

Experimental verification of anti-Kibble-Zurek behavior in a quantum system under a noisy control field

Ming-Zhong Ai, Jin-Ming Cui,^{*} Ran He, Zhong-Hua Qian, Xin-Xia Gao, Yun-Feng Huang,[†]
Chuan-Feng Li[‡] and Guang-Can Guo

CAS Key Laboratory of Quantum Information and CAS Center for Excellence in Quantum Information and Quantum Physics,
University of Science and Technology of China, Hefei 230026, People's Republic of China



(Received 13 August 2020; revised 6 January 2021; accepted 7 January 2021; published 26 January 2021)

The Kibble-Zurek (KZ) mechanism is a universal framework that can in principle describe nonequilibrium phase transition phenomena in any system with the required symmetry properties. However, a conflicting phenomenon termed anti-KZ behavior has been reported in the study of ferroelectric phase transitions, in which slower driving results in more topological defects [S. M. Griffin *et al.*, *Phys. Rev. X* **2**, 041022 (2012)]. Although this finding is significant, related experimental work remains scarce. In this work, we experimentally demonstrate anti-KZ behavior under a noisy control field in three different quantum phase transition protocols using a single trapped $^{171}\text{Yb}^+$ ion. The density of defects is studied as a function of the quench time and the noise intensity. We experimentally verify that the optimal quench time to minimize excitations scales as a universal power law of the noise intensity. Our work provides an experimental verification of anti-KZ behavior in a two-level system.

DOI: [10.1103/PhysRevA.103.012608](https://doi.org/10.1103/PhysRevA.103.012608)

I. INTRODUCTION

The Kibble-Zurek mechanism (KZM), which was originally proposed by Kibble and Zurek to describe early-universe phase transitions [1,2], provides an elegant theoretical framework for exploring the critical dynamics of phase transitions [3]. Its central prediction is that the density of topological defects n , formed when a system goes through a critical point on a time scale τ , follows a universal power law as a function of quench time: $n \propto \tau^{-\beta}$. The power-law exponent $\beta = d\nu/(1+z\nu) > 0$ is determined by the dimensionality of the system d , equilibrium correlation length ν , and dynamic critical exponent z [4]. Notably, in the quantum domain, the KZM provides a useful heuristic for the preparation of ground-state phases of matter in quantum simulation and for adiabatic quantum computation [5]. Although the KZM has many important implications, its experimental verification still requires further work. The KZM has been verified for many systems that undergo classical continuous phase transitions, such as cold atomic gases [6], ion crystals [7,8], and superconductors [9]. In contrast, the KZM for quantum phase transitions, which is usually realized in ultracold atoms or Bose-Einstein Condensate (BEC) systems, still requires experimental demonstration [10–13]. In recent years, the Landau-Zener model has been gradually developed to explore the KZM in two-level systems in optical interferometers [14], trapped ions [15,16], and superconducting circuits [17] because of its ease of manipulation.

While the KZM is believed to be broadly applicable, a conflicting observation has been reported in the study of ferroelectric phase transitions: slower quenching generates more topological defects when approaching the adiabatic limit. This counterintuitive phenomenon, which is opposite the behavior predicted by the standard KZM, is termed anti-Kibble-Zurek (anti-KZ) dynamics [18]. Considerable attention has been devoted to the anti-KZ mechanism in the last few years. The universal properties of quantum quenching in a quantum system coupled with thermal dissipation were theoretically studied by Patane *et al.* and Nalbach *et al.* [19,20], revealing anti-KZ behavior. Meanwhile, del Campo *et al.* found that a thermally isolated system crossing a critical point under a noisy control field also exhibited anti-KZ behavior [21]. To explore whether the anti-KZ behavior appears in other quantum spin models with different scaling exponents, the dynamics of a transverse-field XY chain crossing quantum critical points under noisy control fields were theoretically studied by Gao *et al.* [22]. Recently, anti-KZ behavior was observed in one-dimensional (1D) and two-dimensional (2D) Ising models using a D-Wave quantum simulator [23,24]. A different study on the 1D Ising model using the D-Wave simulator revealed that the anti-KZ behavior results from environmentally induced fluctuations coupled to specific system operators [25]. However, the previous studies are limited to theoretical investigations or to experiments with uncontrolled noisy fields [16,18,21,26,27]. Unfortunately, previously conducted experiments were too limited to verify the most important scaling behavior predicted by theory.

In this paper, we experimentally investigated anti-KZ behavior in quantum phase transitions by applying fully controlled noisy driving fields to a two-level system with Landau-Zener (LZ) crossings in a trapped $^{171}\text{Yb}^+$ ion. We clearly verified three different scaling exponents of the

^{*}jmcui@ustc.edu.cn

[†]hyf@ustc.edu.cn

[‡]cfl@ustc.edu.cn

transverse-field XY chain. The different scaling exponents were determined by quenching across the boundary line between the paramagnetic (PM) and ferromagnetic (FM) phases, quenching across the isolated multicritical (MC) point, and quenching along the gapless line [28–30]. Thanks to the precise control of Gaussian noise, we quantitatively investigated the density of topological defects as a function of the quench time and the intensity of Gaussian noise. The results agree well with the theoretical expectation in Ref. [22], in which the optimal quench time to minimize defects scales as a universal power law of the noise intensity in all three protocols.

