

Cold hybrid electrical-optical ion trap

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Advances in research in fields such as quantum information and quantum chemistry require subtle methods for trapping particles, including ions, neutral atoms, and molecules. Here, we propose a hybrid ion-trapping method that combines a Paul trap with optical tweezers. This trap incorporates both the deep-potential characteristic of the Paul trap and the micromotion-free nature of the optical dipole trap. By modulating the optical dipole trap synchronously with the radio-frequency voltage of the Paul trap, the alternating electrical force in the trap center is fully counteracted, and the micromotion temperature of a cold trapped ion can reach the nanokelvin scale, while the trap depth exceeds 300 K. These features will allow cold collisions between an ion and an atom in the *s*-wave regime and stably trap the produced molecular ion in the cold hybrid system. This provides a unique platform for probing the interactions between the ions and the surrounding neutral particles and enables the investigation of new reaction pathways and reaction products in the cold regime.

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I. INTRODUCTION

At temperatures near absolute zero, cold chemistry can reveal intricate quantum processes within reaction mechanisms, and this has attracted significant attention across both chemical and physical disciplines [1–12]. In the last decade, cold hybrid ion-atom systems have emerged as a new platform for fundamental research focused on controlling cold collisions and quantum chemistry between ions and atoms [13–19]. Binding a neutral atom and an ion, and then stably trapping the produced molecular ion, are prerequisites for exploring the quantum features of such molecular ions. To investigate collision physics in the quantum regime of cold hybrid ion-atom systems, reaching the single partial wave limit (i.e., the *s*-wave limit) is an important step [14]. Although a trapped ion can be cooled close to the quantum collision regime using buffer atomic gases with a large ion-atom mass ratio [20], micromotion in the Paul trap (PT) heats the system away from a stable bound state [21–23], hindering progress in this area. Pioneering experiments using optical dipole traps (ODTs)

to capture ions have been demonstrated [24–27], offering a potential solution to the micromotion heating problem in cold hybrid ion-atom systems; however, because the dipole potential is shallow, it is quite challenging to keep the ion during the reaction and stably trap the produced molecular ion.

In this work, we propose a hybrid electrical-optical ion trap that combines the advantages of the deep-potential feature of a PT and the micromotion-free feature of an ODT. The key idea is to counteract the alternating electrical force around the center of a PT that is responsible for the micromotion problem. This is achieved by introducing ODTs that are modulated synchronously with the radio-frequency (rf) voltage of the PT. This hybrid electrical-optical trap corresponds to a deep PT with a micromotion-free dip at the center, and this potential structure provides an ideal platform for studying low-energy physics such as cold collisions and cold chemistry. Since the center dip provides a micromotion-free area for the trapped ion to interact with a neutral atom, the resulting molecular ion can be stably confined within the deep background potential profile provided by the PT. To analyze the performance of the trap, we consider counteracting 99% of the alternating force, which is experimentally reasonable. The intrinsic micromotion (iMM) energy can be suppressed by 4

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orders of magnitude. Using current voltage-compensation techniques in trapped ion systems [28], the dc stray field can be suppressed below 1 V/m, leading to an ultimate cold-ion excess micromotion (eMM) temperature in the hybrid trap below 1 nK. Under such cold-ion temperatures, many ion-atom *s*-wave scattering process can be investigated. Moreover, by simulating a cold collision process between a Yb^+ ion and a Rb atom, we find that the hybrid trap effectively solves the micromotion heating problem that dissociates the ion-atom bound state during cold collisions. The proposed hybrid trap offers a simple yet powerful platform to study ion-atom collisions in the quantum regime, paving the way for exploring previously inaccessible reactions and enabling manipulation of the resulting molecular ion in cold hybrid systems.

II. SCHEME AND RESULTS

Our scheme employs modulating optical tweezers to counteract the alternating force caused by the rf electrical field. If we consider a hybrid trap consisting of a PT and N modulating ODTs, the total potential of the trap is

$$U(\mathbf{r}, t) = U_E(\mathbf{r}) + \tilde{U}_E(\mathbf{r}) \cos(\Omega t) + \sum_i^N U_{D,i}(\mathbf{r}) [1 + \eta_i \cos(\Omega t)], \quad (1)$$

where $U_E(\mathbf{r})$ and $\tilde{U}_E(\mathbf{r})$ are the potentials generated by the dc and rf fields of the PT, respectively, $U_{D,i}(\mathbf{r})$ is the potential of the i th ODT, and η_i is the modulation depth of the i th ODT, with $|\eta_i| \leq 1$. When the ion is cooled to a low temperature (e.g., via Doppler cooling to several millikelvin), its motional range becomes significantly smaller than the characteristic size of the PT and the ODT. Assuming the centers of all traps overlap each other for simplicity, the potentials around the trap center can be expanded in harmonic form as

$$U_E(\mathbf{r}) = k_0^x x^2 + k_0^y y^2 + k_0^z z^2 + C_E, \quad (2)$$

$$\tilde{U}_E(\mathbf{r}) = k_1^x x^2 + k_1^y y^2 + k_1^z z^2 + \tilde{C}_E, \quad (3)$$

$$U_{D,i}(\mathbf{r}) = K_i^x x^2 + K_i^y y^2 + K_i^z z^2 + C_{D,i}, \quad (4)$$

where C_E , $\tilde{C}_E$, and $C_{D,i}$ are constants. The potentials $U_E(\mathbf{r})$ and $\tilde{U}_E(\mathbf{r})$, generated by electric fields, must satisfy the Laplace equation $\Delta U(\mathbf{r}) = 0$, which leads to the

constraints on the geometric factors

$$k_0^x + k_0^y + k_0^z = 0, \quad (5)$$

$$k_1^x + k_1^y + k_1^z = 0, \quad (6)$$

while $U_{D,i}(\mathbf{r})$ is not restricted by this condition. The alternating force in the hybrid trap is $\tilde{\mathbf{F}}(\mathbf{r}) = \nabla (\tilde{U}_E + \sum \eta_i U_{D,i})$, so eliminating this force leads to the condition

$$k_1^\gamma + \sum_i^N \eta_i K_i^\gamma = 0 \quad (7)$$

for all $\gamma \in \{x, y, z\}$, which can be achieved by setting the modulation depths and geometric shapes of the ODTs.

