

Experimentally detecting dynamical quantum phase transitions in a slowly quenched Ising-chain model

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Recently, research on nonequilibrium dynamics sparked a great amount of topics. One of particular concern is the dynamical quantum phase transition in quenched systems. In this work, we experimentally simulated the slow quenches in a quantum transverse-field Ising model crossing two quantum critical points. We studied the relationship between the appearance of the dynamical quantum phase transition and the quench rate, using the discontinuity of the dynamical topological order parameter (DTOP). In addition, an oscillation of the DTOP is also observed in our experiment. These results provide more insight into the topological aspect of the nonequilibrium dynamics.

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I. INTRODUCTION

With the development of the technologies in quantum simulations, especially the ultracold atoms in optical lattices [1,2] and trapped ions [3–6], the study of the nonequilibrium dynamics of a closed quantum system, particularly the quench dynamics, has become a topic that has attracted much interest and focus in the past years [7–9]. The quenching process is a typical nonequilibrium process, which comes in two different forms. One is called the sudden quench, where the system is first prepared in the ground state of an initial Hamiltonian H_i , followed by an abrupt change to a final Hamiltonian H_f . Subsequently, the system evolves under the final Hamiltonian H_f . The other one is called the slow quench, where the system is also initialized to the ground state of a Hamiltonian H_i , but then the Hamiltonian is slowly changed to the final Hamiltonian H_f (the speed of the change is typically characterized by a quench ratio parameter τ [10]), and the following evolution of the system is also governed by the Hamiltonian H_f .

It is generally recognized that the dynamical quantum phase transition (DQPT) [11,12], which was first introduced in [13] by analogy with the phase transition in thermodynamic systems [14], may occur in some quench processes, especially in the sudden quenches across one critical point. Just like in the equilibrium phase transition where the free energy function goes through a nonanalytical point at the critical temperature, DQPTs take place at a critical time when the Loschmidt echo becomes nonanalytical [15]. DQPTs have been observed in the sudden quenches in both integrable [16–19] and nonintegrable [20,21] systems. However, the situation is much more complicated in the slow quenches due

to the Kibble-Zurek mechanism (KZM). According to the KZM, which was first proposed by Kibble [22] in order to describe the symmetry broken in cosmology and then introduced into condensed matter physics by Zurek [10], when the slow quench crosses the phase boundary, the dynamics are not adiabatic and some topological defects will appear [23–29]. Consequently, the final state of the slow quench is strongly dependent on the speed of the quench. As a result, the appearance of the DQPTs in the slow quenches will be different from that in the sudden quenches [30]. Moreover, when the quench crosses more than one critical point, the appearance of DQPT adds additional complications.

In this study, we experimentally simulated the quenches with different speeds between two paramagnetic phases in the momentum space of the one-dimensional (1D) quantum transverse-field Ising model (QTFIM) and characterized the DQPT with the dynamical topological order parameter (DTOP) [31,32]. By experimentally comparing the DQPT appearing in the sudden quenches and the slow quenches with Loschmidt echo measurements in momentum space, our results show that the DQPT appears when the quench rate is slow enough, whereas the DQPT does not appear in the corresponding sudden quench. In addition, we also observed the oscillation of the DTOP which is different from the behavior of the DTOP when the quenches only cross one critical point (the DTOP increases stepwise by 1 at every quantum dynamical phase transition) [31,32].

II. RESULT

The Hamiltonian of the well-known 1D QTFIM can be written as

$$H = -J \left(\sum_{\langle i,j \rangle}^N \sigma_i^z \sigma_j^z + h \sum_i^N \sigma_i^x \right), \quad (1)$$

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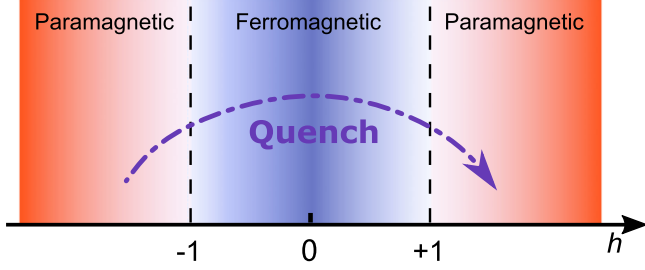


FIG. 1. Phase graph of the QTFIM and the quenching scheme in our experiments. The strength of the transverse field $h(t)$ is quenched from $h(t_i) \ll -1$ to $h(t_f) \gg 1$, and the system is quenched from one paramagnetic phase to another paramagnetic phase.

where σ_i^α ($\alpha = x, y, z$) is the Pauli matrix of the i th spin, $\langle i, j \rangle$ denotes that i and j are the nearest neighbors, N is the number of spins, and h is the strength of the transverse field. Fortunately, this model can be analytically solved through the Fourier transformation and Jordan-Wigner transformation [33], and it can be decoupled to a series of independent two-level subsystems in the momentum space. The Hamiltonian of the decoupled subsystem with momentum (quasimomentum) k can be read as

$$H_k = 2J \begin{pmatrix} -h + \cos k & -i \sin k \\ i \sin k & h - \cos k \end{pmatrix}, \quad (2)$$

where k takes N different values $\{\pm \frac{1}{2} \frac{2\pi}{N}, \dots, \pm(\frac{N}{2} - \frac{1}{2}) \frac{2\pi}{N}\}$. The energy gap of H_k is $E_g = 2\epsilon_k = 2\sqrt{(h - \cos k)^2 + \sin^2 k}$ ($\pm\epsilon_k$ are eigenvalues of H_k); it is only close at $k = 0$ with $h = -1$ or $k = \pi$ with $h = 1$ which corresponds to two critical points of h in this model. Actually, for $|h| < 1$, it is a ferromagnetic phase, and for other values of h , it is a paramagnetic phase. The nonequilibrium dynamics of the sudden quench between the ferromagnetic phase and the paramagnetic phase in the QTFIM have been extensively investigated [34–37] and DQPTs have been found in this system [6, 13].

Generally, the DQPTs are characterized by the Fisher zeros of the Loschmidt amplitude, and $G(t) = \langle \psi_0 | e^{-iHt} | \psi_0 \rangle$ (where $|\psi_0\rangle$ is the wave function of the system at the beginning of the evolution with the Hamiltonian H) [38]. In our current QTFIM, the Loschmidt amplitude can be rewritten as $G(t) = \prod_k G_k(t)$, where $G_k(t)$ is the Loschmidt amplitude in the two-level subsystem with the momentum k . It is clear that the Fisher zeros appear if and only if $G_k(t) = 0$ for some quasimomentum k . Effectively, the initial state in the two-level subsystem with the Hamiltonian H_k can be written as $u_k |\varphi_{k-}\rangle + v_k |\varphi_{k+}\rangle$ (where $|\varphi_{k-}\rangle$ is the ground state and $|\varphi_{k+}\rangle$ is the excited state—they are eigenvectors of the Hamiltonian H_k). As a result, the Loschmidt amplitude $G_k(t) = 0$ (DQPT occurs) if and only if $|u_k|^2 = |v_k|^2 = \frac{1}{2}$.