II. ANTI-KZ BEHAVIOR OF A NOISY TRANSVERSE-FIELD XY CHAIN

According to the Jordan-Wigner (JW) transformation [31–33], a spin-1/2 quantum transverse-field XY chain Hamiltonian can be transformed to the fermion form:

$$H_1 = -J \sum_{l=1}^N [(c_l^\dagger c_{l+1} + c_{l+1}^\dagger c_l) + \gamma (c_l^\dagger c_{l+1}^\dagger + c_{l+1} c_l)] - h \sum_{l=1}^N (2c_l^\dagger c_l - 1), \quad (1)$$

where c_l is obtained by JW transformation, the variable N counts the number of spins, and h measures the strength of the transverse field. We set $J = J_x + J_y$ and $\gamma = (J_x - J_y)/J$, where J_x and J_y represent the anisotropy interactions along the x and y spin directions, respectively. This Hamiltonian can be decoupled into a sum of independent terms $H_1 = \sum_{k \in [0, \pi]} H_m(k)$, where the Hamiltonian density $H_m(k)$ in pseudomomentum space can be written as

$$H_m(k) = -2[\sigma_z(J \cos k + h) + \sigma_x(J\gamma \sin k)], \quad (2)$$

in which σ_z and σ_x are Pauli matrices. The evolution of the generic state $|\psi_k(t)\rangle$ is governed by the Schrödinger equation $i \frac{d}{dt} |\psi_k(t)\rangle = H_m(k, t) |\psi_k(t)\rangle$. This reduces the quantum many-body transverse-field XY chain Hamiltonian to an array of decoupled single spin-1/2 Hamiltonians, which can be experimentally simulated utilizing a two-level system with well-designed LZ crossings (e.g., a trapped-ion qubit).

For the convenience of experimental demonstration, variation in one parameter, $H_m(k)$, was considered. The phase diagram of the transverse-field XY chain, which is spanned by the parameters h/J and γ , was divided into four parts: the quantum PM phase and two long-range FM phases ordering along the x and y directions [respectively denoted by FMx and FMy, as shown in Fig. 1(a)]. The definition of defect density in this transverse-field XY chain after quenching is similar to that for the Ising model [34–36]:

$$n_W = \frac{1}{N_k} \sum_{k \in [0, \pi]} p_k, \quad (3)$$

where N_k is the number of k modes used in the summation of the Hamiltonian $H_m(k)$, and p_k is the probability measured in the excited state $|E_k(\tau)\rangle$ after evolution driven under the k th Hamiltonian from $|G_k(0)\rangle$. Notably, $\{|G_k(t)\rangle, |E_k(t)\rangle\}$ is an adiabatic instantaneous eigenstate of $H_m(k)$. The whole

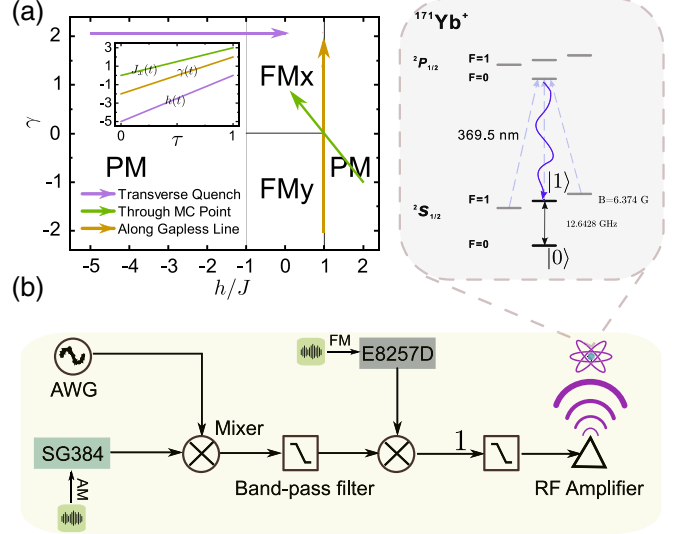


FIG. 1. Phase diagram of the three quantum phase transition protocols and schematic diagram of the experimental device. (a) The phase diagram is divided into the paramagnetic (PM) and ferromagnetic (FM) phases, which are separated by the parameter $h/J = \pm 1$ in the $\gamma - h/J$ frame. The middle FM phase is divided into two parts by the line $\gamma = 0$, with ordering along the x and y directions. The three arrows represent the three quenching protocols, as indicated in the legend. The inset shows the relationships between the quench parameters $h(t)$, $J_x(t)$, and $\gamma(t)$ and the quench time τ in the three quench protocols. (b) The microwave used in our experiments was generated by a mixing wave scheme. The top right illustration shows the energy-level diagram of $^{171}\text{Yb}^+$ ion.

process to measure excitation probability p_k is shown in Fig. 2.

