2 V(x_1 - x_2) \times$$

$$\langle \Phi_0 | \hat{T}[\hat{\psi}^\dagger(x_{1,}t_1) \hat{\psi}^\dagger(x_{2,}t_1) \hat{\psi}(x_{2,}t_1) \hat{\psi}(x_{1,}t_1) \hat{\psi}(x,t) \hat{\psi}^\dagger(x',t')] | \Phi_0 \rangle$$

6 fermionic operators

……

The first few terms ...

- The second-order term :

$$\tilde{G}^{n=2}(x,x';t-t') =$$

$$\frac{i}{2!}\langle\Phi_0 \mid \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2\, \hat{T}\left[\hat{H}_{int}(t_1)\hat{H}_{int}(t_2)\hat{\psi}_I(x,t)\hat{\psi}_I^{\dagger}(x',t')\right] \mid \Phi_0\rangle$$

$$= \frac{i}{2!}\frac{1}{2^2}\int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \int d\,x_1 \int d\,x_2 \int d\,x_3 \int d\,x_4 V(x_1-x_2)V(x_3-x_4) \times$$

$$\langle\Phi_0|\hat{T}[\hat{\psi}^{\dagger}(x_{1,}t_1)\hat{\psi}^{\dagger}(x_{2,}t_1)\hat{\psi}(x_{2,}t_1)\hat{\psi}(x_{1,}t_1) \times$$

$$\hat{\psi}^{\dagger}(x_{3,}t_2)\hat{\psi}^{\dagger}(x_{4,}t_2)\hat{\psi}(x_{4,}t_2)\hat{\psi}(x_{3,}t_2)\hat{\psi}(x,t)\hat{\psi}^{\dagger}(x',t')]|\Phi_0\rangle$$

10 fermionic operators

……

Wick's theorem

The key is to evaluate the expectation value of
the time-ordered products of operators within the non-interacting ground state.

$$\langle \Phi_0 \mid \hat{T}\left[\hat{\psi}(x_{1,}t_1)\hat{\psi}^\dagger(x_{2,}t_2)\hat{\psi}^\dagger(x_{3,}t_3)\cdots\hat{\psi}(x_N,t_N)\right] \mid \Phi_0\rangle$$

Wick's theorem states that this can be factorized into products of pairs; in making all possible pairings of creation and annihilated operators, each pair should be time-ordered.

Wick's theorem

Some simple rules in applying the Wick's theorem:

- The number of creation and annihilation operators must be the same.

- The anticommutation rule needs to be taken care of when moving the operators to form pairs.

- For m creation (or annihilation) operators, there are $m!$ possible ways to form pairs ($m=2n+1$ at the n-th order.)

A simple example

$$\langle \Phi_0 | \hat{T}[\hat{\psi}^\dagger(x_1,t_1)\hat{\psi}^\dagger(x_2,t_2)\hat{\psi}(x_2,t_2)\hat{\psi}(x_1,t_1)] | \Phi_0 \rangle$$

$$= \langle \Phi_0 | \hat{T}[\hat{\psi}^\dagger(x_1,t_1)\hat{\psi}(x_1,t_1)] | \Phi_0 \rangle \langle \Phi_0 | \hat{T}[\hat{\psi}^\dagger(x_2,t_2)\hat{\psi}(x_2,t_2)] | \Phi_0 \rangle$$

$$- \langle \Phi_0 | \hat{T}[\hat{\psi}^\dagger(x_1,t_1)\hat{\psi}(x_2,t_2)] | \Phi_0 \rangle \langle \Phi_0 | \hat{T}[\hat{\psi}^\dagger(x_2,t_2)\hat{\psi}(x_1,t_1)] | \Phi_0 \rangle$$

$$= \left(-iG(x_1,x_1;t=0^-)\right)\left(-iG(x_2,x_2;t=0^-)\right)$$

$$- \left(-iG(x_1,x_2;t_2-t_1)\right)\left(-iG(x_1,x_2;t_1-t_2)\right)$$

$n(x_2)$

The electron density

A simple example

$$\langle \Phi_0 | \hat{T}[\hat{\psi}^\dagger(x_1,t_1)\hat{\psi}^\dagger(x_2,t_2)\hat{\psi}(x_2,t_2)\hat{\psi}(x_1,t_1)] | \Phi_0 \rangle$$

$$= \langle \Phi_0 | \hat{T}[\hat{\psi}^\dagger(x_1,t_1)\hat{\psi}(x_1,t_1)] | \Phi_0 \rangle \langle \Phi_0 | \hat{T}[\hat{\psi}^\dagger(x_2,t_2)\hat{\psi}(x_2,t_2)] | \Phi_0 \rangle$$

$$- \langle \Phi_0 | \hat{T}[\hat{\psi}^\dagger(x_1,t_1)\hat{\psi}(x_2,t_2)] | \Phi_0 \rangle \langle \Phi_0 | \hat{T}[\hat{\psi}^\dagger(x_2,t_2)\hat{\psi}(x_1,t_1)] | \Phi_0 \rangle$$

$$= \left(-iG_0(x_1,x_1;t=0^-)\right)\left(-iG_0(x_2,x_2;t=0^-)\right)$$

$$- \left(-iG_0(x_1,x_2;t_2-t_1)\right)\left(-iG_0(x_1,x_2;t_1-t_2)\right)$$

$n(x_2)$

The electron density

Integrating over the Coulomb interaction, $V(x_1,x_2)\delta(t_1-t_2)$, one gets the Hartree-Fock energy.