Here, we focus on the quenches between two paramagnetic phases which will cross two critical points, depicted as Fig. 1. We can see in the following that there is no DQPT for sudden quench and the DQPTs will appear when the quench speed is slow enough. For simplicity, we suppose that $h_i \ll -1$ in the initial Hamiltonian and $h_f \gg 1$ in the final Hamiltonian. The initial state in the momentum space k is prepared as the ground state of the Hamiltonian $H_{k,i}$. When parameter h in

the Hamiltonian, Eq. (1), is suddenly quenched from h_i to h_f ($H_{k,i}$ is suddenly changed to $H_{k,f}$ at the same time), the Loschmidt amplitude is $G_k(t) = \langle 0 | e^{-iH_{k,f}t} | 0 \rangle = e^{-i\epsilon_k t}$ as the initial state $|0\rangle$ is also the excited eigenstate of $H_{k,f}$ under $h_f \gg 1$. Obviously, $|G| = 1$ and it is never equal to zero for any k . Thus, there is no DQPT in this sudden quench.

For the slow quench, suppose the parameter h is time dependent as $h(t) = t/\tau$, where τ describes the quench rate. Since the quench will cross two critical points where the energy gap is closed, according to the KZM, there will be some complicated nonadiabatic evolution. However, the evolution is quite simple in the momentum space which can be described by N different Landau-Zener (LZ) processes. According to the LZ transition formula [39, 40], the transition probability from the initial ground state $|0\rangle$ to the final excited state can be written as $p_k = e^{-\pi \tau \sin^2 k}$ [41], which is strongly dependent on quench rate τ . After the slow quench, it continues to evolve under the Hamiltonian $H_{k,f}$ and the DQPT will appear if and only if $e^{-\pi \tau \sin^2 k} = \frac{1}{2}$, which leads to $\tau = \frac{\ln 2}{\pi \sin^2 k} > \frac{\ln 2}{\pi} \approx 0.22$ as k is a real number.

In order to detect the appearance of the DQPT in the quenches of the nontopological QTFIM system, we can define the topological quantity DTOP, which is very robust and convenient for experiments [42]. The definition of the DTOP is based on the Pancharatnam geometric phase (PGP) for each momentum k which is an extending concept of Berry's phase into acyclic and nonadiabatic dynamics. Generally, the Loschmidt amplitude is a complex number and its argument is a global phase which includes a dynamical part $\phi_{\text{dyn}}^k(t) = -\int_0^t \langle \psi(s) | H_{k,f} | \psi(s) \rangle ds$ and a PGP part $\phi_{G,k}(t)$ [42]. With the PGP for each momentum k , the topological quantity DTOP can be defined as

$$v_D(t) = \frac{1}{2\pi} \oint_0^\pi \frac{\partial \phi_{G,k}}{\partial k} dk \in \mathbb{Z}, \quad (3)$$

which is an integer and is robust in the experiment. The DQPT can be clearly detected by the discontinuity of the PGP and DTOP. Generally, the DTOP for the quenches crossing one critical point forms a stepwise increasing platform. However, the DTOP for quenches crossing two critical points may demonstrate oscillation between 0 and 1, which indicates that DQPTs successively appear at the discontinuous point. To investigate the quench dynamics of the QTFIM, we experimentally simulate the quenches in the momentum space k . For each k , the slow quench dynamics can be well described by an LZ process and a consecutive evolution. As a result, the whole quench dynamics of the QTFIM can be explored by the dynamics in a series of slow quenches in a qubit system. In the experiment, the LZ model is realized with a $^{171}\text{Yb}^+$ ion confined in a Paul trap [43], consisting of six needles placed on two perpendicular planes, as shown in Fig. 2(a). The hyperfine clock transition in the ground state $S_{1/2}$ manifold is chosen to realize the two-dimensional system (qubit), with energy levels denoted by $|0\rangle \equiv |F=0, m_F=0\rangle$ and $|1\rangle \equiv |F=1, m_F=0\rangle$, as shown in Fig. 2(b). In zero static magnetic field, the splitting between $|0\rangle$ and $|1\rangle$ is 12.642812 GHz. We applied a static magnetic field of 4.66 G to define the quantization axis, which changes the $|0\rangle$ to $|1\rangle$ resonance frequency to 12.642819 GHz. In order to manipulate the hyperfine qubit

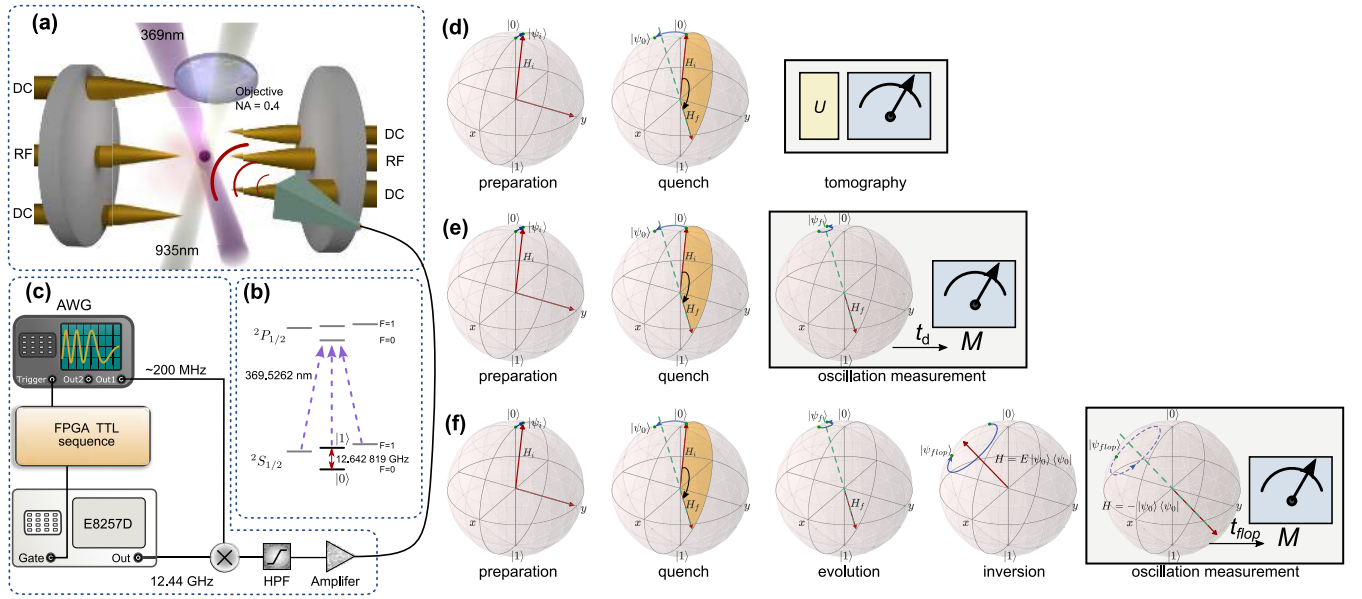


FIG. 2. The method to experimentally detect DQPTs. The dynamics of the QTFIM is decoupled into a series of Landau-Zener processes which can be simulated by a trapped $^{171}\text{Yb}^+$ ion. (a) The Paul trap used in the experiment: the distance between two rf needles is $200\ \mu\text{m}$. A 369-nm laser is applied for Doppler cooling, state initialization, and detection, and the microwave irradiation from a horn antenna is used to manipulate ion spin states. (b) Energy levels of $^{171}\text{Yb}^+$, $|0\rangle \equiv |F=0, m_F=0\rangle$ and $|1\rangle \equiv |F=1, m_F=0\rangle$ of hyperfine states of $S_{1/2}$ constitute the qubit levels in our experiments. (c) Microwave hardware configuration to drive the qubit: a programmed wave generated by the AWG is mixed with a 12.44-GHz wave, filtered with a high-pass filter (HPF), amplified, and sent to the antenna. (d)–(f) The scheme to detect PGP, which includes three stages.

with high control ability, coherent driving is implemented by a wave mixing method [see the scheme in Fig. 2(c)]. First, an arbitrary wave generator (AWG) is programmed to generate signals around 200 MHz. Then, the waveform is mixed with a 12.442819-GHz microwave (generated by Agilent, E8257D) by a frequency mixer. Afterward, a high-pass filter is used to remove the 12.242-GHz component and pass the 12.642-GHz wave, which is then amplified to 2 W and irradiated to the trapped ion by a horn antenna.

