第一章 辐射场的量子理论

1.1 单模光场量子化

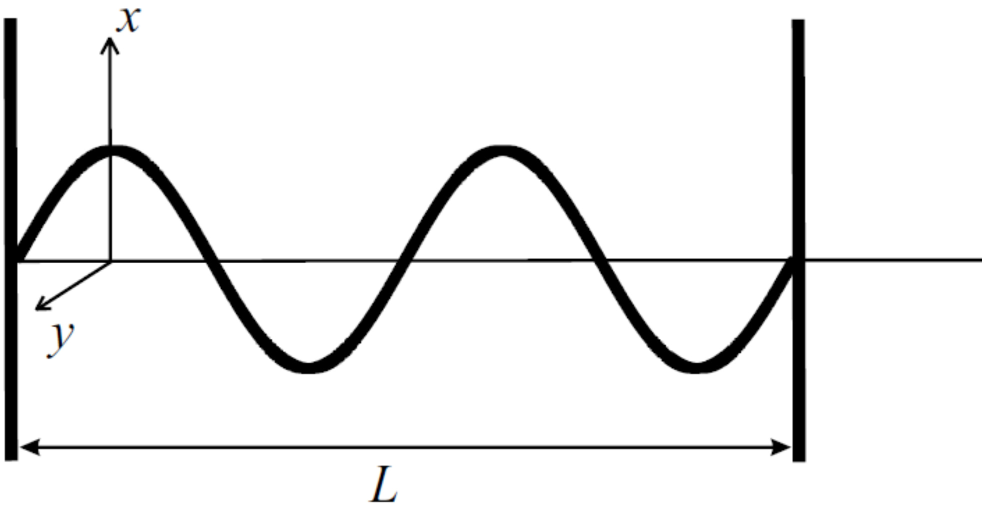


图 1.1 一维（F-P）光腔模型

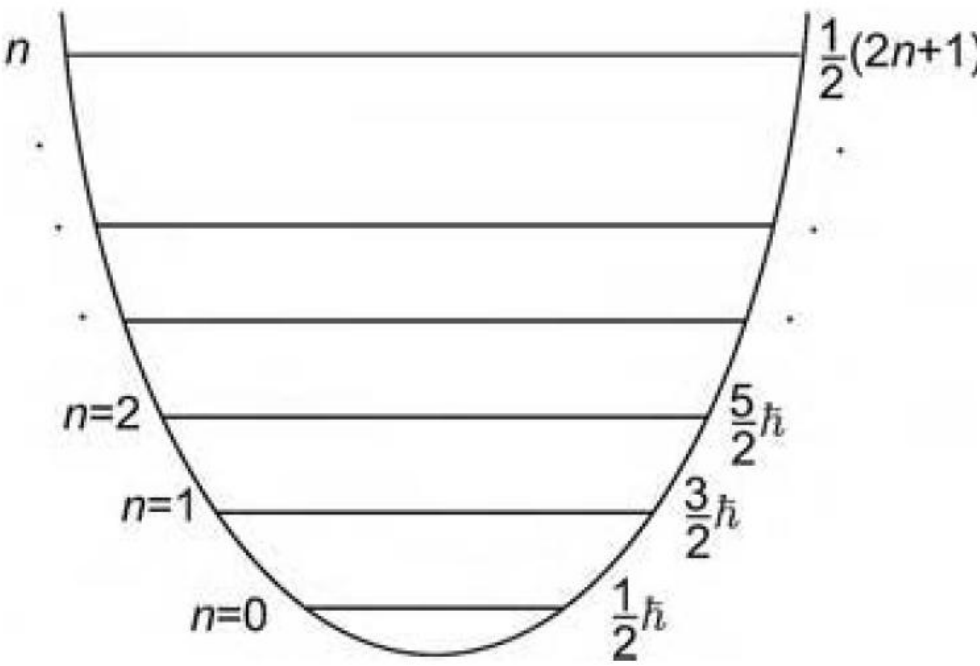
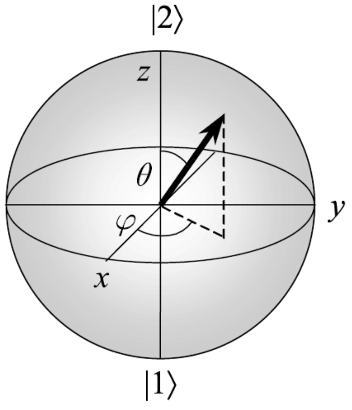