To observe anti-KZ phenomena, driving with noisy control fields in the simulation was considered. White Gaussian noise is a good approximation of ubiquitous colored noise with exponentially decaying correlations; therefore, the noise term $\eta(t)$ was set to correspond to white Gaussian noise with zero mean and second moment $\overline{\eta(t)\eta(t')} = W^2 \delta(t - t')$. Here, W^2 represents the intensity of the noise fluctuation. The quench parameter $f'(t)$ was replaced with the noisy one $f(t)$ in the form of $f(t) = f'(t) + \eta(t)$, where the quench parameter $f'(t)$ is $h(t)$, $J_x(t)$, and $\gamma(t)$ in the transverse quench, anisotropic quench across the MC point, and quench along the gapless line protocols, respectively. All quench parameters $f'(t)$ change linearly with time. The noise control fields in our experiments were produced by the mixing method. The intensity of this noise can be accurately controlled by the modulation parameters. This method can also be applied to other quantum simulation experiments, providing a new way to conduct quantum simulation experiments with noise.

III. EXPERIMENTAL SIMULATIONS

Our experiments were performed using a trapped $^{171}\text{Yb}^+$ ion in a needle trap with the simplified setup shown in Fig. 1(b). Two hyperfine levels of $^{171}\text{Yb}^+$ ion in the $S_{1/2}$ ground state, $|^2S_{1/2}, F=0, m_F=0\rangle$ and $|^2S_{1/2}, F=1, m_F=0\rangle$, were encoded to $|0\rangle$ and $|1\rangle$, respectively. The mi-

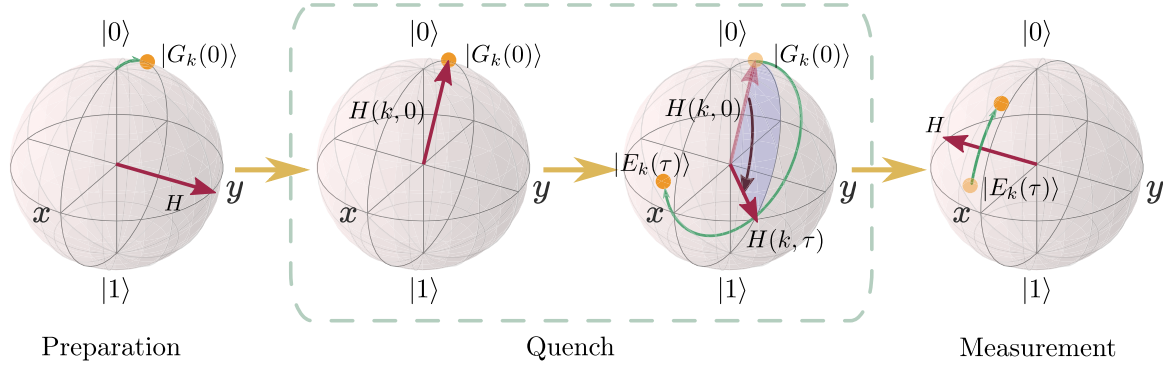


FIG. 2. Scheme for measuring the excitation probability. The quantum critical dynamics of the one-dimensional transverse-field XY chain model is detected by measuring the corresponding Landau-Zener crossing dynamics in each mode. For each mode, a typical process to measure the excitation probability includes preparation, quench, and measurement.

crowave used to drive the ion qubit was generated through two mixing steps as follows. A microwave signal of approximately 200 MHz was generated by a two-channel arbitrary waveform generator and mixed with a 3.0-GHz microwave generated from a radio-frequency signal generator (SG384, Stanford Research Systems). This mixed signal was again mixed with a 9.44-GHz microwave generated by an analog signal generator (E8257D, Agilent) to obtain an arbitrary microwave near 12.64 GHz. This signal was then amplified to approximately 2 W and used to irradiate the trapped ion via a horn antenna. In all experiments, the Rabi time was set to 100 μ s, and all subsequent expressions of τ in the text represent multiples of the Rabi time.