To measure the PGP obtained during the quench processes in the two-level subsystem with momentum k , a scheme that consists of three stages is applied in the experiment. The qubit state after quench and energy gap of $H_{k,f}$ are measured in the first and second stages, respectively. The PGPs are measured in the third stage. The three stages include eight steps:

(1) The qubit state is prepared to the ground state $|\psi_i\rangle$ of the initial Hamiltonian $H_{k,i}$ with $h = -10$.

(2) h is slowly quenched from -10 to 10 with quench rate τ . Meanwhile, the qubit state will evolve to the state $|\psi_0\rangle = a|\varphi_- \rangle + b|\varphi_+ \rangle$, where the coefficients a and b are strongly dependent on the quench rate, and $\{|\varphi_- \rangle, |\varphi_+ \rangle\}$ are the eigenstates of the final Hamiltonian $H_{k,f}$ with $h = 10$. The final state can be expressed in a density matrix $\rho_0 = |\psi_0\rangle\langle\psi_0|$.

(3) Measure the final state ρ_0 in step 2 by a qubit tomography method. From the diagonal element of ρ_0 we can read out the energy level occupancy $|a|^2$ and $|b|^2$.

(4) Prepare an initial state and drive it with $H_{k,f}$; by recording the bright state occupancy at different driven time t , a flopping curve can be acquired. By adjusting the initial state, a flopping curve with a clear contrast can be obtained.

(5) Repeat steps 1 and 2, which will prepare and then quench the state to another ρ_0 .

(6) Keep $H_{k,f}$ to drive the system for some dynamical time t_d to evolve the system to the state $|\psi_f\rangle$.

(7) Cancel out the dynamic phase of step 3, which is realized by driving the qubit under the Hamiltonian $H = E\rho_0$ for another t_d . The Hamiltonian is constructed from the measurement result of step 3, where $E = (|b|^2 - |a|^2)E_g/2$. The current state of the system is denoted as $|\psi_{\text{flop}}\rangle$.

(8) Finally, the qubit is driven under the Hamiltonian $H = E_0\rho_0$ and the Rabi flopping curve with different flopping time t_{flop} is recorded. Since the initial phase of the flopping curve does not depend on E_0 , we set it to $2\pi/t_R$ for convenience.

Steps 1–3 constitute the first stage, which is constructed to obtain the state ρ_0 to be used in the third stage. In the second stage (step 4), the energy gap E_g of $H_{k,f}$ can be obtained by fitting the period of curve, as shown in Fig. 3(a). The third stage includes four steps (steps 5–8); operations of these steps are shown on the Bloch sphere in Fig. 2(d).

Applying the above measurement scheme, a typical image of flopping curves is shown in Fig. 3(b) with $\tau = 0.22$ and $k = 0.49\pi$. Along the horizontal axis a clear stripe structure appears. According to the line at $t_d = 0.24t_R$ in this structure, a flopping curve is drawn as Fig. 3(c). The geometrical phase $\phi_G(k, t_d)$ at $(0.49\pi, 0.24t_R)$ is 6.13 ± 0.03 , which is obtained by fitting the corresponding flopping curve with the formula $A \sin(\Omega t_{\text{flop}} - \phi_G) + B$ [18], where the fitting parameters A , Ω , and B are the amplitude, Rabi frequency, and curve offset, respectively.

Under quench rate $\tau = 0.22$, the 50 subimages of the flopping curves, which correspond to different $k_l = \frac{l\pi}{100}$ with

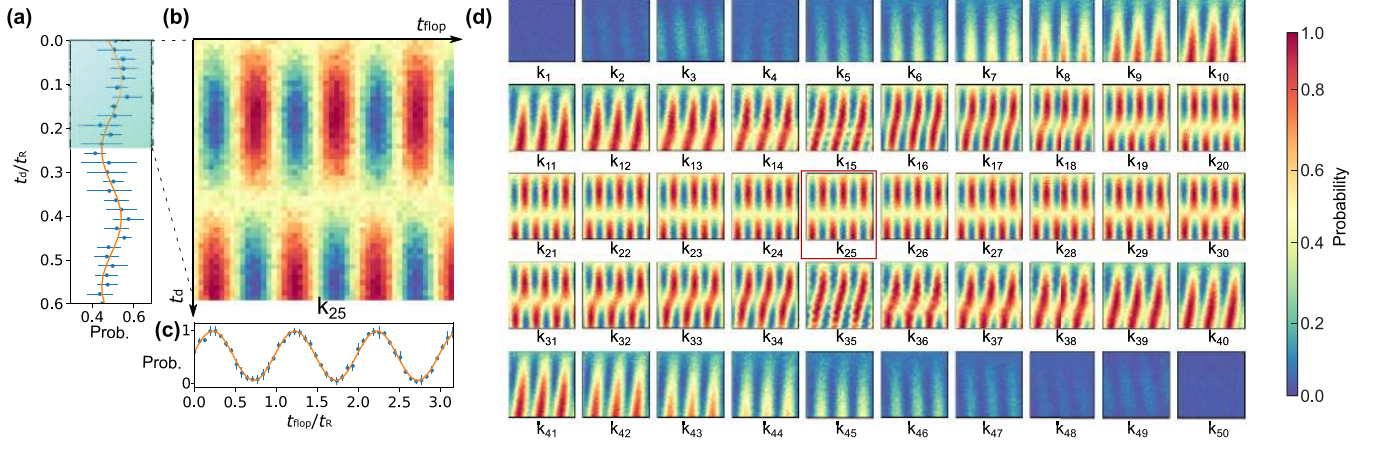


FIG. 3. Measured flopping curves at $\tau = 0.22$ with $N = 100$. (a) The bright state occupancy curve of k_{25} during step 4. The energy gap E_g is obtained by $2\pi/T_d$, where T_d is the period of flopping by data fitting. (b) The subimages corresponding to momentum k_{25} . The vertical axis is the evolution time t_d and the horizontal axis is the flopping time t_{flop} , with column $(t_d, t_{\text{flop}} = 0)$ corresponding to the data points in the shaded zone of (a). The dynamical evolution time t_d and flopping time t_{flop} are normalized with t_R , where $t_R = 120 \mu\text{s}$ is the resonant Rabi time in the experiment. (c) The flopping curve of momentum k_{25} at $t_d = 0.24 t_R$; the PGP ϕ_G for (k, t_d) can be obtained by fitting the corresponding flopping curve with a formula $A \sin(\Omega t_{\text{flop}} - \phi_G) + B$. (d) The array of 5×10 subimages corresponding to the flopping curve data of different momenta ($k_1, \dots, k_{50}$).

