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Article

Experimental realization of nonadiabatic holonomic single-qubit quantum gates with two dark paths in a trapped ion

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ABSTRACT

For circuit-based quantum computation, experimental implementation of a universal set of quantum logic gates with high-fidelity and strong robustness is essential and central. Quantum gates induced by geometric phases, which depend only on global properties of the evolution paths, have built-in noise-resilience features. Here, we propose and experimentally demonstrate nonadiabatic holonomic single-qubit quantum gates on two dark paths in a trapped $^{171}\text{Yb}^+$ ion based on four-level systems with resonant drives. We confirm the implementation with measured gate fidelity through both quantum process tomography and randomized benchmarking methods. Meanwhile, we find that nontrivial holonomic two-qubit quantum gates can also be realized within current experimental technologies. Compared with previous implementations, our experiments share both the advantages of fast nonadiabatic evolution and robustness against systematic errors. Therefore, our experiments confirm a promising method for fast and robust holonomic quantum computation.

1. Introduction

Quantum computer based on quantum mechanics is believed to be more powerful than classical computer in solving some hard problems, such as factorizing large prime number [1] and searching unsorted data [2]. To realize circuit-based quantum computer, a universal set of accurately controllable quantum gates, which include arbitrary single-qubit gates and a nontrivial two-qubit gate [3], are necessary. However, quantum systems usually inevitably suffer from environment-induced noises and operational imperfections, leading to infidelity of quantum evolution. Therefore, implementation of quantum gates with high-fidelity and strong robustness is highly preferred in quantum information processing.

Geometric phases [4–6], which only depend on the global properties of evolution path rather than their evolution details, are naturally applied into the field of quantum computation for noise-resilient quantum manipulation [7–10]. In particular, non-Abelian geometric phases

[5] can naturally induce universal quantum gates for the so-called holonomic quantum computation due to their non-commutativity. Then, holonomic quantum computation based on adiabatic evolution is proposed [11,12] with experimental demonstration [13,14]. However, these adiabatic schemes have to be slow enough, and thus, due to the decoherence of target systems, considerable errors will also accumulate.

To resolve this dilemma, holonomic quantum computation based on nonadiabatic evolutions has been proposed [15], which was theoretically expanded [16–25] and experimentally demonstrated [26–37] in various systems. Unfortunately, the noise-resilience feature of geometric phases is smeared in this type of implementation [38–41]. Recently, nonadiabatic holonomic quantum computation (NHQC) has been proposed [42] with two dark paths based on dressed-state method [43], which share both merits of nonadiabatic evolution and robustness against errors. However, finding representations of dressed states may become rather complicated, especially for high-dimensional quantum systems. On the other hand, due to the challenge of exquisite con-

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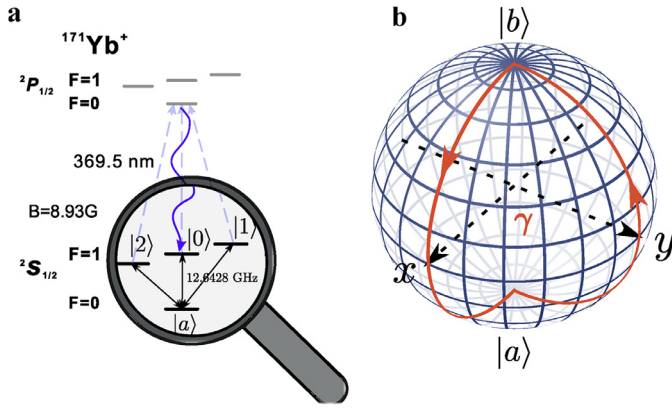


Fig. 1. Realization of single qubit nonadiabatic holonomic quantum gates. (a) Energy level of a $^{171}\text{Yb}^+$ ion. The $\{|0\rangle, |1\rangle\}$ subspace and two auxiliary level $|2\rangle, |a\rangle$ are encoded in the hyperfine level of ground state $S_{1/2}$. Coupling between energy level $|a\rangle$ and $|0\rangle$ ($|1\rangle$ and $|2\rangle$) is realized through resonant microwave. (b) Illustration of the implemented holonomic quantum gates in Bloch sphere of $\{|b\rangle, |a\rangle\}$ subspace. The whole evolution process is divided into two steps, and an additional geometric phase will accumulate in the $|b\rangle$ state after the cyclic evolution. Realization of single qubit nonadiabatic holonomic quantum gates.

trol among multilevel quantum systems, experimental demonstration of NHQC with two dark paths is still lacking.

Here, with the multilevel structure of a trapped $^{171}\text{Yb}^+$ ion, we propose a feasible scheme and for the first time experimentally demonstrate nonadiabatic holonomic single-qubit quantum gates on two dark paths based on a four-level system. The so-called dark path refers to the evolution path of dark state. One dark state is defined as the eigenstate of the system Hamiltonian with zero energy and decoupled, and the other one is defined to fulfill the time-dependent Schrödinger equation with zero expectation value. In the construction of our nonadiabatic holonomic quantum gates, only simple and experimentally accessible microwave controls are needed. Meanwhile, our demonstration merely uses conventional resonant interaction, which could simplify the experimental complexity and decrease the control errors. In our realization, characterized by randomized benchmarking (RB) method, the demonstrated average gate fidelity is above 98.75%, which is mainly restricted by the limited coherence time. Moreover, previous NHQC schemes only have one time-dependent variable of the system Hamiltonian, leading to fixed gate robustness against systematic errors. But our scheme has two time-dependent adjustable variables, providing gate robustness enhancement by proper designing the time-dependent variables. In addition, we demonstrate that our nonadiabatic holonomic gates are more robust against control amplitude errors than previous ones, under the same maximum driving amplitude, which is noteworthy for large-scale quantum computation. Finally, combining with nontrivial nonadiabatic holonomic two-qubit gates, robust universal NHQC can be achieved in the trapped ions setup with current state-of-art technologies.