We first considered the transverse quench protocol, in which only the parameter $h(t)$ is time dependent, as shown in Fig. 1(a). To simulate the quench dynamics under noise fluctuation, white Gaussian noise $\eta(t)$ was added to the time-dependent quench parameter $h(t)$ as described above. The Hamiltonian shown in Eq. (2) can be rewritten as

$$H_m^{(1)}(k, t) = -2[(J_x + J_y)\cos k + h(t)]\sigma_z - 2[(J_x - J_y)\sin k]\sigma_x - 2\eta(t)\sigma_z. \quad (4)$$

This Hamiltonian can be transformed into the standard LZ model with noisy term $H_{LZ}(k, t) = -\frac{1}{2}(\sigma_x + v_{LZ}t_{LZ}\sigma_z) - \frac{1}{2}\eta/(\gamma J \sin k)\sigma_z$ using the substitutions $v_{LZ} = v_h/(2J\gamma \sin k)^2$ and $t_{LZ} = 4J\gamma \sin k(t + J \cos k/v_h)$, where $h(t) = v_h t$, and v_h is the quench velocity. The standard LZ model can be simulated using a trapped-ion qubit as described in Ref. [15]. We first drove the qubit to the ground state $|G_k(0)\rangle$ of the Hamiltonian $H_m^{(1)}(k, 0)$ from the initial state $|0\rangle$. The simulator then evolved under this Hamiltonian. The quench parameter $h(t)$ varied linearly from -5 to zero throughout the quench time τ , while the other two independent parameters were fixed as $J_x = 1$ and $J_y = -1/3$ during the evolution. Finally, the state was driven again to the basis $\{|0\rangle, |1\rangle\}$ (the reverse of the first process) to measure the population probability p_k of the excited state $|E_k(\tau)\rangle$. The white Gaussian noise in this quench protocol was generated via the frequency modulation of the microwave produced by the radio-frequency signal generator utilizing a built-in noise source (the detailed form of this noise is described in Appendix A). Different noise intensities were realized by varying the frequency deviation F during

the frequency modulation method (explained in Appendix B). We decomposed the Hamiltonian $H_1(t)$ into 50 independent terms $H_m(k, t)$ in all three protocols, and the parameter k was sampled 50 times equidistantly from zero to π . The final population probability p_k is shown as a function of k under different noise intensities and quench times in Fig. 3(a). The white Gaussian noise caused a bulge around $k = \pi/2$, which explains the additional defects in this quench process, with stronger noise corresponding to more defects generated.

Here, we consider the anisotropic quench across the MC point protocol, as depicted in Fig. 1(b). The Hamiltonian for each k mode in this case can be rewritten as

$$H_m^{(2)}(k, t) = -2[(J_x(t) + J_y)\cos k + h]\sigma_z - 2[(J_x(t) - J_y)\sin k]\sigma_x - 2\eta(t)[(\sin k)\sigma_x + (\cos k)\sigma_z], \quad (5)$$

where the time-dependent quench parameter $J_x(t)$ ramps from zero to 3. The Hamiltonian $H_m^{(2)}(k, t)$ can be transformed into the standard LZ model and a noisy term using the substitutions $v_{LZ} = v_x/[2(J_y \sin 2k + h \sin k)]^2$ and $t_{LZ} = 4(J_y \sin 2k + h \sin k)[t + (J_y \cos 2k + h \cos k)/v_x]$. Similar to the first protocol, we fixed $h = 2$ and $J_y = 1$ in all experiments for this protocol. Under this condition, the system was initially in the PM phase and then crossed the MC point into the FMx phase. The noise used in this quench protocol was produced by simultaneous frequency modulation using the radio-frequency signal generator and amplitude modulation using the analog signal generator. Different noise intensities were realized by changing the frequency deviation F and modulation depth A proportionally (explained in Appendix B). For convenience, we represented the noise intensity W^2 as a percentage; this percentage was the modulation depth $W^2 = A^2$ for amplitude modulation and the ratio $W^2 = (F/60 \text{ kHz})^2$ for frequency modulation. The probabilities p_k measured in the excited state as a function of k with different noise intensities and quench times are shown in Fig. 3(b).

For the quench along the gapless line protocol, the Hamiltonian for each k mode is

$$H_m^{(3)}(k, t) = -2[(J \cos k + h)\sigma_z + J\gamma(t) \sin k \sigma_x] - 2\eta(t)J \sin k \sigma_x, \quad (6)$$