$l = 1, 3, \dots, 99$, are shown in Fig. 3(d). In the experiment, flopping curves with three different quench rates $\tau = 0.10, 0.22, 0.50$ are measured, respectively. The geometric phases ϕ_G are constructed and shown in Fig. 4. It depicts the evolution of the PGP with different k in the Brillouin zone under different quench rates. There are two π -jumps of PGP during the dynamical evolution for both $\tau = 0.22$ and $\tau = 0.50$ at some time t_d , which means that two DQPTs take place. For the quench rate $\tau = 0.10$, which is below 0.22, no feature of DQPT is observed.

The existence of the DQPT can be explicitly observed by the discontinuity of the DTOP obtained by the measured

PGP. As shown in Fig. 4(g), the DTOP has its first jump at the dynamical evolution step $t_d \approx 0.16 t_R$ with quench rate $\tau = 0.22$, which can be regarded as a sign of DQPT. Then the DTOP jumps back to zero immediately. It means another DQPT occurs. The interval of two DQPTs is dependent on the distance of the two critical momenta k_c . For the quench rate $\tau = 0.50$, which is shown in Fig. 4(h), two DQPTs are clearly shown and a constant DTOP presents between them. If the quench is faster, for instance, $\tau = 0.10$, no DQPT occurs and we cannot see any changes of the DTOP.

Another fascinating prediction made in [30] is the oscillatory behavior of the DTOP when quenches cross two

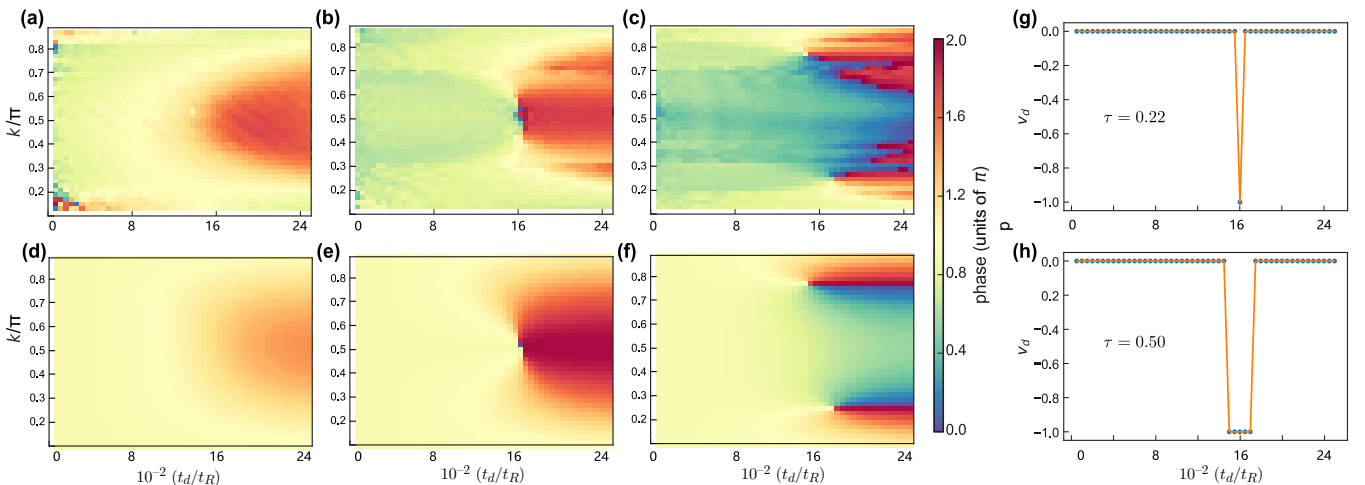


FIG. 4. Pancharatnam geometric phase maps for different evolution time t_d (normalized with Rabi time t_R) and different momentum k and DTOP. Experimentally measured PGP maps for quench rate (a) $\tau = 0.10$, (b) $\tau = 0.22$, and (c) $\tau = 0.50$. Theoretical PGP maps for quench rate (d) $\tau = 0.10$, (e) $\tau = 0.22$, and (f) $\tau = 0.50$. Note, a phase wrapping algorithm is used to format the measured data to the $[0, 2\pi)$ range; when the theoretical data value is near zero or 2π , a small error of measured data can cause a 2π phase jump through the wrapping algorithm, which leads to a mismatch illusion between theoretical and experimental results. DTOPs for (g) $\tau = 0.22$ and (h) $\tau = 0.50$ with experimental (blue dots) and theoretical (orange lines) results, respectively.

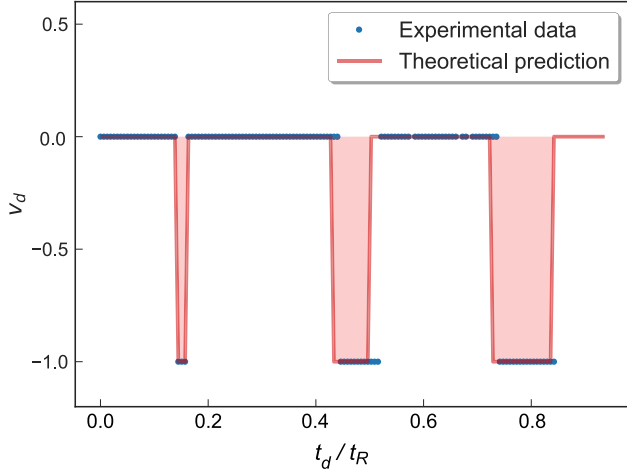


FIG. 5. The theoretical prediction (red line) of DTOP v_d for $\tau = 0.50$ and the corresponding experimental result (blue dots). Their oscillation over evolution time t_d is highly consistent except for a slight time misalignment caused by the Rabi time flutter during the experiment.

quantum critical points. In this situation, there are two DQPT types, corresponding to two critical quasimomenta k_1^* and k_2^* . For each critical momentum k^* , there is a series of DQPT that occurs at critical time $t^* = \frac{\pi}{2\epsilon_{k^*}}$ with period $2t^*$, and the critical times are $t_{n,\pm}^* = t_{\pm}^*(2n+1)$, $n = 0, 1, 2, \dots$, in our case ($\pm$ corresponding to two critical momenta). More importantly, the geometry phases corresponding to k_1^* and k_2^* can be canceled when both of the DQPT types happen. As a result, platforms and oscillation formed. To demonstrate the oscillation phenomenon, we dramatically improved the simulation time in our setup. As shown in Fig. 5 with quench rate $\tau = 0.50$, the DTOP oscillated and formed three platforms with a width ratio 1:3:5, consistent with the theoretical prediction.

III. CONCLUSION

In summary, we have demonstrated an experimental simulation of a 1D QTFIM going through a slow quench scheme. We observed the relationship between the quench rate and the appearance of the DQPT through the discontinuity of the PGP and DTOP. We found that if the quench rate $\tau > 0.22$, there are always two DQPTs happening even though there is no DQPT appearing in the corresponding sudden quenches. We also observed the oscillation of the DTOP when the quench crosses two critical points. Our experiment contributes a method to study DQPTs by detecting the Pancharatnam geometric phase in momentum space which can be extended in more general models [44] and it opens the way to investigate more complicated behavior of the nonequilibrium dynamics.