2. Theoretical model

We first address the realization of arbitrary holonomic single-qubit gates in the $\{|0\rangle, |1\rangle\}$ subspace on $S_{1/2}$ ground-states of a trapped $^{171}\text{Yb}^+$ ion, with $|0\rangle \equiv |^2S_{1/2}, F=1, m_F=0\rangle$, $|1\rangle \equiv |^2S_{1/2}, F=1, m_F=+1\rangle$. The other energy levels on ground states $|2\rangle \equiv |^2S_{1/2}, F=1, m_F=-1\rangle$ and $|a\rangle \equiv |^2S_{1/2}, F=0, m_F=0\rangle$ are treated as two auxiliary states, as shown in Fig. 1a. Our proposal is realized by driving three resonant microwave fields $A_j(t)\cos(\omega_j t + \phi_j)$ ($j=0, 1, 2$) with time-dependent amplitude $A_j(t)$ to the ion. Under this condition, the qubit Hamiltonian can be written as:

$$H = H_0 + \sum_{j=0}^2 \hbar \Omega_j \cos(\omega_j t + \phi_j) |j\rangle\langle a| + \text{H.c.}, \quad (1)$$

where H_0 is Hamiltonian of the free four-level system, Ω_j is the Rabi frequency, ω_j and ϕ_j are frequency and phase of driving microwave respectively. After transformed to the interaction picture with respect to H_0 and applying rotating wave approximation (RWA), assuming $\hbar = 1$ hereafter, the interaction Hamiltonian in Hilbert space $\{|0\rangle, |1\rangle, |2\rangle, |a\rangle\}$ can be written as:

$$H_1(t) = \sum_{j=0}^2 \frac{\Omega_j(t)}{2} e^{-i\phi_j} |j\rangle\langle a| + \text{H.c.}, \quad (2)$$

After setting $\Omega_0(t)/\Omega_1(t) = \tan(\theta/2)$ with θ being a constant angle, this Hamiltonian can be rewritten in a new Hilbert space $\{|d_1\rangle, |b\rangle, |2\rangle, |a\rangle\}$ as:

$$H_d(t) = \frac{\Omega(t)}{2} e^{-i\phi_0} |b\rangle\langle a| + \frac{\Omega_2(t)}{2} |2\rangle\langle a| + \text{H.c.}, \quad (3)$$

in which parameter $\Omega(t) = \sqrt{\Omega_0^2(t) + \Omega_1^2(t)}$, the bright state $|b\rangle = \sin(\theta/2)|0\rangle - e^{i\phi} \cos(\theta/2)|1\rangle$ with $\phi = \phi_0 - \phi_1 + \pi$ and $\phi_2 = 0$. The dark state $|d_1\rangle = -\cos(\theta/2)e^{-i\phi}|0\rangle - \sin(\theta/2)|1\rangle$ is decoupled from the $\{|b\rangle, |2\rangle, |a\rangle\}$ subspace. The quantum dynamical process is induced by two resonant couplings $|b\rangle \leftrightarrow |a\rangle$ and $|2\rangle \leftrightarrow |a\rangle$, while the dark state $|d_1\rangle$ remains unchanged for constant θ and ϕ . The evolution path of $|d_1\rangle$ state forms the first dark path.

Different to the first dark path with zero eigenvalue, another dark path can be simply proved by $\langle d_2(t) | H_d(t) | d_2(t) \rangle = 0$ with zero expectation value, leading the accumulated dynamical phase to be zero, and it could be parameterized by two time-dependent angles α and β as:

$$|d_2(t)\rangle = \cos \alpha (\cos \beta e^{-i\phi_0} |b\rangle - \sin \beta |2\rangle) - i \sin \alpha |a\rangle \quad (4)$$

According to the time dependent Schrödinger equation, the relationship of parameters $|d_2(t)\rangle$ and $H_d(t)$ can be solved as:

$$\begin{aligned} \Omega(t) &= 2(\dot{\beta} \cot \alpha \sin \beta + \dot{\alpha} \cos \beta) \\ \Omega_2(t) &= 2(\dot{\beta} \cot \alpha \cos \beta - \dot{\alpha} \sin \beta) \end{aligned} \quad (5)$$

where the dot represents time differential. In particular, after choosing a proper set of variables $\alpha(t)$, $\beta(t)$ and ϕ_0 , we can inversely engineer the Hamiltonian $H_d(t)$ to induce a target evolution. Then, in this way, after a cyclic evolution, we can design the dark path to induce a target non-Abelian geometric phase on the bright state $|b\rangle$. Under the limitation of cyclic evolution $|d_2(T)\rangle\langle d_2(T)| = |d_2(0)\rangle\langle d_2(0)|$, boundary conditions must be set as $\alpha(T) = \alpha(0) = 0$ and $\beta(T) = \beta(0) = 0$. Meanwhile,

$\langle d_l | H_d(t) | d_k \rangle = 0$ ($l, k = 1, 2$) can always be met when θ is time-independent, i.e., there is also no dynamical phases accumulated during the whole process of evolution. Therefore, we can finally obtain a pure geometric phase to realize target nonadiabatic holonomic quantum gates [15].