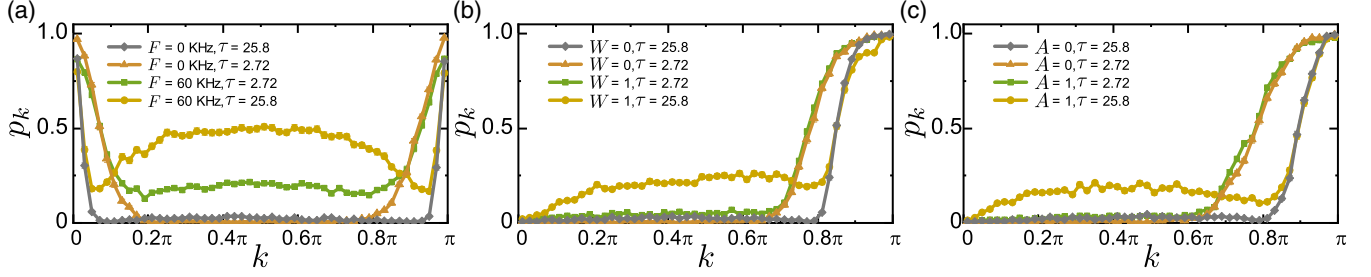


FIG. 3. The probabilities of the excited state p_k as a function of mode k for all three quench protocols. (a) Transverse quench protocol. The blue and red symbols represent the probabilities under the noisy control field with a frequency deviation of 60 kHz. The white Gaussian noise causes excitation near $k = \pi/2$, while the noise-free cases denoted by green and yellow symbols are very flat. (b) Gapless line quenching protocol. (c) MC point quenching protocol. Each protocol is demonstrated with and without noise in the control field. For each point, the experiment was repeated 1000 times, and the error bars indicate standard deviation.

where the time-dependent parameter $\gamma(t)$ ramps from -2 to 2 , while the other parameters are fixed as $h = 1$ and $J = J_x + J_y = 1$. This Hamiltonian can be transformed into the standard LZ model with a noisy term using the substitutions $v_{LZ} = v_\gamma \sin k / [2(\cos k + 1)]^2$ and $t_{LZ} = -4(\cos k + 1)t$. The noise was produced through amplitude modulations in the analog signal generator using built-in Gaussian noise. Figure 3(c) shows the probabilities p_k measured in the excited state as a function of k with different noise intensities and quench times.

The total density of defects, $n_W \approx r_W \tau + c\tau^{-\beta}$, exhibits anti-KZ behavior in all three quench protocols when noise is present. The prefactor c is predicted by the KZM, and r_W characterizes the noise intensity. Moreover, as shown in Fig. 4, the system also shows anti-KZ behavior without additional control noise because of the presence of systematic errors in the

experimental setup (e.g., imperfect microwave pulses, finite coherent time, and state preparation and measurement error). The density of defects without control noise is expressed as $n_0 \approx r_0 \tau + c\tau^{-\beta}$. Since the parameter r_W represents the rate at which the presence of systematic noise and control noise generates excitations, the system noise can be removed by subtracting r_0 from r_W to indicate the efficiency of defects induced by noise under the control field. Here, r_0 is a fitting parameter for the noise-free driving field. We can then define

$$\delta n = n_W - n_0 \approx r_W \tau + c\tau^{-\beta} - (r_0 \tau + c\tau^{-\beta}) = \delta r \tau \quad (7)$$

to represent the defects induced by noise in the control field. The results are shown in Fig. 4. The term $n_w \approx \delta r \tau + c\tau^{-\beta}$ then represents the density of defects produced under the KZM and control noise without systematic error. The optimal quench time τ_{opt} can be approximated by minimizing n_w . As illustrated in Fig. 5, for all three quench protocols, the optimal quench time to minimize defects scales as a power law of the noise intensity W^2 . The linear fitting of $\ln(\tau_{\text{opt}})$ as a function of $\ln(W^2)$ gives $\ln(\tau_{\text{opt}}) \propto \alpha_{\text{fit}} \ln(W^2)$,

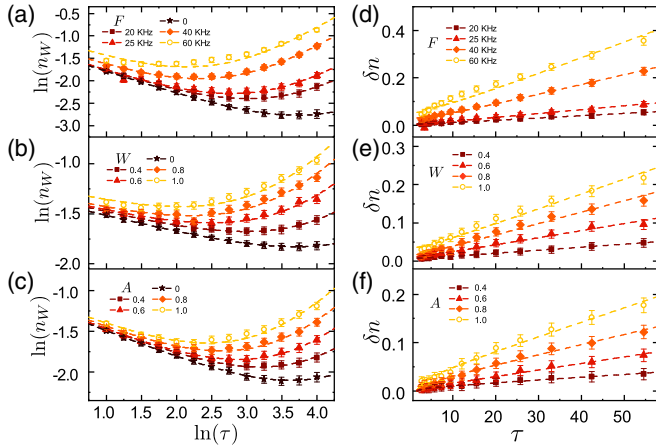


FIG. 4. Anti-KZ behavior of defect production in the three quench protocols. (a)–(c) The density of defects $\ln(n_W)$ as a function of quench time $\ln(\tau)$ for the transverse quench, anisotropic quench across the MC point, and quench along the gapless line protocols, respectively. The defect density first decreases and then increases as quench time increases. For the same quench time, stronger noise results in more defects. Due to the existence of system noise, anti-KZ behavior is observed even when the Gaussian noise is zero. (d)–(f) The Gaussian noise-induced defect densities $\delta n = n_W - n_0$ corresponding to (a)–(c), respectively. These defect densities are proportional to the quench time τ . For each point, the experiment was repeated 1000 times, and the error bars indicate standard deviation.