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APPENDIX A: MAPPING BETWEEN SPIN AND FREE-FERMION REPRESENTATIONS IN THE ISING MODEL

We consider the 1D quantum transverse field Ising model (QTFIM) with a magnetic field h . The QTFIM Hamiltonian is given by

$$H = -J \sum_{m=1}^N (\hat{\sigma}_m^z \hat{\sigma}_{m+1}^z + h \hat{\sigma}_m^x), \quad (\text{A1})$$

where J is the hopping parameter and $\hat{\sigma}_m^\alpha$ is the Pauli matrix along the $\alpha = x, z$ direction at site m . Here we conveniently assume the system size N is even and impose periodic boundary conditions $\hat{\sigma}_{N+1}^z = \hat{\sigma}_1^z$. The Hamiltonian is mapped to a free-fermion model through the Jordan-Wigner transformation [45]

$$\hat{\sigma}_m^z = -(\hat{c}_m^\dagger + \hat{c}_m) \prod_{m' < m} (1 - 2\hat{c}_m^\dagger \hat{c}_{m'}), \quad \hat{\sigma}_m^x = 1 - 2\hat{c}_m^\dagger \hat{c}_m. \quad (\text{A2})$$

Inserting Eq. (A2) into the Ising Hamiltonian [Eq. (A1)], we find that the Hamiltonian in quadratic form using the Fermi operators is

$$H = -J \left[\sum_{m=1}^{N-1} \hat{c}_m^\dagger \hat{c}_{m+1} + \hat{c}_{m+1}^\dagger \hat{c}_m + \hat{c}_m \hat{c}_{m+1} + \hat{c}_m^\dagger \hat{c}_{m+1}^\dagger \right] + h \sum_{m=1}^N (1 - 2\hat{c}_m^\dagger \hat{c}_m) + \hat{\Pi} (\hat{c}_N^\dagger \hat{c}_1 + \hat{c}_N \hat{c}_1 + \hat{c}_1^\dagger \hat{c}_N + \hat{c}_1^\dagger \hat{c}_N^\dagger). \quad (\text{A3})$$

This Hamiltonian includes terms such as $\hat{c}_m \hat{c}_{m+1}$ or $\hat{c}_{m+1}^\dagger \hat{c}_m^\dagger$ that violate the fermion conservation. However, the parity operator $\hat{\Pi} = (-1)^{\sum_{m=1}^N \hat{c}_m^\dagger \hat{c}_m}$ is a constant of motion having the values $+1$ and -1 . We expand $\hat{c}_m$ into the Fourier modes

$$\hat{c}_m = \frac{e^{-i\pi/4}}{\sqrt{N}} \sum_k \exp[im(ka)] \hat{c}_k, \quad (\text{A4})$$

where the pseudomomenta k take values

$$k = \frac{\pi}{Na} (2j-1), \quad j = -\frac{N}{2} + 1, \dots, \frac{N}{2}. \quad (\text{A5})$$

Using the choice of even N and considering the antiperiodic boundary condition $\hat{c}_{N+1} = -\hat{c}_N$, the Hamiltonian maps to

$$H_{\text{Even}} = - \sum_{0 < k < \frac{\pi}{a}} (\hat{c}_k^\dagger \hat{c}_{-k}) H_k \begin{pmatrix} \hat{c}_k \\ \hat{c}_{-k}^\dagger \end{pmatrix}, \quad (\text{A6})$$

where H_k is defined by

$$H_k \equiv 2J(h - \cos(ka))\hat{\sigma}^z + 2J \sin(ka)\hat{\sigma}^x. \quad (\text{A7})$$

As this mapping involves pseudomomenta k , we can treat the mapping space as momentum space. Next, we use the Bogoliubov transformation to map into a new set of fermionic operators $\hat{\gamma}_k$ whose number is conserved. These new operators are defined by a unitary transformation on the pair $\hat{c}_k, \hat{c}_{-k}^\dagger$:

$$\hat{c}_k = u_k \hat{\gamma}_k + v_{-k}^* \hat{\gamma}_{-k}^\dagger,$$

where u_k, v_k are numbers satisfying $u_k^2 + v_k^2 = 1$, $u_{-k} = u_k$, and $v_{-k} = -v_k$. It can be checked that the canonical fermion anticommutation relation for the $\hat{c}_k$ implies that the same relations are also satisfied by $\hat{\gamma}_k$; that is,

$$\{\hat{\gamma}_k, \hat{\gamma}_{k'}^\dagger\} = \delta_{k,k'}, \quad \{\hat{\gamma}_k^\dagger, \hat{\gamma}_{k'}^\dagger\} = \{\hat{\gamma}_k, \hat{\gamma}_{k'}\} = 0.$$

The Hamiltonian can be diagonalized in terms of $\hat{\gamma}_k$,

$$H_{\text{Even}} = \sum_{0 < k < \frac{\pi}{a}} \epsilon_k(h) (\hat{\gamma}_k^\dagger \hat{\gamma}_k + \hat{\gamma}_{-k}^\dagger \hat{\gamma}_{-k} - 1), \quad (\text{A8})$$

where $\epsilon_k(h) = 2J\sqrt{(h - \cos(ka))^2 + \sin^2(ka)}$, with the ground state defined $\hat{\gamma}_k|\mathbf{0}\rangle = 0$.

The Bogoliubov transformation is time dependent, and the Bogoliubov modes $[u_k(t), v_k(t)]$ must satisfy the Heisenberg equation $i\hbar\dot{\hat{c}}_k = [\hat{c}_k, \hat{H}_{\text{Even}}]$, which derives that the evolution of u_k and v_k is in a Landau-Zener form from [46]

$$i\hbar \frac{d}{dt} \begin{bmatrix} v_k \\ \mu_k \end{bmatrix} = 2J \begin{bmatrix} h(t) - \cos(ka) & \sin(ka) \\ \sin(ka) & -h(t) + \cos(ka) \end{bmatrix} \begin{bmatrix} v_k \\ \mu_k \end{bmatrix}. \quad (\text{A9})$$

APPENDIX B: MEASURING THE GEOMETRIC PHASE

We suppose the system is initially quenched to an initial state, and then it continues to evolve to a final state at time t . The Loschmidt amplitude (LA) between the two states is defined as

$$G(t) = \langle \psi_0 | \psi_t \rangle,$$

where $|\psi_0\rangle$ is the initial quenched state and $|\psi_t\rangle$ is the evolved quenched state at time t after the end of the quench. Since the two-dimensional subsystems in momentum space are uncoupled, the LA can be written as

$$G = \prod_k G_k = \prod_k \langle \psi_{k,0} | \psi_{k,t} \rangle.$$

The phase of LA will have a π jump when the DQPT occurs. To measure this phase experimentally, we rewrite the LA. Assuming the final two-dimensional Hamiltonian has two eigenstates $|\varphi_{k-}\rangle, |\varphi_{k+}\rangle$ with eigenvalue $\pm\epsilon_k$, then we obtain

$$|\psi_{k,0}\rangle = u_k |\varphi_{k-}\rangle + v_k |\varphi_{k+}\rangle,$$

$$|\psi_{k,t}\rangle = e^{-iH_{k,t}t} |\psi_{k,t}\rangle = u_k e^{i\epsilon_k t} |\varphi_{k-}\rangle + v_k e^{-i\epsilon_k t} |\varphi_{k+}\rangle,$$