To achieve a universal set of single-qubit holonomic gates, the single-loop method [19,23] is adopted, where the cyclic evolution time T is divided into two equal intervals: $0 \rightarrow T/2$ and $T/2 \rightarrow T$. Specifically, considering the boundary conditions, we set $\alpha(t) = \frac{\pi}{2} \sin^2(\pi t/T)$, $\beta(t) = \eta[1 - \cos \alpha(t)]$ during a cyclic evolution, with η being a constant to decide a target evolution path. When $\eta = 0$, the evolution process is reduced to the same evolution as in previous NHQC scheme [19,23]. In addition to the case of $\eta = 0$, as numerically illustrated in the Appendix, we note that increasing the value of η can enhance the gate robustness against systematic errors, but it also lengthens the gate-time. Thus, considering both gate robustness and decoherence effect with current setup, we choose $\eta = 4$ in the following experimental demonstration, unless otherwise specified. Then, parameters $\Omega(t)$, $\Omega_2(t)$ can be resolved by Eq. (5). During the first interval $t \in [0, T/2]$, we set $\phi_0 = 0$, the corresponding evolution operator is $U_1(T/2, 0) = |d_1\rangle\langle d_1| - i|a\rangle\langle b|$. During the second interval $t \in [T/2, T]$, we set $\phi'_0 = -\gamma$ with γ being an arbitrary constant angle. Then, the evolution operator is $U_2(T, T/2) = |d_1\rangle\langle d_1| + ie^{i\gamma}|b\rangle\langle a|$. Overall, as shown in Fig. 1b, the initial state $|b\rangle$ goes along the same dark path as state $|d_2(t)\rangle$ and acquires a global geometric phase after a cyclic evolution. Meanwhile, the dark state $|d_1(t)\rangle$ is always decoupled.

Consequently, the whole holonomic matrix of the total geometric evolution is given by $U(T, 0) = |d_1\rangle\langle d_1| + e^{i\gamma}|b\rangle\langle b|$ in $\{|d_1\rangle, |b\rangle\}$ subspace. This unitary matrix can be rewritten in the qubit basis $\{|0\rangle, |1\rangle\}$ as:

$$U(\theta, \phi, \gamma) = e^{i\frac{\gamma}{2}} e^{-i\frac{\gamma}{2}\vec{n}\cdot\vec{\sigma}}, \quad (6)$$

where $\vec{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ and $\vec{\sigma}$ is Pauli matrix. The above rotation matrix U describes a rotation operation around the axis $\vec{n}$, by an angle γ , up to a global phase factor $\exp(i\gamma/2)$, with the parameters θ, ϕ, γ determined by the applied microwave fields.

3. Experiment and results

The platform performing our experiments is a $^{171}\text{Yb}^+$ ion trapped in a radio frequency needle trap. Four hyperfine energy levels of ground state $S_{1/2}$ are used to encode the $\{|0\rangle, |1\rangle\}$ computational subspace and $\{|2\rangle, |a\rangle\}$ auxiliary levels. The frequency difference between $|0\rangle$ and $|a\rangle$ states is about $\omega_{0a} = 12.64$ GHz. The $|2\rangle \leftrightarrow |a\rangle$ and $|1\rangle \leftrightarrow |0\rangle$ transition frequencies are about $\omega_{20}(\omega_{10}) = 12.50$ MHz under 8.93 G magnetic field. Three microwaves resonated with $|j\rangle \leftrightarrow |a\rangle$ ($j = 0, 1, 2$) respectively are used to drive the four-energy level system. These microwave drives are generated through mixing method. In more detail, a microwave around 12.44 GHz generated from a radio frequency source is mixed with another microwave around 200 MHz, which is programmable to modulate the driving information. Then this signal is amplified to about 10 W and transmitted to the ion through a microwave horn after a high pass filter. The detailed setup description can be found in [44].

Every experiment is performed in the following process. After 1ms Doppler cooling, the ion is initialized to $|a\rangle$ state through 20 μs optical pumping. Then a resonant microwave between $|0\rangle$ and $|a\rangle$ is applied to rotate the state to $|0\rangle$ state. After that, the well-designed microwaves which include three frequency components are transmitted to the ion to control the evolution of the system. By changing the amplitudes and phases of microwave drive, we can realize different holonomic single qubit quantum gates as follows: $X = U(\frac{\pi}{2}, 0, \pi)$, $H = U(\frac{\pi}{4}, 0, \pi)$, $T = U(0, 0, \frac{\pi}{4})$, and $S = U(0, 0, \frac{\pi}{2})$. Finally, a resonant microwave between $|0\rangle(|1\rangle, |2\rangle)$ and $|a\rangle$ is performed to transform the population to $|a\rangle$ state for state dependent fluorescence detection through a 0.4 numerical aperture (NA) objective. For the purpose of figuring out the whole process of geometric evolution in detail, we investigate the probabilities of states $|0\rangle, |1\rangle, |2\rangle$ and $|a\rangle$ during the state transfer process. These demonstrative results of holonomic X gate and H gates for $\eta = 0$ and $\eta = 4$ performing on $|0\rangle$ initial state are shown in Fig. 2.

The performance of holonomic quantum gates based on two dark paths (HQCTD with $\eta = 4$) can be characterized through quantum process tomography (QPT) method [45]. The corresponding gates duration are 480 μs under the maximum driving strength of $(2\pi)10$ kHz. The QPT method consists of three components, which includes preparing a set of quantum states, sending them through the process and then using quantum state tomography to identify the states. A complete set of basis $\{|0\rangle, |1\rangle, (|0\rangle + |1\rangle)/\sqrt{2}, (|0\rangle - |1\rangle)/\sqrt{2}, (|0\rangle + i|1\rangle)/\sqrt{2}, (|0\rangle - i|1\rangle)/\sqrt{2}\}$ are prepared through pre-rotations with the help of auxiliary energy level $|a\rangle$. The final process matrix S_{exp} could be reconstructed through maximum likelihood estimation method according to the results of quantum state tomography [46]. The whole pipeline of QPT is shown in Fig. 3a. The final fidelity of each process is calculated according to $F_{\text{QPT}} = |\text{Tr}(S_{\text{exp}} S_{\text{the}}^\dagger)|$, where S_{the} is the theoretically predicted process matrix. Four nonadiabatic holonomic single-qubit quantum gates with $\eta = 4$ are characterized as $F_X = 96.48\%$, $F_H = 97.70\%$, $F_T = 97.24\%$ and $F_S = 97.51\%$ respectively and the corresponding process matrix is shown in Fig. 3b.

