Accurately calculation high quality factor of whispering-gallery modes with boundary element method

Chang-Ling Zou[1], Yong Yang[1], Yun-Feng Xiao[2], Chun-Hua Dong[1], Zheng-Fu Han[1*] & Guang-Can Guo[1]

[1] *Key Lab of Quantum Information of CAS, University of Science and Technology of China, Anhui, Hefei, 230026, P. R. China*

[2] *State Key Laboratory for Artificial Microstructure and Mesoscopic Physics, Peking University Beijing 100083, P. R. China*

*Email: zfhan@ustc.edu.cn

Introduction:

The boundary element method (BEM) could efficiently solve optical modes in deformed microcavity[1], but limited to calculate quality (Q) factors with *Q=-Re(kR)/2Im(kR)* for high Q modes[2]. Here, we introduced the Poyting Vector method to accurately calculate the Q factor of ultra-high Q whispering gallery modes.

Method:

Take the circular cavity as an example, one analytically solved high Q mode is: $TM_{56,1}$ with $kR = 42.665 - 7.307*10^{-7}i$, Q=2.92*10^7. We can not calculate the Q by BEM in our personal computer with 2G memory. However, we found that the field distribution by BEM is very similar to the exact mode field. In order to compare them, we introduced the normalized field overlap integral:

$$F = \left|\int d\Omega \psi_A \psi_B\right| / \left(\sqrt{\int d\Omega \psi_A \psi_A}\sqrt{\int d\Omega \psi_B \psi_B}\right),$$

where the ψ_A is analytical result and ψ_B is numerical result by BEM, and we can defined the error of field distribution as E=1-F, which is presented in left figure. For even N=100, the differences is smaller than 0.001.

The quality factor of a resonator could be derived as $Q = -\omega \frac{I(t)}{dI(t)/dt}$. The energy of the mode $I = \iint w d\Omega$ could be obtained by integrating the energy density $\overline{w} = \varepsilon |E|^2/2$. And the energy loss is $\frac{dI(t)}{dt} = \oint_S \vec{p} \bullet \vec{v} dl$, could be obtained by integrating the Poyting Vector $\vec{p} = \vec{E} \times \vec{H}$ on the boundary. Finally, we got

$$Q = k^2 \frac{\iint_\Omega n^2 |E|^2 d\Omega}{\oint_S \mathrm{Im}(E^* \partial_v E) dl}$$ for TM modes, and

$$Q = k^2 \frac{\iint_\Omega |H|^2 d\Omega}{\oint_S \mathrm{Im}(H^* \partial_v H) dl}$$ for TE modes.

where, $\partial_v = \vec{v} \bullet \nabla$ is the normal derivative on the boundary.

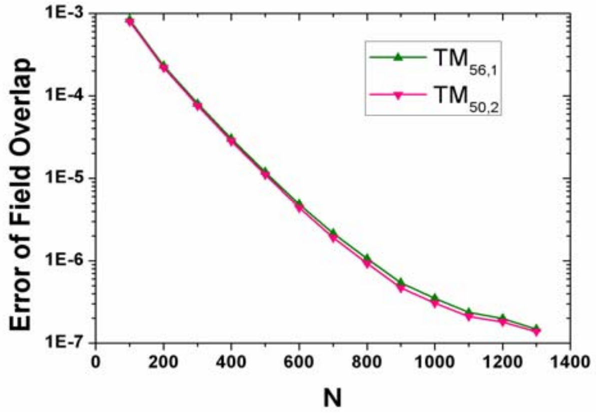
Result:

We calculated the Q factor of WGMs in circle and quadrupole cavity, and compared with the analytical and Finite Element Method (FEM) respectively. In conclusion, we can accurately calculate the high Q factor of whispering gallery modes in microcavities.

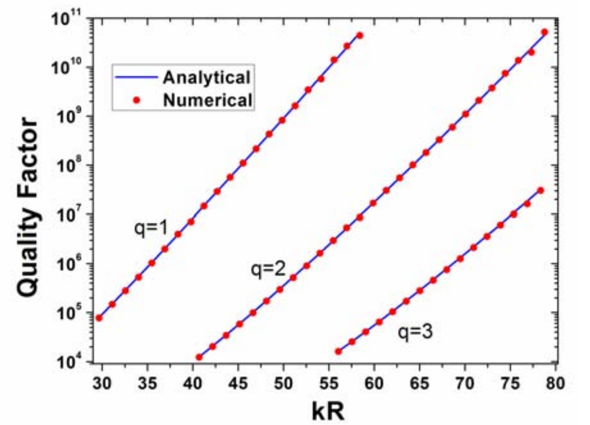
Reference:

[1] J. Wiersig, *J. Opt. A: Pure Appl. Opt.* **5**, 53-60 (2003)

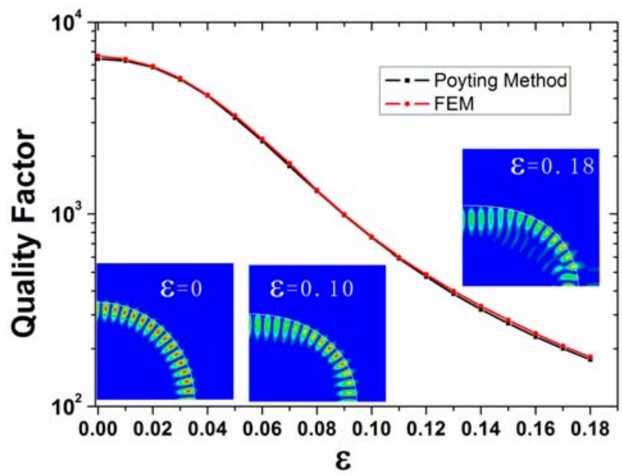
[2] Arnd Bäcker et. al. *Phys. Rev. A* **79**, 063804 (2009)



The error of field overlap E=1-F v.s. the boundary elements number on quarter of circle N.



Compare the calculation of Q factor of TM mode of circle cavity with the analytical result, with N=600.



The *Q* factor of TM WGMs in Quadrupole cavity calculated by the numerical Poyting Vector method compares to the Finite Element Method.