and

$$G_k(t) = |u_k|^2 e^{i\epsilon_k t} + |v_k|^2 e^{-i\epsilon_k t}.$$

It is worth noting that the phase of $G_k(t)$ includes both geometrical and dynamical parts. To obtain the pure geometrical

phase, we deducted the dynamical part:

$$\phi_{G,k,t} = \angle(G_k(t)) - (|v_k|^2 - |u_k|^2)\epsilon_k t.$$

To measure the pure geometrical phase experimentally, we designed a method to deduct the dynamical phase. The basic idea of the method is to evolve the system under a designated Hamiltonian, which will generate a unitary operator to cancel the dynamical part. Then the LA contains only the pure geometrical phase:

$$G_{k,G}(t) = e^{-i(|v_k|^2 - |u_k|^2)\epsilon_k t} G_k(t).$$

Letting $E = (|v_k|^2 - |u_k|^2)\epsilon_k$ and defining $H_c = E|\psi_{k,0}\rangle\langle\psi_{k,0}|$, it can be deduced that

$$\begin{aligned} G_{k,G}(t) &= e^{-iEt} \langle \psi_{k,0} | \psi_{k,t} \rangle = \langle \psi_{k,0} | e^{-iEt} | \psi_{k,t} \rangle \\ &= \langle \psi_{k,0} | e^{-iH_c t} | \psi_{k,t} \rangle = \langle \psi_{k,0} | \psi_{g,k,t} \rangle, \end{aligned}$$

where $|\psi_{g,k,t}\rangle = e^{-iH_c t} |\psi_{k,t}\rangle$. By performing an extra unitary operator, the dynamical phase can be canceled.

To construct the Hamiltonian $H_c = E|\psi_{k,0}\rangle\langle\psi_{k,0}|$, we can first measure the initial quenched state $\rho_0 = |\psi_{k,0}\rangle\langle\psi_{k,0}|$ by a quantum tomography method, which we further introduce in Appendix E. To measure the phase of $G_{k,G}(t)$, we can use the time-resolved state tomography method [18]. As the evolved quenched state $|\psi_{k,t}\rangle$ can be decomposed,

$$|\psi_{k,t}\rangle = G_k(t) |\psi_{k,0}\rangle - 2iu_k v_k \sin(\epsilon_k t) |\psi_{k,0}^\perp\rangle,$$

where $|\psi_{k,0}^\perp\rangle = v_k^* |\varphi_{k-}\rangle - u_k^* |\varphi_{k+}\rangle$ is the orthogonal state of $|\psi_{k,0}\rangle$. If we continue the evolution under H_c after the dynamical phase is canceled, the probability of the excited state is

$$\begin{aligned} &\langle \varphi_{k+} | \psi_{g,k,t}, t' \rangle \langle \psi_{g,k,t}, t' | \varphi_{k+} \rangle \\ &= |v_k|^2 (|G_k|^2 + 4|u_k|^4 \sin^2(\epsilon_k t) \\ &\quad + 4|G_k||u_k|^2 \sin(\epsilon_k t) \sin(Et' + \phi_{G,k,t})), \end{aligned} \quad (\text{B1})$$

where t' is the additional time used to drive after the evolved quenched state. By recording the excitation probability of the final states at different t' , we can get the phase by fitting the recorded data with

$$P(t')_{k,t} = A_{k,t} \sin(2\pi t'/T + \phi_{G,k,t}) + B_{k,t}.$$

Applying the measurement scheme in the main text, the flopping curves can be measured with different momentum k and dynamical evolution time t_d . Then a fast Fourier transform is performed on the experimental data to analyze the geometrical phase $\phi_{G,k,t}$. The biggest challenge is that the amplitude $A_{k,t}$ will be negative and cause an extra π phase if we ignore the sign of it. With the help of the phase continuity in k -space, the sign of each amplitude can be uniquely determined. We use this method to solve the extra π -phase problem.

APPENDIX C: DQPT IN MOMENTUM SPACE

Supposing the initial state in the two-dimensional subsystem with momentum k is

$$|\psi_{k,0}\rangle = u_k |\varphi_{k-}\rangle + v_k |\varphi_{k+}\rangle, \quad (\text{C1})$$

where $|u_k|^2 + |v_k|^2 = 1$ and $|\varphi_{k-}\rangle, |\varphi_{k+}\rangle$ is the ground (excited) state of the final Hamiltonian $H_{k,f}$. Then, the evolution

under the Hamiltonian $H_{k,f}$ only adds some time-dependent phases into each basis, and thus we can get the Loschmidt amplitude as

$$G_k(t) = |u_k|^2 e^{i\epsilon_k t} + |v_k|^2 e^{-i\epsilon_k t}, \quad (\text{C2})$$

where $\pm\epsilon_k$ are the eigenvalues of $H_{k,f}$. The Fisher zeros of the Loschmidt amplitude can be further found as

$$t_n(k) = \frac{1}{2\epsilon_k} [-i \ln \left(\frac{|v_k|^2}{|u_k|^2} \right) + (2n+1)\pi]. \quad (\text{C3})$$

Obviously, t_n is a real number if and only if $|u_k|^2 = |v_k|^2 = \frac{1}{2}$. In another words, when the former condition is satisfied, the DQPT will appear.

For the sudden quench, the condition is straightforward and only dependent on the initial state. However, for a slow quench, there is a nonadiabatic evolution across the phase boundary, where the transition between the ground state and the excited state will happen. As a result, the appearance of the DQPT is dependent on the quench rate τ . According to the LZ transition formula [39,40], the transition probability from the ground state to the excited state during the slow quench, from $H_{k,i}$ to $H_{k,f}$, can be written as

$$p_k = |u_k|^2 = e^{-\pi\tau \sin^2 k}. \quad (\text{C4})$$

As mentioned before, if the DQPT appears in the following evolution (governed by $H_{k,f}$), the amplitude of the excited state satisfies $e^{-\pi\tau \sin^2 k} = \frac{1}{2}$, which means $\tau > \frac{\ln 2}{\pi} \approx 0.22$.

In the sudden quench, there is no DQPT. The ground state for $H_{i,f}$ is

$$|g_{i,f}\rangle_k = \frac{\sin k}{\sqrt{2\epsilon_{k(i,f)}^2 - 2(h_{i,f} - \cos k)\epsilon_{k(i,f)}}} |\varphi_{k,-}\rangle + i \frac{h_{i,f} - \cos k - \epsilon_{k(i,f)}}{\sqrt{2\epsilon_{k(i,f)}^2 - 2(h_{i,f} - \cos k)\epsilon_{k(i,f)}}} |\varphi_{k,+}\rangle. \quad (\text{C5})$$

After the sudden quench the excitation probability is $1 - |v_k|^2$, where v_k is

$$v_k = \frac{\sin^2 k + [h_i - \cos k - \epsilon_{k,i}][h_f - \cos k - \epsilon_{k,f}]}{2\sqrt{[\epsilon_{k,i}^2 - (h_i - \cos k)\epsilon_{k,i}][\epsilon_{k,f}^2 - (h_f - \cos k)\epsilon_{k,f}]}}, \quad (\text{C6})$$

Under the condition $h_i = -h_f$, v_k take the maximum when $k = \pi/2$, and $v_{\max} = \frac{1}{\epsilon_{\frac{\pi}{2}}} < \frac{\sqrt{2}}{2}$, which means the excitation probability is always larger than $\frac{1}{2}$. Furthermore, a larger $|h_{i,f}|$ will lead to higher excitation probability. Therefore, in a sudden quench from one para-magnetic phase to another paramagnetic phase, the DQPT will never take place.