Another method to characterize the performance of holonomic quantum gates which does not depend on perfect state preparation and measurements is randomized benchmarking (RB) [47]. We perform both a reference RB experiment and an interleaved RB experiment to characterize the average quantum gate fidelity and individual quantum

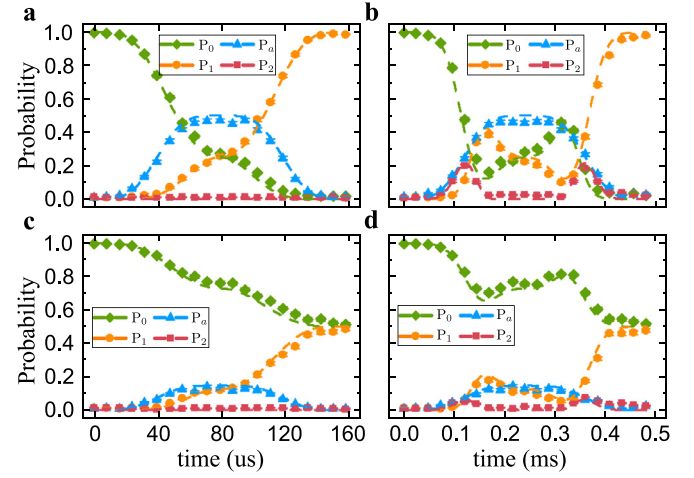


Fig. 2. States populations dynamics during holonomic quantum gates performing on $|0\rangle$ initial state. (a) and (c) are the probabilities of $|0\rangle, |1\rangle, |2\rangle$ and $|a\rangle$ in conventional holonomic X and H gates ($\eta = 0$), respectively. (b) and (d) are the probabilities of $|0\rangle, |1\rangle, |2\rangle$ and $|a\rangle$ in holonomic X and H gates based on two dark paths ($\eta = 4$), respectively. The experimental results (dots) agree well with the theoretical simulation (dash lines). The error bars indicate the standard deviation, each data point is averaged over 2000 experiments.

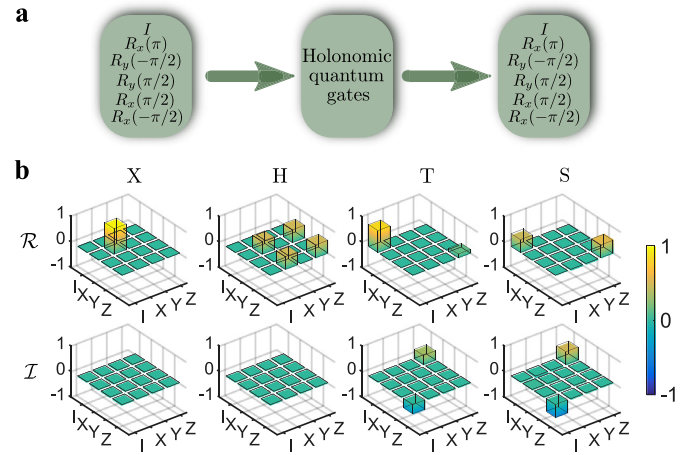


Fig. 3. QPT results of holonomic single-qubit quantum gates. (a) The whole pipeline of QPT. (b) The bar charts of the real and imaginary parts of process matrix of X, H, T and S gates. The solid black outlines are for the theoretically predicted results.

gate fidelity. Standard RB method is based on uniformly random Clifford operations. The Clifford group is defined as the group of unitary that normalize the Pauli group: $C = \{V \in U_2 | V P V^\dagger = P\}$, where P is the single qubit Pauli matrix. We chose a Clifford generating set of $s = \{I, X_{\pm\pi/2}, X_\pi, Y_{\pm\pi/2}, Y_\pi\}$. Each Clifford gate in a random sequence is performed by a random choice from the set of minimal length constructions of that gate using pyGSTI, an open-source python implementation package of Gate Set Tomography [48]. The probabilities of excited state with the function of number of Cliffords m are shown in Fig. 4. Both types of curves are fitted with the function $F = A r^m + B$ to obtain the error per Clifford gate r_{ref} and r_{int} for reference and interleaved RB, respectively. The fidelity of average holonomic quantum gate and individual quantum gate can be calculated according to $F_{\text{ave}} = 1 - \frac{1-r_{\text{ref}}}{2}$ and $F_{\text{gate}} = 1 - (1 - \frac{r_{\text{int}}}{r_{\text{ref}}})/2$. The results for four specific holonomic quantum gates are $F_X = 98.03\%$, $F_H = 98.95\%$, $F_T = 98.55\%$, $F_S = 98.64\%$ and the average gate fidelity is $F_{\text{ave}} = 98.75\%$. The error of state preparation is about 0.5% while the error of measurement is about 0.6% [49] in our

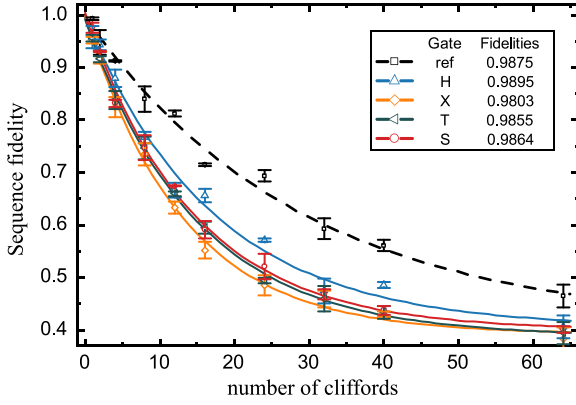


Fig. 4. RB results of holonomic single qubit quantum gates. Both sequences fidelities with the number of Clifford gates m are fitted with $F = Ar^m + B$. The fidelity for each sequence length is measured for 20 different random sequences with the standard deviation from the mean plotted as the error bars. The inset is sequences of reference RB and interleaved RB experiments.