Supposing we use astigmatic Gaussian spots to generate the required dipole traps. The intensity profile of a focused astigmatic Gaussian beam along axis c in the coordinate system $(\hat{a}, \hat{b}, \hat{c})$ can be written as

$$I(\mathbf{r}) = I_0 \frac{w_{0a}}{w_a(c)} \frac{w_{0b}}{w_b(c)} \exp \left\{ -2 \left[\frac{a^2}{w_a(c)^2} + \frac{b^2}{w_b(c)^2} \right] \right\}, \quad (8)$$

where $w_{a/b}(c) = w_{0,a/b} \sqrt{1 + (c/z_{Ra/b})^2}$ are the beam waists for the a and b axes, i.e., the short and long axes, respectively; $z_{Ra/b} = \pi w_{0,a/b}^2 / \lambda$ are the Rayleigh ranges; and λ is the wavelength. As the potential of the ODT is proportional its intensity, $U_{D,i}(\mathbf{r}) \propto I_{D,i}(\mathbf{r})$, expanding the intensity near the zero point gives the potential coefficients

$$K_i^a = \frac{2U_{0i}}{w_{0,ai}^2}, \quad K_i^b = \frac{2U_{0i}}{w_{0,bi}^2}, \quad (9)$$

$$K_i^c = \frac{U_{0i}\lambda^2}{2\pi^2} \left(\frac{1}{w_{0,ai}^4} + \frac{1}{w_{0,bi}^4} \right),$$

where $U_{0i} = \pi c^2 \Gamma / 2\omega_0^3 (2/\Delta_2 + 1/\Delta_1) I_{0i}$ [29]. Finding an appropriate set of $\{\eta_i U_{0i}, w_{0,ai}, w_{0,bi}\}$ to satisfy Eqs. (7) and (9) will eliminate the alternation force at the trap center.

Figures 1(c) and 1(d) show two feasible configurations to demonstrate the scheme in a symmetric linear PT with four rods or blades [30,31]. For simplicity, we take Fig. 1(c) as an example, where two optical tweezers with focused astigmatic Gaussian spots are applied to the trapped ion with similar beam waists, i.e., $w_{0,a1} = w_{0,a2} = w_a$, $w_{0,b1} = w_{0,b2} = w_b$, $U_{01} = U_{02} = U_0$, and the long axes of two ODTs are perpendicular to each other, i.e., $\hat{a}_1 \perp \hat{a}_2 \parallel \hat{y}$ and $\hat{b}_1 \perp \hat{b}_2 \parallel \hat{x}$. The symmetric linear PT condition requires $k_1^x = -k_1^y = k_1$, $k_1^z = 0$. Suppose $\eta_1 = -\eta_2 = 1$, indicating that the modulations between the two

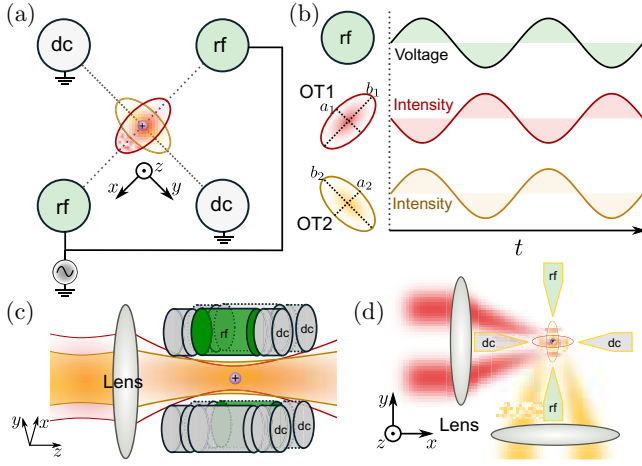


FIG. 1. Scheme of a hybrid electrical-optical ion trap. (a) The trap consists of a Paul trap (e.g., a four-rod trap or a blade trap) and two modulating optical dipole traps (ODTs) with differently oriented astigmatic spots. (b) Phase relationship between the rf voltage of the Paul trap and the optical intensities of the two ODTs. (c),(d) Two feasible realizations of the hybrid trap.

ODTs have a π phase difference [as shown in Fig. 1(b)]. The configurations of the ODTs can then be given by

$$\begin{aligned} k_1 &= k_a - k_b, \\ K_1^x &= K_2^y = k_a, \\ K_1^y &= K_2^x = k_b, \end{aligned} \quad (10)$$

where $k_{a,b} = 2U_0/w_{a,b}^2$.

The pseudopotential of the hybrid trap can be decomposed into a direct potential (DP) and an alternating potential (AP),

$$\Psi(\mathbf{r}) = \Phi_0(\mathbf{r}) + \frac{1}{4m\Omega^2} \|\nabla \Phi_1(\mathbf{r})\|^2, \quad (11)$$

where $\Phi_0(\mathbf{r}) = U_E(\mathbf{r}) + \sum_i^N U_{D,i}(\mathbf{r})$ and $\Phi_1(\mathbf{r}) = \tilde{U}_E(\mathbf{r}) + \sum_i^N \eta_i U_{D,i}(\mathbf{r})$ are the direct and alternating potentials, respectively. For a symmetric linear PT [30], $k_0^x = k_0^y = -2k_0^z = -2\kappa QV_{dc}/Z_0^2$, $k_1^x = -k_1^y = QV_{rf}/2R^2$, and $k_1^z = 0$, where κ is a geometric factor of the trap, Q is the charge of the ion, V_{dc} and V_{rf} are the dc and rf voltages applied to the trap, respectively, R is the perpendicular distance from the trap axis to the trap electrodes, and Z_0 is the distance from the trap center to the endcap electrodes. The pseudopotential can be calculated by substituting Eqs. (8)–(10) into Eq. (11). When the alternating force caused by the rf electrical field is completely compensated by the modulating ODTs, which we call the zero alternating trapping condition (ZATC), we can derive $U_0 = QV_{rf}w_a^2w_b^2/4R^2(w_a^2 - w_b^2)$. To make the condition realistic, a low rf voltage in the PT and small ODT spots should be used to minimize

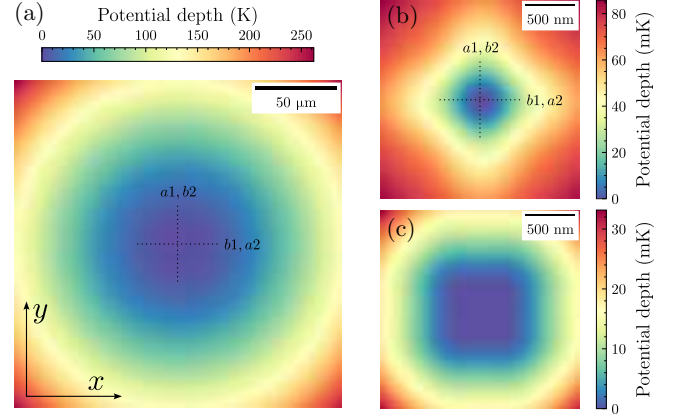


FIG. 2. Pseudopotential of a hybrid trap for $^{171}\text{Yb}^+$. (a) Total pseudopotential of the trap, indicating a deep potential. (b),(c) Constant and alternating parts of the pseudopotential near the trap center, respectively. The alternating pseudopotential is flat near the center, which indicates that the alternating force is zero. The simulation conditions are $\Omega = 2\pi \times 2$ MHz, $V_{rf} = 10$ V, $\kappa = 0.6$, $V_{dc} = 1$ V, $R = 0.5$ mm, $Z_0 = 1$ mm, $\omega_x = \omega_y = 2\pi \times 180$ kHz, and $\omega_z = 2\pi \times 131$ kHz for the Paul trap, laser wavelength: 554.6 nm, $w_a = w_b/3 = 478$ nm can be generated through an objective with a numerical aperture (NA) of 0.6, and $U_0/h = -622$ MHz for the optical dipole traps.

the optical power required for the ODTs. Consequently, a lower rf frequency Ω is preferred to generate a deeper pseudopotential under the low rf voltage. A numerical simulation result is shown in Fig. 2. Over a large range, it acts as a PT with a deep potential [Fig. 2(a)]. At the center of the trap, it acts as a tight ODT [Fig. 2(b)]. The alternating pseudopotential in Fig. 2(c) is flat, indicating that the alternating force is zero. From these simulation results, the required laser power is 0.7 W at 554.6 nm, assuming an objective with numerical aperture $\text{NA} = 0.6$ is used to generate the required ODTs.