However, if the quench is slow enough, for $k \neq 0, \pi$ the subsystem can evolve adiabatically. According to the continuity of the excitation probability with quench rate, there exists a specific τ_c that for $\tau > \tau_c$, the DQPT can appear, i.e., for some $|u_k|^2$ equal $\frac{1}{2}$.

For large h and $\tau > \frac{\ln 2}{\pi}$, there are always two critical momenta k^* and $\pi - k^*$, where

$$\sin k^* = \sqrt{\frac{\ln 2}{\pi\tau}}. \quad (\text{C7})$$

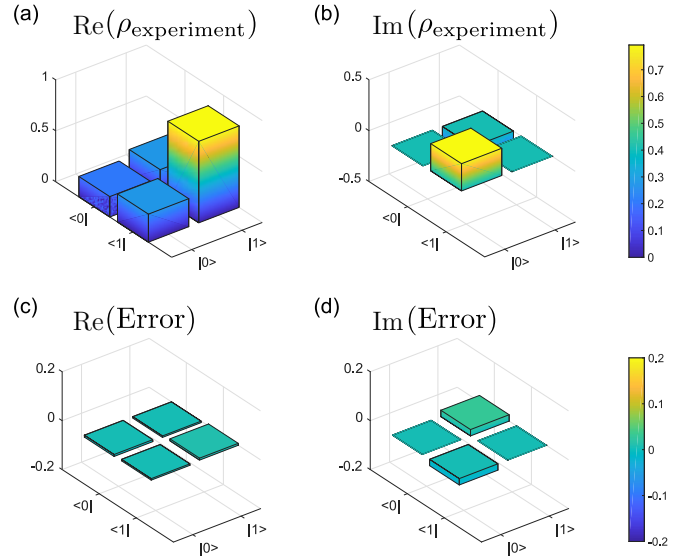


FIG. 6. A typical tomography result for the initial quenched state at ($\tau = 0.22$, $k = 0.66$). (a) and (b) represent the real part of the result; (c) and (d) represent the imaginary part of the result. The preparation fidelity is calculated as 99.4% from the tomography result.

Those momenta correspond to two critical times of the DQPTs:

$$t_+^* = \frac{\pi}{2\epsilon_{k^*}}, \quad t_-^* = \frac{\pi}{2\epsilon_{\pi-k^*}}. \quad (\text{C8})$$

APPENDIX D: DRIVING WAVEFORM OF AWG

We suppose the driving microwave from the AWG is $A \cos(\omega_c t + \Phi(t))$, where A is the amplitude, $f_c = \omega_c/2\pi$ is the carrier frequency, and $\Phi(t)$ is an arbitrary phase function. After the frequency mixer with a local oscillator, the waveform is

$$A \cos(\omega_c t + \Phi(t)) \cos(\omega_o t) = A/2 (\cos((\omega_o + \omega_c)t + \Phi(t)) + \cos((\omega_o - \omega_c)t - \Phi(t))).$$

When the wave passes through the high-pass filter, the waveform becomes $A/2 \cos((\omega_o + \omega_c)t + \Phi(t))$. In the experiment, we choose the $\omega_o + \omega_c$ to resonate with the qubit transition $\omega_Q = \omega_o + \omega_c$, so that the final driving waveform is $\cos(\omega_Q t + \Phi(t))$. In this condition, the Hamiltonian of the driving qubit after the rotating wave approximation is

$$H(t) = -\frac{\hbar}{2} [\Omega_R |1\rangle\langle 0| e^{i\Phi(t)} + \Omega_R^* |0\rangle\langle 1| e^{-i\Phi(t)}], \quad (\text{D1})$$

where Ω_R is the Rabi frequency of the driven qubit. Theoretically, we need to start a quench process from infinitely negative time, which is unrealizable experimentally. Actually, we quench the state from a finite negative time with a relative large detuning $-\Delta_0$ with $\Delta_0 \gg \Omega_R$, as a substantially equivalent method. In this circumstance, the state should be prepared on the ground state of the Hamiltonian with large detuning before the quench process, which can be realized by rotating $|0\rangle$ along a vector in the equatorial plane. The phase function $\Phi(t)$ corresponds to the azimuthal angle of the rotation axis

on the Bloch sphere. The Hamiltonian in Eq. (D1) represents a qubit rotation operation when the $\Phi(t)$ of a pulse is a constant. The pulse length ($t = \theta/\Omega_R$) for preparation is determined by the polar angle θ of the ground state. In the quench stage, $\Phi(t)$ changes with some quench rate v : $\Phi(t) = \frac{1}{2}vt^2$. The composited $\Phi(t)$ in Eq. (D1) with the above preparation and quench processes can be derived in a piecewise function

$$\Phi(t) = \begin{cases} \Phi_1 + \frac{\pi}{2} & (t_0, t_1) \\ \frac{1}{2}vt^2 & (t_1, t_2), \end{cases}$$

where $t_0 = t_1 - (\pi - \theta_1)/\Omega_R$, $t_1 = -\Delta_0/v$, $\theta_1 = \arctan \frac{\Omega_R}{v t_1}$, and $\Phi_1 = \frac{1}{2}v t_1^2$.

APPENDIX E: QUANTUM STATE TOMOGRAPHY

The state of the qubit is determined by a destructive readout through the resonant fluorescence and reconstructed using state tomography. Any single-qubit density matrix $\hat{\rho}$ can be uniquely represented by four parameters $\{S_0, S_1, S_2, S_3\}$:

$$\hat{\rho} = \frac{1}{2} \sum_{i=0}^3 S_i \hat{\sigma}_i, \quad (\text{E1})$$

where the $\hat{\sigma}_i$ matrices are

$$\begin{aligned} \hat{\sigma}_0 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \hat{\sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\ \hat{\sigma}_2 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \hat{\sigma}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \end{aligned} \quad (\text{E2})$$

and the S_i values are given by $S_i = \text{Tr}\{\hat{\sigma}_i \hat{\rho}\}$. We construct these four parameters according to specific pairs of projective measurements:

$$\begin{aligned} S_0 &= P_{|0\rangle} + P_{|1\rangle}, \\ S_1 &= P_{|X\rangle_+} - P_{|X\rangle_-}, \\ S_2 &= P_{|Y\rangle_+} - P_{|Y\rangle_-}, \\ S_3 &= P_{|0\rangle} - P_{|1\rangle}, \end{aligned} \quad (\text{E3})$$

where $P_{|\psi\rangle}$ is the measured probability that the state is in $|\psi\rangle$, $|X\rangle_{\pm} = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$, and $|Y\rangle_{\pm} = \frac{1}{\sqrt{2}}(|0\rangle \pm i|1\rangle)$. Then the density matrix of the final state can be reconstructed according to Eq. (E1). We tested the state ψ_0 for all parameters, and confirmed that the preparation fidelity of ψ_0 is above 99%. For convenience, we show the tomography result of the initial quenched state at ($\tau = 0.22$, $k = 0.66$) in Fig. 6.