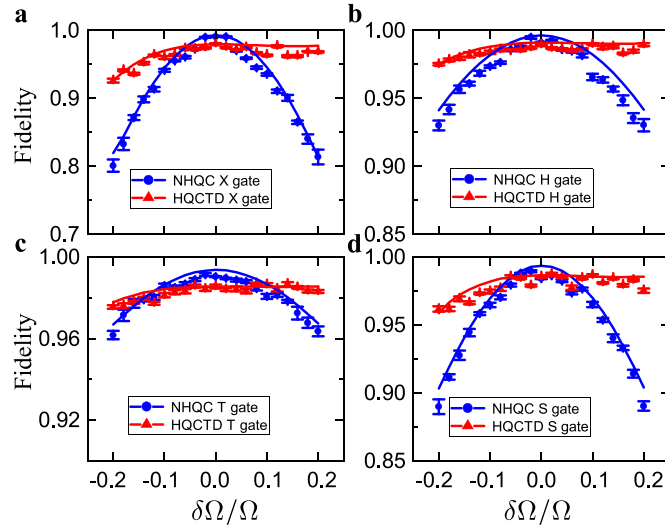


Fig. 5. Noise-resilient feature of different single-qubit holonomic quantum gates. The comparison of HQCTD (with $\eta = 4$) and NHQC (with $\eta = 0$) of X, H, T and S gates with different Rabi frequency error are shown in (a), (b), (c) and (d), respectively. The performances of NHQC scheme have advantages than HQCTD scheme in around zero Rabi frequency error. However, as the error increases, the HQCTD performs better than NHQC. The experimental results (dots) agree well with the theoretical simulation (solid lines) when considering decoherence of the system.

setup. The total state preparation and measurement errors is about 1.1%, which is in agreement with the difference of gate-fidelities between RB and QPT in our experiment.

With the demonstrated holonomic quantum gates, we further explore the robustness of HQCTD against Rabi frequency error $\delta\Omega$. We compare the performance of HQCTD and the corresponding conventional three level holonomic quantum gates (NHQC with $\eta = 0$) with the exactly same other parameters. As shown in Fig. 5, in all demonstrative single quantum gates, the NHQC has advantages than HQCTD if the Rabi frequency error is smaller than 0.1Ω . However, as the Rabi frequency error $\delta\Omega$ increases, the HQCTD will perform better than conventional NHQC undoubtedly. The poor performance of HQCTD in small error is mainly limited to coherence time of our system, which is about $800 \mu\text{s}$ for $|1\rangle \leftrightarrow |a\rangle$ and $|2\rangle \leftrightarrow |a\rangle$ Zeeman transition. Notice that the HQCTD scheme always performs better than NHQC scheme without considering the decoherence (the numerical simulations can be found in appendix).

In addition, with the technical development, the improvement of hardware can effectively prolong coherence times of a quantum system [50]. Then our method could be a powerful tool to overcome the errors from imperfect operations and crosstalk between qubits (e.g., Raman beam intensity drift in trapped ion system [51] and control crosstalk in superconducting qubits [52]). We note that, the gate robustness from two- and three-level systems [53,54,37] can also be improved by optimal control technique with different theoretical methods, which has similar gate performance compared with our theoretical scheme and experimental demonstration.

For the purpose of universal quantum computation, two qubits entangling operation is also necessary. We propose a feasible two-qubit control phase gate scheme based on internal state (spin) $\{|0\rangle, |1\rangle\}$ and motional state (phonon) $\{|0_p\rangle, |1_p\rangle\}$ of the ion. The coupling between spin and phonon of the ion could be realized through two photons Raman process. The corresponding Hamiltonian is

$$\hat{H} = \hat{H}_m + \hat{H}_e + \hat{H}_i, \quad (7)$$

where $\hat{H}_m$ is the motional Hamiltonian along one trap axis, $\hat{H}_e$ describes the internal electronic level structure of the ion, and $\hat{H}_i$ is the Hamiltonian of the interactions mediated by the applied light fields. After transformed into the interaction picture and applied rotating-wave approximation, the interaction Hamiltonian will be

$$\hat{H}(t) = (\hbar/2)\Omega_0\sigma_+ \{1 + i\eta_p(\hat{a}e^{-i\omega t} + \hat{a}^\dagger e^{i\omega t})\}e^{i(\phi_p - \delta t)} + \text{H.c.}, \quad (8)$$

which contains three resonances according to $\delta = 0, \pm\nu$ (carry, blue and red transitions respectively). The parameters η_p is Lamb-Dicke coefficient, ν is the phonon frequency, Ω_0 and ϕ_p are Rabi frequency and phase of the interaction.

A resonant carry transition between $|a0_p\rangle$ and $|20_p\rangle$ and a resonant blue sideband transition between $|a0_p\rangle$ and $|11_p\rangle$ will result in the Hamiltonian:

$$H(t) = \frac{1}{2}\Omega_1(t)e^{-i\phi(t)}|11_p\rangle\langle a0_p| + \Omega_2(t)|20_p\rangle\langle a0_p| + \text{H.c.}, \quad (9)$$

where the Ω_1 and Ω_2 are assigned according to Eq. 5, with the parameter $\theta = 0$. We set $\alpha(t) = \frac{\pi}{2}\sin^2(\frac{\pi t}{T})$, $\beta(t) = 4[1 - \cos(\alpha(t))]$ during a cyclic evolution. Then a geometric gate $\text{diag}(e^{i\gamma}, e^{-i\gamma})$ in the subspace $\{|11_p\rangle, |a0_p\rangle\}$ could be realized. When only considering the two-qubit computational subspace $\{|00_p\rangle, |01_p\rangle, |10_p\rangle, |11_p\rangle\}$, the resulting unitary operation corresponds to a controlled-phase gate with a conditional phase γ will be achieved as $U(\gamma) = \text{diag}(1, 1, 1, e^{i\gamma})$. In our realistic platform, $\eta_p = 0.1$, $\nu_x = 2.4 \text{ MHz}$ and with 20 mW average power in each laser beams, the effective Rabi frequency of blue sideband is about $100 \mu\text{s}$, resulting corresponding two-qubit control phase gate time is about $500 \mu\text{s}$. In addition, one can use the phonon as the quantum bus to realize entanglement between two spins at different ions. That is, we can be firstly entangled ion 1 and the phonon using $U(\gamma)$ when their both at the superposition state of $(|0\rangle + |1\rangle)/\sqrt{2}$, then another spin can be entangled with the same phonon similarly. Finally, after a measurement on the phonon in the basis of $(|0\rangle_p \pm |1\rangle_p)/\sqrt{2}$, the two ions can be entangled together, via the phonon bus.