Next, we will discuss the cold trapped ion's temperature limit in the hybrid trap. We assume that the ion oscillates near $\mathbf{u}_0$ in the hybrid trap under a stray electrical field $\mathbf{E}$ and has been cooled to a low temperature $|\mathbf{u} - \mathbf{u}_0| \ll w_a$. The motion equation can then be approximated as a Mathieu equation near $\mathbf{u}_0$, as follows:

$$\ddot{u}_l + [a_l(\mathbf{u}_0) + 2q_l(\mathbf{u}_0) \cos(\Omega t)] \frac{\Omega^2}{4} u_l = \frac{Q \cdot E_l}{m}, \quad (12)$$

where $a_l(\mathbf{u}) = 4\hat{e}_l \cdot \nabla \Phi_0(\mathbf{u})/m\Omega^2 u_l$ and $q_l(\mathbf{u}) = 2\hat{e}_l \cdot \nabla \Phi_1(\mathbf{u})/m\Omega^2 u_l$. The equilibrium position $\mathbf{u}_0$ can be numerically solved by iterating

$$u_{0,l} = \frac{4Q \cdot E_l}{m\Omega^2 [a_l(\mathbf{u}) + q_l^2(\mathbf{u})/2]}. \quad (13)$$

The final temperature caused by excess micromotion is

$$T_{\text{eMM}} = \frac{m\Omega^2}{16k_B} \sum_l q_l^2(\mathbf{u}_0) u_{0,l}^2. \quad (14)$$

Using the above numerical method, we can study the ion's eMM temperature under a large electrical field. Figure 3(a) shows a comparison of the trapped ion's eMM temperature in three different traps under a stray electrical field along the (1, 1, 1) direction. The stray field will shift the trap center of the PT part of the hybrid trap, while the ODT remains unaffected. The total pseudopotential is thus a double well under a small stray field. The ion stays in the tight ODT under a small stray field, leading to a low eMM temperature even without modulating the ODTs. As the stray field is increased, the ion will eventually be pushed from the ODT into the PT, and the hybrid trap's eMM temperature approaches that of the PT when the stray field exceeds 5 V/m.

To gain further insights into the relationships between the ion eMM temperature and experimental parameters [e.g., the misalignment between the PT and the ODT δx , stray electrical field E , residual alternating force (RAF) parameter $\Delta q = 4(k_1 + k_a - k_b)/m\Omega^2$], we studied one-dimensional motion for simplicity. Considering the condition of a small misalignment distance $\delta x \ll w_0$, the approximated ion eMM temperature can be derived as (see Appendix B)

$$T_{\text{eMM}} \approx \frac{m\Omega^2}{16k_B} \left(\Delta q + \frac{(\delta x - x_0)^2}{x_b^2} \right)^2 x_0^2, \quad (15)$$

where $x_0 \approx -Q \cdot E / (k_0 + 2k_a + 2k_b)$ is the ion's position offset under the stray electrical field, and $x_b^2 = m\Omega^2 / 8(k_b/w_b^2 - k_a/w_a^2)$.

Figure 3(b) demonstrates that the ion eMM temperature can reach the 100 nK level, even with a misalignment of approximately 50 nm and a RAF ratio of $\Delta q/q_0 = 5\%$ (where $q_0 = 4k_1/m\Omega^2$ is the q factor of the bare Paul trap) under a stray field of 1 V/m along the x axis. Single-particle localization microscopy has demonstrated sub-10-nm positioning accuracy in ion imaging systems [32–34]. By implementing a dual-objective configuration on a high-optical-access ion trap [35]—where opposing objectives are used, with one generating optical tweezers and the other simultaneously imaging both the ions and the tweezer spots—we can achieve a colocalization accuracy of less than 50 nm. This approach not only significantly reduces alignment complexity but also relaxes the trap alignment requirements in our scheme. Additionally, the ion's iMM temperature and energy, which scale as Δq^2 (see Appendix B), can be reduced by 4 orders of magnitude with a 1% RAF ratio. The iMM temperature for the motional ground state of the bare Paul trap, calculated

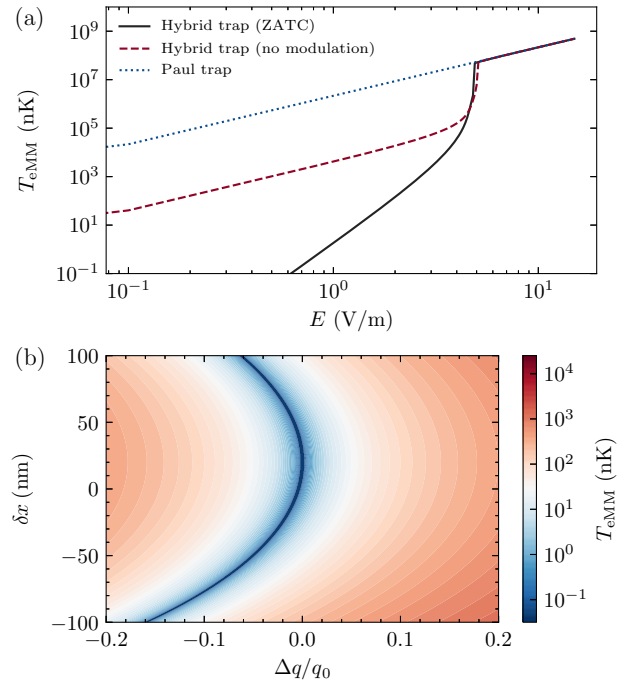


FIG. 3. Ion motion simulation for trapping temperature in a stray electrical field (SEF). (a) Excess micromotion (eMM) temperature of a trapped ion under different SEFs. The hybrid trap provides a lower temperature than a PT under the same SEF. When the hybrid trap works at the zero alternating trapping condition (ZATC), the ion's eMM temperature reaches the nK level under an SEF of 1 V/m. The direction of the SEF for the simulation is (1, 1, 1). (b) Ion's eMM temperature simulation under experimental imperfections, in which δx is the misalignment between the Paul trap and the ODT and Δq is the uncompensated alternating force factor. Intrinsic micromotion (iMM) is also suppressed, as the iMM temperature scales as Δq^2 . The figure is calculated under an SEF of 1 V/m along the x axis. The hybrid trap and Paul trap parameters are the same as those in Fig. 2.

using the parameters in Fig. 2, is 2.7 μK . In contrast, for the hybrid trap with a RAF ratio of 1%, it is significantly reduced to 58.3 pK.