Evaluating the first-order Green function

$$\tilde{G}^{n=1}(x,x';t-t') = -\frac{1}{2}\int_{-\infty}^{\infty} dt_1 \int d\,x_1 \int d\,x_2 V(x_1 - x_2) \times$$

$$\langle \Phi_0 \mid \hat{T}\left[\hat{\psi}^\dagger(x_1,t_1)\hat{\psi}^\dagger(x_2,t_1)\hat{\psi}(x_2,t_1)\hat{\psi}(x_1,t_1)\hat{\psi}(x,t)\hat{\psi}^\dagger(x',t')\right] \mid \Phi_0\rangle$$

There are six ways of forming pairs!

$$(1) = \left(-iG_0\left(x_1,x_1;0^-\right)\right)\left(-iG_0\left(x_2,x_2;0^-\right)\right)\left(iG_0(x,x';t-t')\right)$$

$$(2) = -\left(-iG_0\left(x_1,x_2;0^-\right)\right)\left(-iG_0\left(x_2,x_1;0^-\right)\right)\left(iG_0(x,x';t-t')\right)$$

$$(3) = -\left(-iG_0(x,x_1;t-t_1)\right)\left(-iG_0\left(x_2,x_2;0^-\right)\right)\left(iG_0\left(x_1,x';t_1-t'\right)\right)$$

$$(4) = \left(-iG_0\left(x_1,x_2;0^-\right)\right)\left(-iG_0(x,x_1;t-t_1)\right)\left(iG_0\left(x_2,x';t_2-t'\right)\right)$$

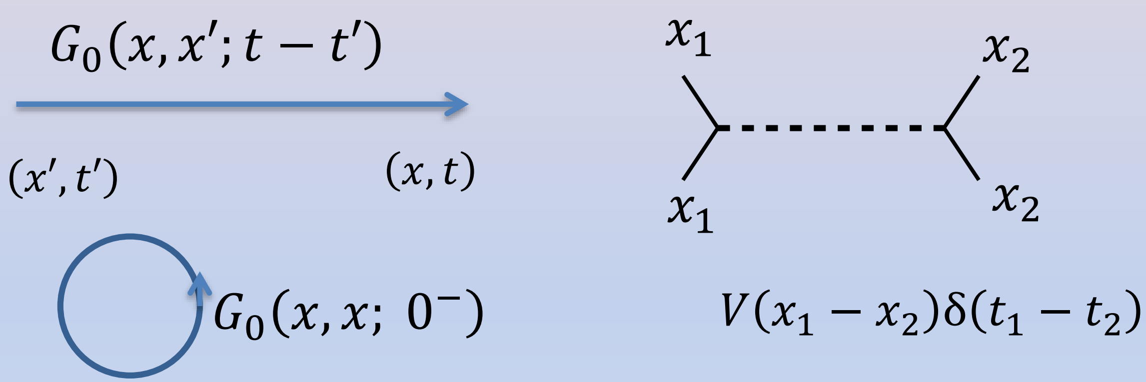
$$(5) = -\left(-iG_0(x,x_2;t-t_1)\right)\left(-iG_0\left(x_1,x_1;0^-\right)\right)\left(iG_0\left(x_2,x';t_1-t'\right)\right)$$

$$(6) = \left(-iG_0\left(x_2,x_1;0^-\right)\right)\left(-iG_0(x,x_2;t-t_1)\right)\left(iG_0\left(x_1,x';t_2-t'\right)\right)$$

It is very tedious and lengthy to evaluate these terms algebraically; not the way to go!

The way out: Feynman diagrams

Using pictures to represent the algebraic expressions!



Advantages of Feynman diagrams:

- Intuitive, and reflects the underlying physical processes
- Reveals simple rules for accounting for physically relevant contributions.

The cancellation theorem

The Green function in the numerator $\tilde{G}$ factorizes into connected and disconnected parts !

$$\tilde{G}(x,x';t-t') = -i\sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int_{-\infty}^{\infty}dt_1\cdots\int_{-\infty}^{\infty}d\,t_n$$

$$\times\langle\Phi_0\,|\,\hat{T}\big[\hat{H}_{int}(t_1)\cdots\hat{H}_{int}(t_n)\hat{\psi}_I(x,t)\hat{\psi}_I^{\dagger}(x',t')\big]\,|\,\Phi_0\rangle_{\text{connected}}$$

$$\times\langle\Phi_0\,|\,S(\infty,-\infty)\,|\,\Phi_0\rangle$$

The closed part of disconnected diagrams

The true Green function:

$$G(x,x';t-t') = \frac{\tilde{G}(x,x';t-t')}{\langle\Phi_0\,|\,S(\infty,-\infty)\,|\,\Phi_0\rangle} = -i\sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int_{-\infty}^{\infty}dt_1\cdots\int_{-\infty}^{\infty}d\,t_n$$

$$\times\langle\Phi_0\,|\,\hat{T}\big[\hat{H}_{int}(t_1)\cdots\hat{H}_{int}(t_n)\hat{\psi}_I(x,t)\hat{\psi}_I^{\dagger}(x',t')\big]\,|\,\Phi_0\rangle_{\text{connected}}$$

Only connected diagrams need to be taken into account!

The counting factor

For each connected diagram at order *n*, there are $2^n n!$ many diagrams that have the same "topological structure". They differ only in the permutation of the dummy integration variables, and hence yield the same contribution, killing the $1/n!$ prefactor in front (and also 2^n from the Coulomb interaction).

Only topologically different diagrams need to be taken into account!

- For $n=1$, the number of topologically different connected diagrams is 2.
- For $n=2$, this number is 10. (Remember the total number is 120)

The next steps ...

- Dyson equation
- Irreducible self-energy