4. Conclusion

In conclusion, we have experimentally demonstrated arbitrary robust nonadiabatic holonomic single-qubit quantum gates using four hyperfine energy levels of an ion. Both QPT and RB methods are used to characterize the performance of these quantum gates. The superior against Rabi frequency error of the realized quantum gates is verified through comparison with the corresponding conventional NHQC gates. The distinct advantage of these holonomic quantum gates illustrates that they are promising candidates for robust quantum computation. Finally, aiming at a universal robust NHQC, we also propose a scheme for non-trivial two-qubit control phase gate, which can be realized with an ion qubit and its phonon qubit. Therefore, our work validates the feasibility towards robust NHQC in the trapped ions platform.

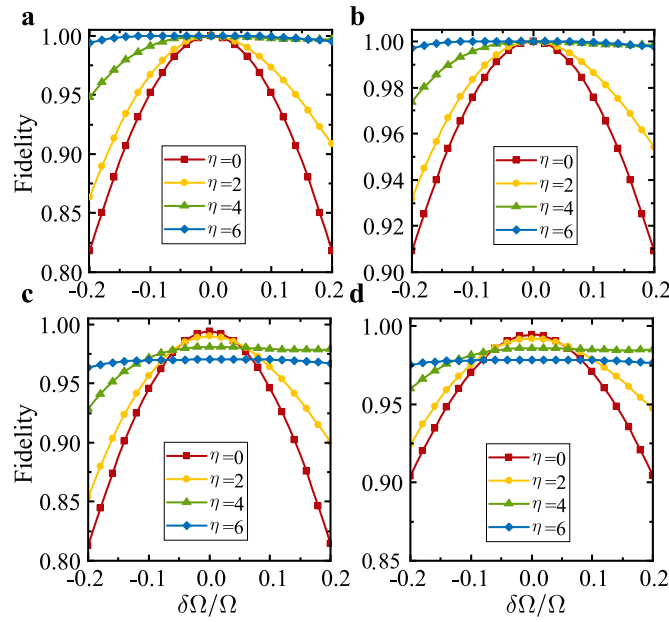


Fig. 6. Noise-resilient feature of different single-qubit holonomic quantum gates. The comparisons of HQCTD (with $\eta = 2, 4, 6$) and NHQC (with $\eta = 0$) for X and S gates with different Rabi frequency error are shown in (a) and (b) without decoherence, (c) and (d) with decoherence, respectively.

Declaration of Competing Interest

The authors declare that they have no conflicts of interest in this work.

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Appendix

Here, we add numerical simulations to illustrate that our scheme has better gate robustness against systematic errors. Different from previous NHQC schemes, our scheme has two time-dependent variables, and we can proper design them to significantly enhance the gate robustness. To faithfully verify our scheme, we consider a quantum system suffers systematic Rabi frequency error, i.e., $H_d^\delta(t) = (1 + \delta\Omega/\Omega)H_d(t)$. We compare the performance of holonomic quantum gates of X and S gates based on two dark paths (HQCTD with $\eta = 2, 4, 6$) and the corresponding conventional holonomic quantum gates (NHQC with $\eta = 0$) with the exact same other parameters. For ideal case without decoherence, X and S gates in our HQCTD scheme perform better than NHQC scheme, as shown in Fig. 6a, b. While the performances of NHQC scheme have advantages over HQCTD scheme around zero Rabi frequency error with decoherence, as shown in Fig. 6c, d, and the HQCTD performs better than NHQC as the error increasing. Based on above numerical simulations and considering both gate robustness and decoherence effect with current setup, we choose $\eta = 4$ in our experimental demonstration.

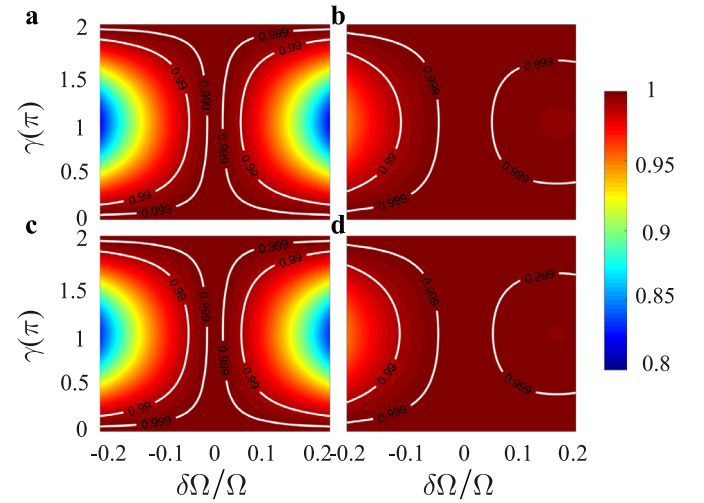


Fig. 7. Noise-resilient feature for both X- and Z-axis rotations. The comparisons of NHQC (with $\eta = 0$) and HQCTD (with $\eta = 4$) without decoherence are shown in (a) and (b) for X-axis rotations, (c) and (d) for Z-axis rotations, with different Rabi frequency error, respectively.