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- [1] I. Bloch, J. Dalibard, and S. Nascimbene, *Nat. Phys.* **8**, 267 (2012).
 - [2] C. Gross and I. Bloch, *Science* **357**, 995 (2017).
 - [3] K. Kim, M.-S. Chang, S. Korenblit, R. Islam, E. E. Edwards, J. K. Freericks, G.-D. Lin, L.-M. Duan, and C. Monroe, *Nature* **465**, 590 (2010).
 - [4] B. P. Lanyon, C. Hempel, D. Nigg, M. Müller, R. Gerritsma, F. Zähringer, P. Schindler, J. T. Barreiro, M. Rambach, G. Kirchmair, M. Hennrich, P. Zoller, R. Blatt, and C. F. Roos, *Science* **334**, 57 (2011).
 - [5] R. Blatt and C. F. Roos, *Nat. Phys.* **8**, 277 (2012).
 - [6] J. Zhang, G. Pagano, P. W. Hess, A. Kyprianidis, P. Becker, H. Kaplan, A. V. Gorshkov, Z.-X. Gong, and C. Monroe, *Nature* **551**, 601 (2017).
 - [7] M. Eckstein, M. Kollar, and P. Werner, *Phys. Rev. Lett.* **103**, 056403 (2009).
 - [8] B. Sciolla and G. Biroli, *Phys. Rev. Lett.* **105**, 220401 (2010).
 - [9] P. Calabrese, F. H. L. Essler, and M. Fagotti, *Phys. Rev. Lett.* **106**, 227203 (2011).
 - [10] W. H. Zurek, *Nature* **317**, 505 (1985).
 - [11] M. Heyl, *Rep. Prog. Phys.* **81**, 054001 (2018).
 - [12] J. Lang, B. Frank, and J. C. Halimeh, *Phys. Rev. Lett.* **121**, 130603 (2018).
 - [13] M. Heyl, A. Polkovnikov, and S. Kehrein, *Phys. Rev. Lett.* **110**, 135704 (2013).
 - [14] S. Sachdev, *Quantum Phase Transitions* (Cambridge University Press, Cambridge, UK, 2011).
 - [15] B. Žunkovič, M. Heyl, M. Knap, and A. Silva, *Phys. Rev. Lett.* **120**, 130601 (2018).
 - [16] S. Vajna and B. Dóra, *Phys. Rev. B* **89**, 161105(R) (2014).
 - [17] T. Tian, Y. Ke, L. Zhang, S. Lin, Z. Shi, P. Huang, C. Lee, and J. Du, *Phys. Rev. B* **100**, 024310 (2019).
 - [18] N. Fläschner, D. Vogel, M. Tarnowski, B. Rem, D.-S. Lühmann, M. Heyl, J. Budich, L. Mathey, K. Sengstock, and C. Weitenberg, *Nat. Phys.* **14**, 265 (2018).
 - [19] X.-Y. Guo, C. Yang, Y. Zeng, Y. Peng, H.-K. Li, H. Deng, Y.-R. Jin, S. Chen, D. Zheng, and H. Fan, *Phys. Rev. Appl.* **11**, 044080 (2019).
 - [20] S. Sharma, S. Suzuki, and A. Dutta, *Phys. Rev. B* **92**, 104306 (2015).
 - [21] P. Jurcevic, H. Shen, P. Hauke, C. Maier, T. Brydges, C. Hempel, B. P. Lanyon, M. Heyl, R. Blatt, and C. F. Roos, *Phys. Rev. Lett.* **119**, 080501 (2017).
 - [22] T. W. Kibble, *J. Phys. A: Math. Gen.* **9**, 1387 (1976).
 - [23] V. Ruutu, V. Eltsov, A. Gill, T. Kibble, M. Krusius, Y. G. Makhlin, B. Placais, G. Volovik, and W. Xu, *Nature* **382**, 334 (1996).
 - [24] S. Ulm, J. Roßnagel, G. Jacob, C. Degünther, S. Dawkins, U. Poschinger, R. Nigmatullin, A. Retzker, M. Plenio, F. Schmidt-Kaler *et al.*, *Nat. Commun.* **4**, 2290 (2013).
 - [25] A. Polkovnikov, *Phys. Rev. B* **72**, 161201(R) (2005).
 - [26] C. Bäuerle, Y. M. Bunkov, S. Fisher, H. Godfrin, and G. Pickett, *Nature* **382**, 332 (1996).
 - [27] J.-M. Cui, Y.-F. Huang, Z. Wang, D.-Y. Cao, J. Wang, W.-M. Lv, L. Luo, A. d. Campo, Y.-J. Han, C.-F. Li, and G.-C. Guo, *Sci. Rep.* **6**, srep33381 (2016).
 - [28] X.-Y. Xu, Y.-J. Han, K. Sun, J.-S. Xu, J.-S. Tang, C.-F. Li, and G.-C. Guo, *Phys. Rev. Lett.* **112**, 035701 (2014).
 - [29] W. H. Zurek, U. Dorner, and P. Zoller, *Phys. Rev. Lett.* **95**, 105701 (2005).
 - [30] S. Sharma, U. Divakaran, A. Polkovnikov, and A. Dutta, *Phys. Rev. B* **93**, 144306 (2016).
 - [31] X.-Y. Xu, Q.-Q. Wang, S.-J. Tao, W.-W. Pan, Z. Chen, M. Jan, Y.-T. Zhan, K. Sun, J.-S. Xu, Y.-J. Han, C.-F. Li, and G.-C. Guo, *Phys. Rev. Research* **1**, 033039 (2019).

- [32] X.-Y. Xu, Q.-Q. Wang, M. Heyl, J. C. Budich, W.-W. Pan, Z. Chen, M. Jan, K. Sun, J.-S. Xu, Y.-J. Han *et al.*, [Light Sci. Appl.](#) **9**, 7 (2020).
- [33] B. Damski, [Phys. Rev. Lett.](#) **95**, 035701 (2005).
- [34] F. Iglói and H. Rieger, [Phys. Rev. Lett.](#) **85**, 3233 (2000).
- [35] A. Silva, [Phys. Rev. Lett.](#) **101**, 120603 (2008).
- [36] D. Rossini, A. Silva, G. Mussardo, and G. E. Santoro, [Phys. Rev. Lett.](#) **102**, 127204 (2009).
- [37] L. Foini, L. F. Cugliandolo, and A. Gambassi, [Phys. Rev. B](#) **84**, 212404 (2011).
- [38] F. Andraschko and J. Sirker, [Phys. Rev. B](#) **89**, 125120 (2014).
- [39] C. Zener, [Proc. R. Soc. London Ser. A](#) **137**, 696 (1932).
- [40] L. D. Landau, [Phys. Z. Sowjetunion](#) **2**, 46 (1932).
- [41] C. Wittig, [J. Phys. Chem. B](#) **109**, 8428 (2005).
- [42] J. C. Budich and M. Heyl, [Phys. Rev. B](#) **93**, 085416 (2016).
- [43] S. Olmschenk, K. C. Younge, D. L. Moehring, D. N. Matsukevich, P. Maunz, and C. Monroe, [Phys. Rev. A](#) **76**, 052314 (2007).
- [44] F. Franchini, *An Introduction to Integrable Techniques for One-Dimensional Quantum Systems* (Springer, Berlin, 2017), Vol. 940.
- [45] E. Barouch and B. M. McCoy, [Phys. Rev. A](#) **3**, 786 (1971).
- [46] J. Dziarmaga, [Phys. Rev. Lett.](#) **95**, 245701 (2005).