Finally, we investigate the collision process between an ion and an atom in the hybrid trap. For this study, the laser wavelength of the ODT must be carefully selected based on the specific ion-atom pair. The alternating ODT for the trapped ion should not contribute to the trapping potential for the atom. This requires the ODT wavelength to be chosen such that it does not create a trapping potential for the atom, ensuring that the ac Stark shift of the atomic ground state is minimized. If this condition is not met, the alternating ODT will drive the atom into micromotion as it approaches the ion, resulting in unwanted dynamics. The wavelengths listed in Table I are carefully chosen to satisfy this requirement while balancing the trade-off between ODT power and detuning from atomic and ionic resonances. Table I lists several laser wavelengths for various

TABLE I. ODT laser wavelengths for hybrid ion traps in different ion-atom systems. Here, E_k is a characteristic energy scale of the ion-atom interaction [14], which is associated with the s -wave scattering limit. The wavelengths are chosen to ensure that the alternating ODT operates at a magic wavelength for the atom, minimizing the ac Stark shift of the atomic ground state. This prevents the ODT from driving the atom into micromotion as it approaches the ion.

Ion	Atom	λ_{ODT} (nm)	P (W)	E_k/k_B (nK)
Yb^+	Rb	420.6	0.24	44.7
Yb^+	Rb	422.3	0.25	44.7
Yb^+	Rb	787.4	1.48	44.7
Yb^+	Yb	554.63	0.70	44.7
Ca^+	Na	589.46	0.48	1370.7
Ca^+	Li	670.97	1.09	10601.5
Sr^+	Rb	787.41	0.94	77.8

ion-atom combinations, along with the corresponding laser power under the same hybrid trap configuration as Fig. 2.

We take Rb and Yb^+ in this investigation, following the method presented in Ref. [21], with details provided in Appendix C. When an ion moves in a PT without any disturbance, the rf field does no net work on the ion over a single rf period; however, in a cold hybrid ion-atom system, the polarized atom will exert an attraction force on the ion via the attraction potential $V(r) = -C_4/r^4$, thus disturbing the energy-conserving cycle. As a result, the ion-atom system gains net energy from the rf field, increasing the temperature of the system. A characteristic length scale $R = \sqrt[3]{2C_4/m_i\omega^2}$ is used, as defined when the interaction potential $V(r)$ is equal in magnitude to the trap harmonic potential, where $\omega = \Omega\sqrt{a + q^2/2}/2$ is the secular frequency of the ion near the trap center. In a PT [Fig. 4(b)], the ion gains significant energy from the rf field when the atom approaches, and after one collision, the atom escapes. In the hybrid trap [Fig. 4(a)], since the alternating force induced by the rf field near the trap center is compensated, the ion-atom system nearly undergoes an energy-conserving process and forms a weakly bound ion-atom pair, maintaining a low temperature. In the hybrid trap, the corresponding characteristic energy scale is no longer restricted by the rf heating but determined by the atomic gas temperature, enabling access to the regimes of the s -wave limit or quantized ion motion in the cold hybrid system. The work done by the RAF during a collision depends on the phase ϕ at the time of the close-range collision, which can be written as (see Appendix D)

$$W = \frac{1}{2}\mu\Omega^2 R^2 \sin\phi \cdot \left(\Delta q\eta_1 + \frac{R^4}{r_0^2 x_b^2} \eta_2 \right),$$

where $\mu = m_i m_a / (m_i + m_a)$ is the reduced mass, $\eta_1 = 2/3 \int_0^{\Omega t_1} (A - B\tau^{1/3}) \cdot \tau^{-2/3} \sin\tau \cdot d\tau$ and $\eta_2 = 2/3 \int_0^{\Omega t_1} (A - B\tau^{1/3})^3 \cdot \tau^{-2/3} \sin\tau \cdot d\tau$ are integrals with parameters

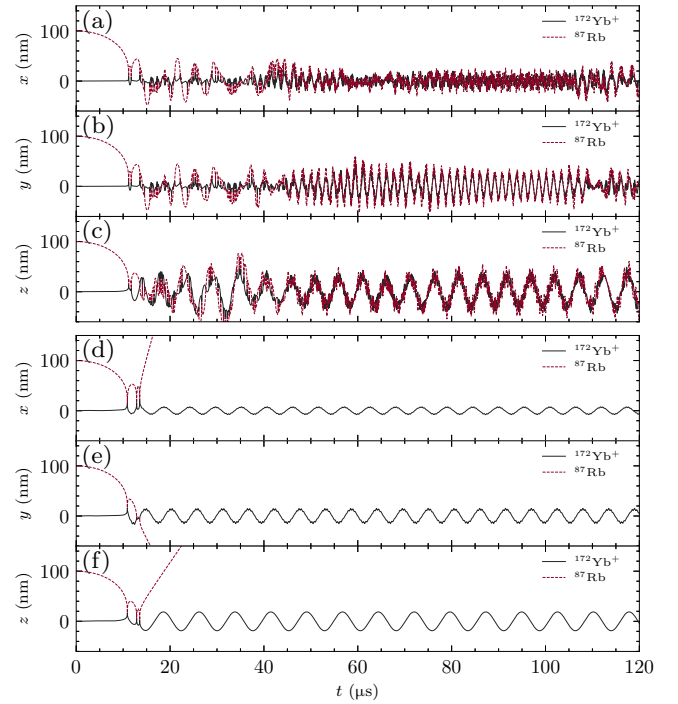


FIG. 4. (a)–(c) Numerical simulations of an ion-atom (Rb and Yb^+) collision in a hybrid trap and (d)–(f) a bare Paul trap. For comparison, the hybrid trap uses the same electric field configuration as in Fig. 2. A Rb approaches an Yb^+ ion initially located in the trap center. In the Paul trap, the ion is heated by the rf field and quickly escapes, whereas in the hybrid trap, the ion and atom can remain weakly bound. The harmonic frequencies of two traps are $\omega_{\text{Hybrid}} = 2\pi \times 0.84$ MHz and $\omega_{\text{Paul}} = 2\pi \times 0.18$ MHz, leading to the corresponding characteristic lengths $R_{\text{Hybrid}} = 33.4$ nm and $R_{\text{Paul}} = 55.9$ nm. The simulation method is adapted from Ref. [21].

$A = r_c r_0 / R^2$, $B = \mu r_0^2 / m_i R^2$, the ion's displacement from the trap center is r_c for the collision position, and the characteristic length $r_0 = \sqrt[3]{18C/\mu\Omega^2}$. Figure 4(a) shows that r_c is distributed in the range 0–20 nm for multi hard-core collisions, indicating that the maximum work done by the RAF for one collision is of the order of tens of nK k_B .

III. CONCLUSION AND DISCUSSION

In summary, we have proposed a hybrid trap consisting of a PT and modulated ODTs that takes advantage of the deep-potential feature of the PT while eliminating the micromotion problem in the cold hybrid ion-atom system. This trap can suppress the micromotion temperature to the nanokelvin level, offering a new route to reaching the quantum collision regime in ion-atom mixtures. Our proposed schemes are quite feasible, as an ion trap system with high optical accessibility has already been demonstrated in Ref. [35]. Compared with previous optical trapping methods for ions where the PT is switched off [24–27],

our scheme can stably trap both elementary and molecular ions throughout the cold ion-atom collision process due to the always-on PT. This provides an ideal platform for studying cold ion-atom collisions and reactions, as well as investigating the properties of the produced molecular ions.