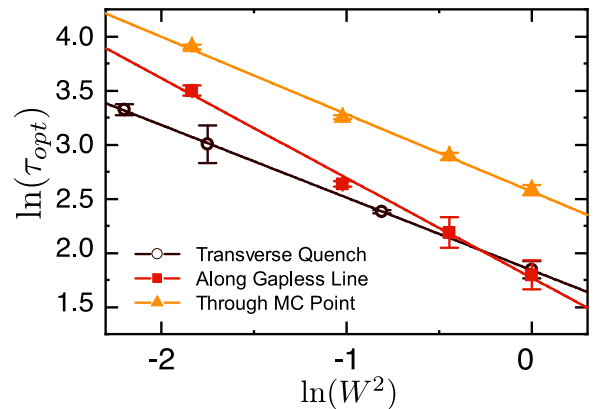


FIG. 5. The linear relationship between the logarithm of optimal quench time $\ln(\tau_{\text{opt}})$ and the logarithm of noise intensity $\ln(W^2)$ gives $\ln(\tau_{\text{opt}}) \propto \alpha_{\text{fit}} \ln(W^2)$. The fitting parameters are $\alpha_{\text{fit}}^1 = -0.67 \pm 0.01$ for the transverse quench protocol, similar to the analytical result $\alpha_{\text{theory}}^1 = -0.67$; $\alpha_{\text{fit}}^2 = -0.92 \pm 0.05$ for the quench through the MC point protocol with $\alpha_{\text{theory}}^2 = -0.86$; and $\alpha_{\text{fit}}^3 = -0.71 \pm 0.03$ for the quench along the gapless line protocol with $\alpha_{\text{theory}}^3 = -0.75$. Error bars indicate standard deviation.

and the resulting fitting parameters for all three cases agree well with the analytical result: $\alpha_{\text{fit}}^1 = -0.67 \pm 0.01$ for the transverse quench protocol with $\alpha_{\text{theory}}^1 = -2/3 = 0.67$, $\alpha_{\text{fit}}^2 = -0.92 \pm 0.05$ for the quench through the MC point protocol with $\alpha_{\text{theory}}^2 = -6/7 = -0.86$, and $\alpha_{\text{fit}}^3 = -0.71 \pm 0.03$ for the quench along the gapless line protocol with $\alpha_{\text{theory}}^3 = -3/4 = -0.75$ [22].

IV. CONCLUSION

We have experimentally demonstrated the anti-KZ behavior in three quantum phase transition protocols under a white Gaussian noisy control field using a single trapped ion. The experimental results provide powerful evidence for the anti-KZ phenomenon. We also show that the optimal quench time to minimize defect density (τ_{opt}) scales as a universal power law of the noise intensity W^2 for all three protocols, which highlights the limitations of adiabatic protocols such as quantum annealing.

ACKNOWLEDGMENTS

This work was supported by the National Key Research and Development Program of China (Grants No. 2017YFA0304100 and No. 2016YFA0302700), the National Natural Science Foundation of China (Grants No. 11874343, No. 61327901, No. 11774335, No. 11474270, No. 11734015, No. 11874343, and No. 11821404), Key Research Program of Frontier Sciences, CAS (Grant No. QYZDY-SSW-SLH003), the Fundamental Research Funds for the Central Universities (Grants No. WK2470000026 and No. WK2470000018), Anhui Initiative in Quantum Information Technologies (Grants No. AHY020100 and No. AHY070000), and the National Program for Support of Topnotch Young Professionals (Grant No. BB2470000005).

APPENDIX A: GAUSSIAN NOISE

White Gaussian noise is a basic noise model in information theory used to mimic the effects of many random processes that occur in nature. The white Gaussian noise used in our experiments was generated from microwave signal generators (E8257D and SG384) with a constant spectral density and a Gaussian distribution. To verify the form of this noise, we sampled the white Gaussian noise from these devices; the processed results are shown in Fig. 6.

APPENDIX B: CONTROLLABLE NOISE

Suppose that $\eta(t)$ is white Gaussian noise with zero mean and the second moment $\langle \eta(t)\eta(\tau) \rangle = W^2\delta(\tau)$, where the parameter W^2 represents the intensity of the noise. According to the Wiener-Khinchin-Kolmogorov theorem [37], the power spectral density (PSD) of this noise is the Fourier transform of its autocorrelation:

$$\langle \eta(t)\eta(\tau) \rangle = \int \eta(t)\eta(t-\tau) dt = \int \frac{d\omega}{2\pi} |g(\omega)|^2 e^{-i\omega\tau}, \quad (\text{B1})$$

where $g(\omega)$ is the PSD. For white Gaussian noise, the PSD is a constant, and $\delta(\tau) = \frac{1}{2\pi} \int e^{i\omega\tau} d\omega$. Thus, we can conclude that $g(\omega) = W$.