图 1.2 简谐振子能级图

Bloch Sphere

The Bloch representation was originally developed by Felix Bloch in 1946 to model NMR phenomena, and was first adapted to two-level atoms by Feynman et al. in 1957. (R. P. Feynman, et al. J. Appl. Phys. **28**, 49 (1957).)



Bloch sphere:

The direction of the Bloch vector can be specified either in Cartesian coordinates (x, y, z) or spherical polar coordinates (r, θ, ϕ), with

$$x = r \sin\theta \cos\phi,$$

$$y = r \sin\theta \sin\phi,$$

$$z = r \cos\theta,$$

Vector: **a**={x,y,z}; |**a**|=r

The requirement that the vector has unit length is satisfied when

$$r^2 = (x^2 + y^2 + z^2) = 1$$

Two-level system:

An arbitrary superposition state of a two-level system will have a wave function of the form given by eqn 9.2, namely:

$$|\psi\rangle = c_1|1\rangle + c_2|2\rangle$$

The normalization condition on the wave function requires that:

$$|c_1|^2 + |c_2|^2 = 1$$

$$|\psi\rangle = |1\rangle \rightarrow (0, 0, -1),\ \theta = \pi$$

$$|\psi\rangle = |2\rangle \rightarrow (0, 0, 1),\ \theta = 0$$

An arbitrary state is given in

Cartesian coordinates as:

$$x = 2\mathrm{Re}(c_1c_2),$$

$$y = 2\mathrm{Im}(c_1c_2),$$

$$z = |c_2|^2 - |c_1|^2.$$

In polar coordinates this simplifies to:

$$c_1 = \sin(\theta/2),$$

$$c_2 = e^{i\phi}\cos(\theta/2)$$

$$|\psi\rangle = \sin(\theta/2)|1\rangle + e^{i\phi}\cos(\theta/2)|2\rangle$$

Density matrix:

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1+z & x+iy \\ x-iy & 1-z \end{pmatrix} = \frac{1}{2}(I + \vec{a}\cdot\vec{\sigma})$$

Pauli matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The eigenvalues:

$$g_1 = \frac{1}{2}\left[1 + \sqrt{x^2 + y^2 + z^2}\right] = \frac{1}{2}[1 + |a|]$$

$$g_2 = \frac{1}{2}\left[1 + \sqrt{x^2 + y^2 + z^2}\right] = \frac{1}{2}[1 - |a|]$$

Pure state:

$\rho=|\psi\rangle\langle\psi|$**:** $\mathrm{tr}(\rho^2)= (1+|\mathbf{a}|^2)/2=1 \rightarrow |\mathbf{a}|=r=1$; on the surface of the sphere.

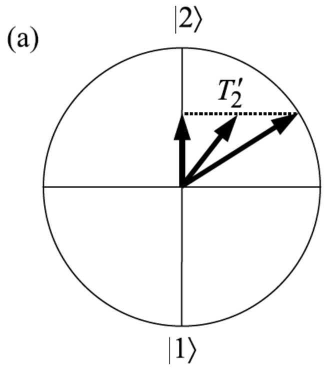
Mixed state:

$\mathrm{tr}(\rho^2)= (1+|\mathbf{a}|^2)/2<1 \rightarrow |\mathbf{a}|<1$; in the sphere.

Maximally mixed state:

$|\mathbf{a}|=0$, $\mathrm{tr}(\rho^2)=1/2$, at the center of the sphere.

Pure dephasing:



T_1 Decay:

(b)