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J.-M.C. and S.-J.S. contributed equally to this work.

DATA AVAILABILITY

The data that support the findings of this article are openly available [36].

APPENDIX A: ION DYNAMICS IN THE HYBRID TRAP POTENTIAL

The electrical potential of a classical Paul trap is

$$U_{\text{Paul}}(\mathbf{r}, t) = U_E(\mathbf{r}) + \tilde{U}_E(\mathbf{r}) \cos(\Omega t), \quad (\text{A1})$$

where $U_E(\mathbf{r})$ and $\tilde{U}_E(\mathbf{r})$ are the potentials generated by the dc and rf fields, respectively. For a symmetric linear Paul trap with four rods or blades, the electrical potentials can be expressed as [30]

$$U_E(\mathbf{r}) = k_0 z^2 - \frac{1}{2} k_0 (x^2 + y^2), \quad (\text{A2})$$

$$\tilde{U}_E(\mathbf{r}) = k_1 (x^2 - y^2) + V_{\text{rf}}/2, \quad (\text{A3})$$

where $k_0 = \kappa Q V_{\text{dc}}/Z_0^2$ and $k_1 = Q V_{\text{rf}}/2R^2$, in which κ is a geometric factor of the trap; Q is the charge of the ion; V_{dc} and V_{rf} are the dc and rf voltages applied to the trap, respectively; R is the perpendicular distance from the trap axis to the trap electrodes; and Z_0 is the distance from the trap center to the end-cap electrodes. The equations of motion for a single ion of mass m in the above potential are given by

the Mathieu equations

$$\ddot{u}_l + [a_{0,l} + 2q_{0,l} \cos(\Omega t)] \frac{\Omega^2}{4} u_l = 0, \quad (\text{A4})$$

where $\mathbf{u} = u_x \hat{x} + u_y \hat{y} + u_z \hat{z}$ is the position of the ion and

$$a_{0,x} = a_{0,y} = -\frac{1}{2} a_{0,z} = a_0 = \frac{4k_0}{m\Omega^2}, \quad (\text{A5})$$

$$q_{0,x} = -q_{0,y} = q_0 = \frac{4k_1}{m\Omega^2}, \quad (\text{A6})$$

$$q_{0,z} = 0. \quad (\text{A7})$$

The potential generated by the hybrid trap is

$$U(\mathbf{r}, t) = U_E(\mathbf{r}) + \tilde{U}_E(\mathbf{r}) \cos(\Omega t) + \sum_{i=1} U_{D,i}(\mathbf{r}) [1 + \eta_i \cos(\Omega t)] \quad (\text{A8})$$

$$= \Phi_0(\mathbf{r}) + \Phi_1(\mathbf{r}) \cos(\Omega t), \quad (\text{A9})$$

where $U_{D,i}(\mathbf{r})$ is the potential of the i th ODT, η_i is the modulation depth of the ODT, and $\Phi_0(\mathbf{r}) = U_E(\mathbf{r}) + \sum_i U_{D,i}(\mathbf{r})$ and $\Phi_1(\mathbf{r}) = \tilde{U}_E(\mathbf{r}) + \sum_i \eta_i U_{D,i}(\mathbf{r})$ are the direct and alternating potentials, respectively. If we use the two-dipole trap scheme in the main text and the zero alternating trapping condition (ZATC), the modulation parameters will be $\eta_1 = -\eta_2 = 1$. The potentials of the ODTs are [29]

$$U_D(\mathbf{r}) = \frac{\pi c^2 \Gamma}{2\omega_0^3} \left(\frac{2}{\Delta_2} + \frac{1}{\Delta_1} \right) I(\mathbf{r}), \quad (\text{A10})$$

where $I(\mathbf{r})$ is the intensity of the specified ODT, Γ is the damping rate of the P state, Δ_2 is the laser detuning from the $S_{1/2} \rightarrow P_{3/2}$ transition, Δ_1 is the laser detuning from the $S_{1/2} \rightarrow P_{1/2}$ transition, and ω_0 is the ODT laser frequency.

The intensity profile of a focused astigmatic Gaussian beam along axis $\hat{c}$ in coordinate $(\hat{a}, \hat{b}, \hat{c})$ can be written as

$$I(\mathbf{r}) = I_0 \frac{w_{0a}}{w_a(c)} \frac{w_{0b}}{w_b(c)} \exp \left\{ -2 \left[\frac{a^2}{w_a(c)^2} + \frac{b^2}{w_b(c)^2} \right] \right\}, \quad (\text{A11})$$

where $w_{a/b}(c) = w_{0,a/b} \sqrt{1 + (c/z_{Ra/b})^2}$ are the beam waists for the a and b axes, i.e., the short and long axes, respectively; $z_{Ra/b} = \pi w_{0,a/b}^2/\lambda$ are the Rayleigh ranges; and λ is the ODT laser wavelength. Applying the scheme with two orthogonal astigmatic Gaussian beam spots at

$z = 0$ in the main text, the potentials are

$$\begin{aligned}\Phi_0(x, y, 0) &= k_0 z^2 - \frac{1}{2} k_0 (x^2 + y^2) \\ &+ U_0 \exp(-2x^2/w_a^2 - 2y^2/w_b^2) \\ &+ U_0 \exp(-2x^2/w_b^2 - 2y^2/w_a^2),\end{aligned}\quad (\text{A12})$$

$$\begin{aligned}\Phi_1(x, y, 0) &= k_1 (x^2 - y^2) + U_0 \exp(-2x^2/w_a^2 - 2y^2/w_b^2) \\ &- U_0 \exp(-2x^2/w_b^2 - 2y^2/w_a^2),\end{aligned}\quad (\text{A13})$$

where $U_0 = \pi c^2 \Gamma(\Delta_2 + 2\Delta_1) I_0 / 2\omega_0^2 \Delta_2 \Delta_1$. Expanding $\Phi_1(x, y, 0)$ near the trap center ($x = 0, y = 0$) and setting the alternating force $\nabla \Phi_1$ to zero, we can derive the ZATC

$$U_{0,\text{ZATC}} = \frac{k_1 w_a^2 w_b^2}{2(w_a^2 - w_b^2)}.\quad (\text{A14})$$

For the other arbitrary U_0 , there will be residual alternating forces (RAFTs) near the trap center. In this case, we can define a RAF parameter $\Delta q = 4(k_1 + k_a - k_b)/m\Omega^2$ (where $k_{a,b} = 2U_0/w_{a,b}^2$; see Sec. B 2). Comparing with the

original Paul trap, we define the RAF ratio of the hybrid trap as $\beta = \Delta q/q_0$.