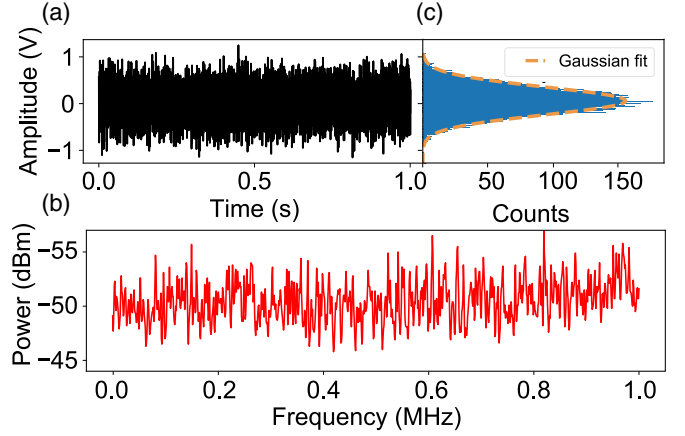


FIG. 6. Detailed information about white Gaussian noise generated from the microwave signal generator. (a) Time domain information about the noise. (b) Frequency domain information of the white Gaussian noise collected by a spectrum analyzer. The spectral density is a constant that reaches approximately 1 MHz, indicating that the noise is white noise. (c) Histogram of noise amplitudes in (a). We fit this histogram with a Gaussian formula, resulting in the parameters $\mu = -6.27 \times 10^{-3}$ and $\sigma^2 = 0.1$.

In the transverse quench protocol, the white Gaussian noise was added to the σ_z part of the Hamiltonian and the noisy Hamiltonian $V^{(1)} = -2\eta(t)\sigma_z$. Frequency modulation as a cosine signal with white Gaussian noise can be written as

$$y_1(t) = \cos((\omega_{\text{res}} + FG(t))t), \quad (\text{B2})$$

where ω_{res} is the resonant frequency of the internal state of the ion, F is the modulation deviation, and the dimensionless $G(t)$ is the noise signal. In the standard Hamiltonian of a two-level system of an ion, the detuning frequency $\Delta = FG(t) \propto \eta(t)$. The PSD of this noise is the Fourier transform of its autocorrelation:

$$g(\omega) = \int_0^{1\text{MHz}} dt \eta(t) e^{i\omega t}. \quad (\text{B3})$$

The bandwidth of this white Gaussian noise is up to 1 MHz, which far exceeds the Rabi frequency (approximately 10 kHz in our experiments). Thus, we can conclude that $F \propto W$, and that the change in modulation deviation equals the change in noise intensity.

In the quenching along the gapless line protocol, the white Gaussian noise is added to the σ_x part of the Hamiltonian and the noisy Hamiltonian $V^3 = -2\eta(t)J\text{sink}\sigma_x$. Amplitude modulation as a cosine signal with white Gaussian noise can be written as

$$y_2(t) = [1 + AG(t)] \cos(\omega_{\text{res}}t), \quad (\text{B4})$$

where A is the modulation depth. In the Hamiltonian of an ion, the Rabi frequency $\Omega \propto AG(t)$; that is, $AG(t) \propto \eta(t)$. According to Eq. (B3), the final strength of noise is proportional to the modulation depth A .

As for the second quench protocol, the white Gaussian noise was added to both the σ_x and σ_z parts of the Hamiltonian. Thus, frequency modulation and amplitude modulation were conducted simultaneously.