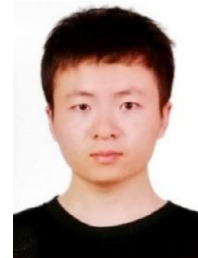
Note that, our scheme also favors technical development, which makes the decoherence effect to be less important.

In addition, to further illustrate the generality of our proposal for HQCTD with $\eta = 4$ case, we compare the gate performance without decoherence of all X-axis rotations (with $\hat{n} = (1, 0, 0)$ in Eq. 6 in main text) and all Z-axis rotations (with $\hat{n} = (0, 0, 1)$) in the case of $\eta = 0$ and $\eta = 4$, respectively. As shown in Fig. 7, the HQCTD (with $\eta = 4$) is more robust against Rabi frequency error than NHQC (with $\eta = 0$) in both X- and Z-axis rotations. As arbitrary single-qubit gate can be constructed from X-axis and Z-axis rotations, this does show our HQCTD proposal is more robust against conventional NHQC case. Actually, the robustness of all other axis rotations in our proposal can also be numerically demonstrated. Therefore, our HQCTD scheme possesses stronger robustness in a more general case.

References

- [1] P.W. Shor, Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer, *SIAM J. Comput.* 26 (5) (1997) 1484–1509.
- [2] L.K. Grover, Quantum computers can search arbitrarily large databases by a single query, *Phys. Rev. Lett.* 79 (23) (1997) 4709.
- [3] M.J. Bremner, C.M. Dawson, J.L. Dodd, et al., Practical scheme for quantum computation with any two-qubit entangling gate, *Phys. Rev. Lett.* 89 (24) (2002) 247902.
- [4] M.V. Berry, Quantal phase factors accompanying adiabatic changes, *Proc. Royal Soc. Lond. A* 392 (1802) (1984) 45–57.
- [5] F. Wilczek, A. Zee, Appearance of gauge structure in simple dynamical systems, *Phys. Rev. Lett.* 52 (24) (1984) 2111.
- [6] Y. Aharonov, J. Anandan, Phase change during a cyclic quantum evolution, *Phys. Rev. Lett.* 58 (16) (1987) 1593.
- [7] P. Solinas, P. Zanardi, N. Zanghi, Robustness of non-Abelian holonomic quantum gates against parametric noise, *Phys. Rev. A* 70 (4) (2004) 042316.
- [8] S.L. Zhu, P. Zanardi, Geometric quantum gates that are robust against stochastic control errors, *Phys. Rev. A* 72 (2) (2005) 020301.
- [9] P. Solinas, M. Sassetti, P. Truini, et al., On the stability of quantum holonomic gates, *New J. Phys.* 14 (2012) 093006.
- [10] M. Johansson, E. Sjöqvist, L.M. Andersson, et al., Robustness of nonadiabatic holonomic gates, *Phys. Rev. A* 86 (6) (2012) 062322.
- [11] P. Zanardi, M. Rasetti, Holonomic quantum computation, *Phys. Lett. A* 264 (2-3) (1999) 94–99.
- [12] L.M. Duan, J.I. Cirac, P. Zoller, Geometric manipulation of trapped ions for quantum computation, *Science* 292 (5522) (2001) 1695–1697.
- [13] K. Toyoda, K. Uchida, A. Noguchi, et al., Realization of holonomic single-qubit operations, *Phys. Rev. A* 87 (5) (2013) 052307.
- [14] F. Leroux, K. Pandey, R. Rehbi, et al., Non-Abelian adiabatic geometric transformations in a cold Strontium gas, *Nat. Commun.* 9 (1) (2018) 3580.
- [15] E. Sjöqvist, D.M. Tong, L.M. Andersson, et al., Non-adiabatic holonomic quantum computation, *New J. Phys.* 14 (10) (2012) 103035.
- [16] G.F. Xu, J. Zhang, D.M. Tong, et al., Nonadiabatic holonomic quantum computation in decoherence-free subspaces, *Phys. Rev. Lett.* 109 (17) (2012) 170501.