To study the three-dimensional dynamics of a single ion in the hybrid trap, we can perform numerical simulations by retaining the full form of the potentials. Taking the equation along the x axis as an example, the equation of motion driven by forces is

$$m\ddot{x} = -\frac{\partial U(\mathbf{r}, t)}{\partial x} = -\frac{\partial \Phi_0}{\partial x} - \frac{\partial \Phi_1}{\partial x} \cos(\Omega t),\quad (\text{A15})$$

which leads to

$$\ddot{x} + [a_x(x, y, z) + 2q_x(x, y, z) \cos(\Omega t)] \frac{\Omega^2}{4} x = 0,\quad (\text{A16})$$

where

$$\begin{aligned}a_x(x, y, z) &= \frac{4}{m\Omega^2} \frac{\partial \Phi_0}{x \partial x}, \\ q_x(x, y, z) &= \frac{2}{m\Omega^2} \frac{\partial \Phi_1}{x \partial x}\end{aligned}$$

are the Mathieu equation parameters, dependent on the ion's position. The position-dependent Mathieu equation parameters in three dimensions can be deduced as

$$\begin{aligned}a_x(x, y, z) &= \frac{4}{m\Omega^2} \left[-k_0 - U_0 g(z) \left(f_1(x, y, z) \frac{4}{w_a^2(z)} + f_2(x, y, z) \frac{4}{w_b^2(z)} \right) \right], \\ a_y(x, y, z) &= \frac{4}{m\Omega^2} \left[-k_0 - U_0 g(z) \left(f_1(x, y, z) \frac{4}{w_b^2(z)} + f_2(x, y, z) \frac{4}{w_a^2(z)} \right) \right], \\ a_z(x, y, z) &= \frac{4}{m\Omega^2} [2k_0 + U_0 g(z) (f_1 M_1 + f_2 M_2) + U_0 N(z) (f_1 + f_2)],\end{aligned}\quad (\text{A17})$$

and

$$\begin{aligned}q_x(x, y, z) &= \frac{2}{m\Omega^2} \left[2k_1 + U_0 g(z) \left(f_1(x, y, z) \frac{4}{w_a^2(z)} - f_2(x, y, z) \frac{4}{w_b^2(z)} \right) \right], \\ q_y(x, y, z) &= \frac{2}{m\Omega^2} \left[-2k_1 + U_0 g(z) \left(f_1(x, y, z) \frac{4}{w_b^2(z)} - f_2(x, y, z) \frac{4}{w_a^2(z)} \right) \right], \\ q_z(x, y, z) &= \frac{2}{m\Omega^2} [U_0 g(z) (f_1 M_1 - f_2 M_2) + U_0 N(z) (f_1 - f_2)],\end{aligned}\quad (\text{A18})$$

where

$$f_1(x, y, z) = \exp[-2x^2/w_a^2(z) - 2y^2/w_b^2(z)],\quad (\text{A19})$$

$$f_2(x, y, z) = \exp[-2x^2/w_b^2(z) - 2y^2/w_a^2(z)],\quad (\text{A20})$$

$$g(z) = \frac{1}{\sqrt{1 + (z/z_{Ra})^2} \sqrt{1 + (z/z_{Rb})^2}},\quad (\text{A21})$$

$$N(z) = -g(z) \left(\frac{1}{z_{Ra}^2 + z^2} + \frac{1}{z_{Rb}^2 + z^2} \right),\quad (\text{A22})$$

$$M_1(x, y, z) = \frac{4x^2}{w_{0,a}^2 z_{Ra}^2 \alpha_a^4(z)} + \frac{4y^2}{w_{0,b}^2 z_{Rb}^2 \alpha_b^4(z)}, \quad (\text{A23})$$

$$M_2(x, y, z) = \frac{4x^2}{w_{0,b}^2 z_{Rb}^2 \alpha_b^4(z)} + \frac{4y^2}{w_{0,a}^2 z_{Ra}^2 \alpha_a^4(z)}, \quad (\text{A24})$$

and $w_{a/b}^2(z) = w_{0,a/b}^2 \alpha_{a/b}^2(z)$, $\alpha_{a/b}(z) = \sqrt{1 + (z/z_{Ra/b})^2}$, $z_{Ra/b} = \pi w_{0,a/b}^2 / \lambda$.

In the presence of a stray electrical field $\mathbf{E} = E_x \hat{x} + E_y \hat{y} + E_z \hat{z}$, the equations of motion will be

$$\ddot{x} + [a_x(x, y, z) + 2q_x(x, y, z) \cos(\Omega t)] \frac{\Omega^2}{4} x = \frac{Q \cdot E_x}{m}, \quad (\text{A25})$$

$$\ddot{y} + [a_y(x, y, z) + 2q_y(x, y, z) \cos(\Omega t)] \frac{\Omega^2}{4} y = \frac{Q \cdot E_y}{m}, \quad (\text{A26})$$

$$\ddot{z} + [a_z(x, y, z) + 2q_z(x, y, z) \cos(\Omega t)] \frac{\Omega^2}{4} z = \frac{Q \cdot E_z}{m}. \quad (\text{A27})$$

APPENDIX B: ION MICROMOTION TEMPERATURE

1. Micromotion temperature of a trapped ion

The Mathieu equation when a dc field $\mathbf{E}$ shifts the equilibrium position of the ion is written by Berkeland *et al.* [30] as

$$\ddot{u}_l + [a_l + 2q_l \cos(\Omega t)] \frac{\Omega^2}{4} u_l = \frac{Q \cdot E_l}{m},$$

and the solution is

$$u_l(t) = (u_{0,l} + u_{1,l} \cos(\omega_l t + \varphi_l)) \left(1 + \frac{q_l}{2} \cos(\Omega t)\right),$$

where the equilibrium position

$$u_{0,l} \cong \frac{4Q \cdot E_l}{m\Omega^2(a_l + q_l^2/2)}, \quad (\text{B1})$$

and $u_{1,l}$ is the secular motion amplitude. The average kinetic energy due to motion along $\hat{u}_l$ is

$$\begin{aligned} E_{k,l} &= \frac{1}{4} m u_{1,l}^2 (\omega_l^2 + \frac{1}{8} q_l^2 \Omega^2) + \frac{m\Omega^2}{16} q_l^2 u_{0,l}^2 \\ &= \frac{1}{4} m u_{1,l}^2 \omega_l^2 + \frac{1}{32} m q_l^2 u_{1,l}^2 \Omega^2 + \frac{m\Omega^2}{16} q_l^2 u_{0,l}^2 \\ &= E_{0,l} + E_{\text{iMM},l} + E_{\text{eMM},l}, \end{aligned} \quad (\text{B2})$$

where the second and third terms represent the kinetic energy due to intrinsic micromotion (iMM) and excess

TABLE II. Numerical simulation parameters for trapping $^{171}\text{Yb}^+$. The shared trap parameters are $\Omega = 2\pi \times 2$ MHz (rf frequency), $\kappa = 0.6$, $V_{\text{dc}} = 1$ V, $V_{\text{rf}} = 10$ V, $R = 0.5$ mm, and $Z_0 = 1$ mm.