- [1] T. W. Kibble, *J. Phys. A: Math. Gen.* **9**, 1387 (1976).
- [2] W. H. Zurek, *Nature* **317**, 505 (1985).
- [3] A. D. Campo and W. H. Zurek, in *Symmetry and Fundamental Physics: Tom Kibble at 80* (World Scientific, Singapore, 2014), pp. 31–87.
- [4] P. Laguna and W. H. Zurek, *Phys. Rev. Lett.* **78**, 2519 (1997).
- [5] S. Suzuki, in *Quantum Quenching, Annealing and Computation* (Springer, Berlin, 2010), pp. 115–143.
- [6] N. Navon, A. L. Gaunt, R. P. Smith, and Z. Hadzibabic, *Science* **347**, 167 (2015).
- [7] S. Ulm, J. Roßnagel, G. Jacob, C. Degünther, S. T. Dawkins, U. G. Poschinger, R. Nigmatullin, A. Retzker, M. B. Plenio, F. Schmidt-Kaler, and K. Singer, *Nat. Commun.* **4**, 2290 (2013).
- [8] K. Pyka, J. Keller, H. L. Partner, R. Nigmatullin, T. Burgermeister, D. M. Meier, K. Kuhlmann, A. Retzker, M. B. Plenio, W. H. Zurek, A. del Campo, and T. E. Mehlstäubler, *Nat. Commun.* **4**, 2291 (2013).
- [9] R. Monaco, J. Mygind, and R. J. Rivers, *Phys. Rev. Lett.* **89**, 080603 (2002).
- [10] D. Chen, M. White, C. Borries, and B. DeMarco, *Phys. Rev. Lett.* **106**, 235304 (2011).
- [11] S. Braun, M. Friesdorf, S. S. Hodgman, M. Schreiber, J. P. Ronzheimer, A. Riera, M. Del Rey, I. Bloch, J. Eisert, and U. Schneider, *Proc. Natl. Acad. Sci. USA* **112**, 3641 (2015).
- [12] M. Anquez, B. A. Robbins, H. M. Bharath, M. Boguslawski, T. M. Hoang, and M. S. Chapman, *Phys. Rev. Lett.* **116**, 155301 (2016).
- [13] A. Keesling, A. Omran, H. Levine, H. Bernien, H. Pichler, S. Choi, R. Samajdar, S. Schwartz, P. Silvi, S. Sachdev *et al.*, *Nature* **568**, 207 (2019).
- [14] X.-Y. Xu, Y.-J. Han, K. Sun, J.-S. Xu, J.-S. Tang, C.-F. Li, and G.-C. Guo, *Phys. Rev. Lett.* **112**, 035701 (2014).
- [15] J.-M. Cui, Y.-F. Huang, Z. Wang, D.-Y. Cao, J. Wang, W.-M. Lv, L. Luo, A. Del Campo, Y.-J. Han, C.-F. Li *et al.*, *Sci. Rep.* **6**, 33381 (2016).
- [16] J.-M. Cui, F. J. Gómez-Ruiz, Y.-F. Huang, C.-F. Li, G.-C. Guo, and A. del Campo, *Commun. Phys.* **3**, 44 (2020).
- [17] V. Cappello, L. Marchetti, P. Parlanti, S. Landi, I. Tonazzini, M. Cecchini, V. Piazza, and M. Gemmi, *Sci. Rep.* **6**, 1 (2016).
- [18] S. M. Griffin, M. Lilienblum, K. T. Delaney, Y. Kumagai, M. Fiebig, and N. A. Spaldin, *Phys. Rev. X* **2**, 041022 (2012).
- [19] D. Patane, A. Silva, L. Amico, R. Fazio, and G. E. Santoro, *Phys. Rev. Lett.* **101**, 175701 (2008).
- [20] P. Nalbach, S. Vishveshwara, and A. A. Clerk, *Phys. Rev. B* **92**, 014306 (2015).
- [21] A. Dutta, A. Rahmani, and A. del Campo, *Phys. Rev. Lett.* **117**, 080402 (2016).
- [22] Z.-P. Gao, D.-W. Zhang, Y. Yu, and S.-L. Zhu, *Phys. Rev. B* **95**, 224303 (2017).
- [23] B. Gardas, J. Dziarmaga, W. H. Zurek, and M. Zwolak, *Sci. Rep.* **8**, 4539 (2018).
- [24] P. Weinberg, M. Tylutki, J. M. Rönkkö, J. Westerholm, J. A. Åström, P. Manninen, P. Törmä, and A. W. Sandvik, *Phys. Rev. Lett.* **124**, 090502 (2020).
- [25] Y. Bando, Y. Susa, H. Oshiyama, N. Shibata, M. Ohzeki, F. J. Gómez-Ruiz, D. A. Lidar, S. Suzuki, A. del Campo, and H. Nishimori, *Phys. Rev. Res.* **2**, 033369 (2020).
- [26] R. Puebla, A. Smirne, S. F. Huelga, and M. B. Plenio, *Phys. Rev. Lett.* **124**, 230602 (2020).
- [27] S.-F. Liou and K. Yang, *Phys. Rev. B* **97**, 235144 (2018).
- [28] V. Mukherjee, U. Divakaran, A. Dutta, and D. Sen, *Phys. Rev. B* **76**, 174303 (2007).
- [29] U. Divakaran, V. Mukherjee, A. Dutta, and D. Sen, *J. Stat. Mech.* (2009) P02007.
- [30] U. Divakaran, A. Dutta, and D. Sen, *Phys. Rev. B* **78**, 144301 (2008).
- [31] E. Lieb, T. Schultz, and D. Mattis, *Ann. Phys.* **16**, 407 (1961).
- [32] J. E. Bunder and R. H. McKenzie, *Phys. Rev. B* **60**, 344 (1999).
- [33] T. Caneva, R. Fazio, and G. E. Santoro, *Phys. Rev. B* **76**, 144427 (2007).
- [34] J. Dziarmaga, *Adv. Phys.* **59**, 1063 (2010).
- [35] J. Dziarmaga, *Phys. Rev. Lett.* **95**, 245701 (2005).
- [36] W. H. Zurek, U. Dorner, and P. Zoller, *Phys. Rev. Lett.* **95**, 105701 (2005).
- [37] A. Khintchine, *Math. Ann.* **109**, 604 (1934).