- [17] G.F. Xu, C. Liu, P.Z. Zhao, et al., Nonadiabatic holonomic gates realized by a single-shot implementation, *Phys. Rev. A* 92 (5) (2015) 052302.
- [18] E. Sjoqvist, Nonadiabatic holonomic single-qubit gates in off-resonant systems, *Phys. Lett. A* 380 (1-2) (2016) 65–67.
- [19] E. Herterich, E. Sjoqvist, Single-loop multiple-pulse nonadiabatic holonomic quantum gates, *Phys. Rev. A* 94 (5) (2016) 052310.
- [20] Z.Y. Xue, F.L. Gu, Z.P. Hong, et al., Nonadiabatic holonomic quantum computation with dressed-state qubits, *Phys. Rev. Appl.* 7 (5) (2017) 054022.
- [21] G.F. Xu, P.Z. Zhao, D.M. Tong, et al., Robust paths to realize nonadiabatic holonomic gates, *Phys. Rev. A* 95 (5) (2017) 052349.
- [22] G.F. Xu, P.Z. Zhao, T.H. Xing, et al., Composite nonadiabatic holonomic quantum computation, *Phys. Rev. A* 95 (3) (2017) 032311.
- [23] Z.P. Hong, B.J. Liu, J.Q. Cai, et al., Implementing universal nonadiabatic holonomic quantum gates with transmons, *Phys. Rev. A* 97 (2) (2018) 022332.
- [24] G.F. Xu, D.M. Tong, E. Sjoqvist, Path-shortening realizations of nonadiabatic holonomic gates, *Phys. Rev. A* 98 (5) (2018) 052315.
- [25] N. Ramberg, E. Sjoqvist, Environment-assisted holonomic quantum maps, *Phys. Rev. Lett.* 122 (14) (2019) 140501.
- [26] A.A. Abdulalikhov, J.M. Fink, K. Juliusson, et al., Experimental realization of non-Abelian non-adiabatic geometric gates, *Nature* 496 (7446) (2013) 482–485.
- [27] Y. Xu, W. Cai, Y. Ma, et al., Single-loop realization of arbitrary nonadiabatic holonomic single-qubit quantum gates in a superconducting circuit, *Phys. Rev. Lett.* 121 (11) (2018) 110501.
- [28] G. Feng, G. Xu, G. Long, Experimental realization of nonadiabatic holonomic quantum computation, *Phys. Rev. Lett.* 110 (19) (2013) 190501.
- [29] H. Li, Y. Liu, G. Long, Experimental realization of single-shot nonadiabatic holonomic gates in nuclear spins, *Sci. Chin. Phys. Mech. Astro.* 60 (8) (2017) 080311.
- [30] Z. Zhu, T. Chen, X. Yang, et al., Single-loop and composite-loop realization of nonadiabatic holonomic quantum gates in a decoherence-free subspace, *Phys. Rev. Appl.* 12 (2) (2019) 024024.
- [31] C. Zu, W.B. Wang, L. He, et al., Experimental realization of universal geometric quantum gates with solid-state spins, *Nature* 514 (7520) (2014) 72–75.
- [32] S. Arroyo-Camejo, A. Lazariev, S.W. Hell, et al., Room temperature high-fidelity holonomic single-qubit gate on a solid-state spin, *Nat. Commun.* 5 (1) (2014) 4870.
- [33] Y. Sekiguchi, N. Niikura, R. Kuroiwa, et al., Optical holonomic single quantum gates with a geometric spin under a zero field, *Nat. Photon.* 11 (5) (2017) 309–314.
- [34] B.B. Zhou, P.C. Jerger, V.O. Shkolnikov, et al., Holonomic quantum control by coherent optical excitation in diamond, *Phys. Rev. Lett.* 119 (14) (2017) 140503.
- [35] N. Ishida, T. Nakamura, T. Tanaka, et al., Universal holonomic single quantum gates over a geometric spin with phase-modulated polarized light, *Opt. Lett.* 43 (10) (2018) 2380–2383.
- [36] K. Nagata, K. Kuramitani, Y. Sekiguchi, et al., Universal holonomic quantum gates over geometric spin qubits with polarised microwaves, *Nat. Commun.* 9 (1) (2018) 3227.
- [37] M.Z. Ai, S. Li, Z. Hou, et al., Experimental realization of nonadiabatic holonomic single-qubit quantum gates with optimal control in a trapped ion, *Phys. Rev. Appl.* 14 (5) (2020) 054062.
- [38] S.B. Zheng, C.P. Yang, F. Nori, Comparison of the sensitivity to systematic errors between nonadiabatic non-Abelian geometric gates and their dynamical counterparts, *Phys. Rev. A* 93 (3) (2016) 032313.
- [39] J. Jing, C.H. Lam, L.A. Wu, Non-Abelian holonomic transformation in the presence of classical noise, *Phys. Rev. A* 95 (1) (2017) 012334.
- [40] B.J. Liu, X.K. Song, Z.Y. Xue, et al., Plug-and-play approach to nonadiabatic geometric quantum gates, *Phys. Rev. Lett.* 123 (10) (2019) 100501.
- [41] S. Li, T. Chen, Z.Y. Xue, Fast holonomic quantum computation on superconducting circuits with optimal control, *Adv. Quantum Technol.* 3 (3) (2020) 2000001.
- [42] B.J. Liu, Z.H. Huang, Z.Y. Xue, et al., Superadiabatic holonomic quantum computation in cavity QED, *Phys. Rev. A* 95 (6) (2017) 062308.
- [43] A. Baksic, H. Ribeiro, A.A. Clerk, Speeding up adiabatic quantum state transfer by using dressed states, *Phys. Rev. Lett.* 116 (23) (2016) 230503.
- [44] J.M. Cui, Y.F. Huang, Z. Wang, et al., Experimental trapped-ion quantum simulation of the Kibble-Zurek dynamics in momentum space, *Sci. Rep.* 6 (1) (2016) 33381.
- [45] I.L. Chuang, M.A. Nielsen, Prescription for experimental determination of the dynamics of a quantum black box, *J. Mod. Opt.* 44 (11-12) (1997) 2455–2467.
- [46] M. Jezek, J. Fiurasek, Z. Hradil, Quantum inference of states and processes, *Phys. Rev. A* 68 (1) (2003) 012305.
- [47] E. Knill, D. Leibfried, R. Reichle, et al., Randomized benchmarking of quantum gates, *Phys. Rev. A* 77 (1) (2008) 012307.
- [48] E. Nielsen, K. Rudinger, T. Proctor, et al., Probing quantum processor performance with pyGSTI, *Quantum Sci. Technol.* 5 (2020) 044002.
- [49] J.M. Cui, M.Z. Ai, R. He, et al., Experimental demonstration of suppressing residual geometric dephasing, *Sci. Bull.* 64 (23) (2019) 1757–1763.
- [50] P. Wang, C.Y. Luan, M. Qiao, et al., Single ion qubit with estimated coherence time exceeding one hour, *Nat. Commun.* 12 (1) (2021) 233.
- [51] C.J. Ballance, T.P. Harty, N.M. Linke, et al., High-fidelity quantum logic gates using trapped-ion hyperfine qubits, *Phys. Rev. Lett.* 117 (6) (2016) 060504.
- [52] D.M. Abrams, N. Didier, S.A. Caldwell, et al., Methods for measuring magnetic flux crosstalk between tunable transmons, *Phys. Rev. Appl.* 12 (6) (2019) 064022.
- [53] A. Ruschhaupt, X. Chen, D. Alonso, et al., Optimally robust shortcuts to population inversion in two-level quantum systems, *New J. Phys.* 14 (9) (2012) 093040.
- [54] T. Yan, B.J. Liu, K. Xu, et al., Experimental realization of nonadiabatic shortcut to non-Abelian geometric gates, *Phys. Rev. Lett.* 122 (8) (2019) 080501.



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