	Paul trap	Hybrid trap
a_x	-8.575×10^{-3}	
a_y	-8.575×10^{-3}	
a_z	17.15×10^{-3}	
q_x	0.286	
q_y	-0.286	
q_z	0	
$\omega_x/2\pi$	179.7 kHz	840.2 kHz
$\omega_y/2\pi$	179.7 kHz	840.2 kHz
$\omega_z/2\pi$	131.0 kHz	131.0 kHz

micromotion (eMM) along $\hat{u}_l$, respectively. The temperature caused by the eMM in all directions thus reads

$$T_{\text{eMM}} = \sum_l \frac{E_{\text{eMM},l}}{k_B} = \frac{m\Omega^2}{16k_B} \sum_l q_l^2 u_{0,l}^2. \quad (\text{B3})$$

Here, T_{eMM} represents the lowest attainable temperature caused by the excess micromotion of an ion in the trap. This can in theory be zero if we set $u_{0,l} = 0$; however, experimental techniques for adjusting the voltages on dc electrodes limit the range of achievable T_{eMM} values. The temperature caused by the iMM is

$$T_{\text{iMM}} = \sum_l \frac{E_{\text{iMM},l}}{k_B} = \frac{m\Omega^2}{32k_B} \sum_l q_l^2 u_{1,l}^2. \quad (\text{B4})$$

Even in the condition that the secular motion is cooled to its ground state (when $u_{1,l} = \sqrt{\hbar/2m\omega_l}$) and the eMM is well compensated, the ion still has a temperature limit caused by the iMM

$$T_{0,\text{iMM}} = \frac{\hbar\Omega^2}{64k_B} \sum_l \frac{q_l^2}{\omega_l}. \quad (\text{B5})$$

Using the trap parameters in Table II, the $T_{0,\text{iMM}}$ value of the Paul trap is 2.73 μK . Considering a RAF ratio of $\beta = \Delta q/q_0 = 1\%$ for a hybrid trap, which is achievable in experiments, the corresponding $T_{0,\text{iMM}}$ is 58.3 pK.

We now consider the case where the Mathieu equation parameters are in a position-dependent form: $a_l(\mathbf{u})$, $q_l(\mathbf{u})$. If the ion oscillates near $\mathbf{u}_0$ in the hybrid trap under a stray electrical field $\mathbf{E}$, and it has been cooled to a low temperature such that $|\mathbf{u} - \mathbf{u}_0| \ll w_a$, then the Mathieu equation can be approximated as

$$\ddot{u}_l + [a_l(\mathbf{u}_0) + 2q_l(\mathbf{u}_0) \cos(\Omega t)] \frac{\Omega^2}{4} u_l = \frac{Q \cdot E_l}{m}, \quad (\text{B6})$$

where the equilibrium position $\mathbf{u}_0$ can be numerically solved by iterating

$$u_{0,l}^{i+1} = \frac{4Q \cdot E_l}{m\Omega^2 [a_l(\mathbf{u}_0^i) + q_l^2(\mathbf{u}_0^i)/2]},$$

with an initial position $\mathbf{u}_0^0 = (0, 0, 0)$. Once the equilibrium position $\mathbf{u}_0$ is solved, we can obtain the temperature caused by excess micromotion

$$T_{\text{eMM}} = \frac{m\Omega^2}{16k_B} \sum_l q_l^2(\mathbf{u}_0) u_{0,l}^2. \quad (\text{B7})$$

2. Approximate expression for ion's eMM temperature

We take the Paul trap center as the origin of the coordinates, assuming full modulation and a misalignment between the Paul trap and the ODT of δx . We define $\Delta x = x - \delta x$. Then, we can approximate the Mathieu equation parameters at the position $(x, 0, 0)$ as follows:

$$\begin{aligned} a_x(x) &= \frac{4}{m\Omega^2} \left[-k_0 - U_0 \left(e^{-2\Delta x^2/w_a^2} \frac{4}{w_a^2} + e^{-2\Delta x^2/w_b^2} \frac{4}{w_b^2} \right) \right] \\ &\approx -\frac{4}{m\Omega^2} [k_0 + 2k_a + 2k_b - 4(k_a/w_a^2 + k_b/w_b^2) \Delta x^2] \\ &= a - (x - \delta x)^2/x_a^2, \end{aligned} \quad (\text{B8})$$

$$\begin{aligned} q_x(x) &= \frac{2}{m\Omega^2} \left[2k_1 + U_0 \left(e^{-2\Delta x^2/w_a^2} \frac{4}{w_a^2} - e^{-2\Delta x^2/w_b^2} \frac{4}{w_b^2} \right) \right] \\ &\approx \frac{4}{m\Omega^2} [k_1 + (k_a - k_b) + 2(k_b/w_b^2 - k_a/w_a^2) \Delta x^2] \\ &= \Delta q + (x - \delta x)^2/x_b^2, \end{aligned} \quad (\text{B9})$$

where $k_{a,b} = 2U_0/w_{a,b}^2$, $a = -4(k_0 + 2k_a + 2k_b)/m\Omega^2$, $\Delta q = 4(k_1 + k_a - k_b)/m\Omega^2$, $x_a^2 = -m\Omega^2/16(k_a/w_a^2 + k_b/w_b^2)$, and $x_b^2 = m\Omega^2/8(k_b/w_b^2 - k_a/w_a^2)$. The trap center will shift to an equilibrium position under a stray field E

$$x_0 = \frac{4Q \cdot E}{m\Omega^2 (a + \Delta q^2/2)} \approx \frac{4Q \cdot E}{m\Omega^2 a} = -\frac{Q \cdot E}{k_0 + 2k_a + 2k_b}.$$

Combined with Eq. (B3), the approximated eMM temperature expression in this situation reads

$$\begin{aligned} T_{\text{eMM}} &\approx \frac{m\Omega^2}{16k_B} q_x^2(x_0, 0, 0) x_0^2 \\ &\approx \frac{m\Omega^2}{16k_B} \left(\Delta q + \frac{(x_0 - \delta x)^2}{x_b^2} \right)^2 x_0^2, \end{aligned}$$

which clearly illustrates the roles that the imperfections Δq and δx play in restricting the lowest attainable temperature.

In this expression, we approximate the iterated equilibrium position u_0 to x_0 , which represents the equilibrium position when no imperfections are presented. If we want the expression to be more precise, we can simply substitute x_0 into the right-hand side of the iteration formula in Eq. (14), and then let u_0 be equal to the first iteration solution $x_1 = 4Q \cdot E / [m\Omega^2 (a_x(x_0, 0, 0) + q_x^2(x_0, 0, 0)/2)]$, which gives

$$T_{\text{eMM}} \approx \frac{m\Omega^2}{16k_B} \left(\Delta q + \frac{(x_1 - \delta x)^2}{x_b^2} \right)^2 x_1^2.$$

APPENDIX C: NUMERICAL SIMULATION OF ION-ATOM COLLISIONS IN THREE DIMENSIONS

In this section, a numerical simulation method used for calculating the classical low-energy three-dimensional ion-atom collision trajectory is introduced. We simulated and compared two situations: one where the collision occurs in a normal Paul trap and the other where it happens in the hybrid trap we propose. In the following content, we explain how to calculate the collision process in the hybrid trap, and the situation in the Paul trap can be similarly and easily deduced.

An ^{87}Rb atom of mass m_a approaches an $^{172}\text{Yb}^+$ ion of mass m_i (initially resting in the center of the hybrid trap) from an arbitrary position (in the following content, we take the initial position as $x = 100$ nm, $y = 100$ nm, $z = 100$ nm without loss of generality) due to the attractive force between the two, the potential of which originates from the interaction between the ion and polarized atom. This potential has the form $-C_4/r^4$, where C_4 represents the polarizability of the atom and the value of C_4 is taken from Ref. [14]. For our selected ^{87}Rb - $^{172}\text{Yb}^+$ pair, $C_4 = 160E_h a_0^4$, where E_h is the Hartree energy and a_0 is the Bohr radius. At short ranges, we add a repulsive r^{-6} term to simulate a hard-core potential, where the hard-core collision distance is of the order of nanometers, and the coefficient C_6 is taken as a fraction of C_4 as $C_6 = (10a_0)^2 \times C_4$ in our simulation. Therefore, the total ion-atom potential reads

$$V(r) = -C_4/r^4 + C_6/r^6.$$

Since we intentionally select the magic wavelength $\lambda = 422.3$ nm of the ^{87}Rb atom as the ODT laser wavelength, the only potential experienced by the atom when approaching or moving away is the $V(R)$ described above. Thus, the equation of motion of the atom reads

$$m_a \frac{d^2 \mathbf{r}_a}{dt^2} = -\frac{4C_4}{(\mathbf{r}_a - \mathbf{r}_i)^6} (\mathbf{r}_a - \mathbf{r}_i) + \frac{6C_6}{(\mathbf{r}_a - \mathbf{r}_i)^8} (\mathbf{r}_a - \mathbf{r}_i), \quad (\text{C1})$$

where $\mathbf{r}_a = (x_a, y_a, z_a)$ and $\mathbf{r}_i = (x_i, y_i, z_i)$ are the 3D positions of the atom and the ion, respectively.

Combining Eq. (A16) and the attractive force from the polarized atom, the equation of motion of the ion along the x direction is

$$m_i \frac{d^2 x_i}{dt^2} = -\frac{m_i \Omega^2}{4} [a_x(r_i) + 2q_x(r_i) \cos(\Omega t + \phi)] x_i + \frac{4C_4}{(r_a - r_i)^6} (x_a - x_i) \quad (\text{C2})$$

and the equations of motion of the ion along the y and z direction can be written similarly.

We use the explicit Runge-Kutta method of order 8 to solve the equations of motion by combining Eqs. (A17), (A18), (C1) and (C2). Figure 4 shows a simulated collision trajectory of an atom-ion pair in the x , y , and z directions. The lifetime of the bound atom-ion pair produced in the hybrid trap is significantly prolonged due to the low rf heating near the hybrid trap center.

APPENDIX D: WORK DONE BY RF FIELD ON THE ION

When an ion moves in a Paul trap without any disturbance, the rf field does zero work on the ion for an rf period; however, in a cold hybrid ion-atom system, the polarized atom will exert an attraction force on the ion [due to the attraction potential $V(r) = -C_4/r^4$] and thus disturb the conserved cycle of energy transfer. Finally, the ion-atom system gains net energy from the rf field, which increases the temperature of the system.

Here, we calculate the work done by the rf field on the ion when an atom approaches the ion and a collision occurs. Suppose the collision point is at r_c , and the collision happens at $t = 0$. When the attraction force between the ion-atom pair dominates, the distance between the ion and atom follows $r(t) = (18C_4/\mu)^{1/6} |t|^{1/3}$, so the trajectory of the ion becomes $r_i(t) = r_c - \mu r(t)/m_i$, where $\mu = m_i m_a / (m_i + m_a)$ is the reduced mass of the ion-atom pair. We can write the work done by the rf field on the ion when the atom is close enough to perturb the motion of ion in the trap as

$$W = \int_{-t_1}^{t_1} 2q(r_i) \cdot \frac{m_i \Omega^2}{4} r_i \cos(\Omega t + \phi) r_i dt,$$

where the integration range $t = -t_1$ to t_1 is the time during which the perturbation from the atom dominates, and we can take $t_1 \approx 0.8$ for approximation [21]. By substituting Eq. (B9) and the trajectory of the ion, we obtain the work done by the rf field during one collision as

$$W = W_1 + W_2,$$

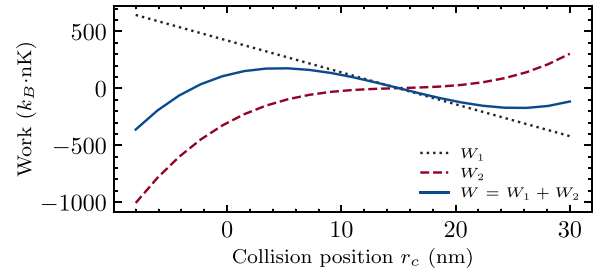


FIG. 5. Maximum work done by the rf field during one collision at $\sin \phi = 1$.

where

$$W_1 = \frac{1}{3} \mu \Omega^2 R^2 \sin \phi \cdot \Delta q \int_0^{\Omega t_1} (A - B\tau^{1/3}) \cdot \tau^{-2/3} \sin \tau \cdot d\tau,$$

$$W_2 = \frac{1}{3} \mu \Omega^2 R^2 \sin \phi \cdot \frac{R^4}{r_0^2 x_b^2} \int_0^{\Omega t_1} (A - B\tau^{1/3})^3 \cdot \tau^{-2/3} \sin \tau \cdot d\tau,$$

where $A = r_c r_0 / R^2$, $B = \mu r_0^2 / m_i R^2$, and $r_0 = \sqrt[6]{18C_4 / \mu \Omega^2}$. Here, W_1 comes from the uncompensated alternating force Δq , and W_2 comes from the dependence of q on position. For the first collision, when the atom approaches the ion from infinity, the collision happens at $r_{c,0} = 1.11(m_a/m_i)^{5/6} R \approx 20.8$ nm [21]. The numerical results give the maximum work heating as $W_1/k_B \approx -85.2$ nK and $W_2/k_B \approx 27.8$ nK for $\Delta q = -0.001$ at $\sin \phi = 1$. Here, we intentionally choose Δq as negative because we want $q(r)$ to be nearly 0 at the collision points, not at the origin.

After the first collision at $r_{c,0}$, the subsequent collision position $r_{c,i}$ for the i th collision is unpredictable. From Figs. 4(a) and 4(b), we can see that the collisions are most likely to occur at approximately 0–20 nm at micromotion directions (x and y axes). Figure 5 illustrates the relationship between $W = W_1 + W_2$ and r_c for $\Delta q = -0.001$, from which we can see that by compensating Δq to nearly zero, the work done by residual rf field during one collision is of the order of tens of nK k_B . Therefore, the relative energy between an ion-atom pair could easily fall below the characteristic energy scale E_k after one or more collisions, making the formation of a bound state (such as a molecular ion) possible.

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