

Polarization nondegenerate fiber Fabry-Perot cavities with large tunable splittings

Jin-Ming Cui, Kun Zhou, Ming-Shu Zhao, Ming-Zhong Ai, Chang-Kang Hu, Qiang Li, Bi-Heng Liu, Jin-Lan Peng, Yun-Feng Huang, Chuan-Feng Li, and Guang-Can Guo

Citation: [Appl. Phys. Lett.](#) **112**, 171105 (2018); doi: 10.1063/1.5024798

View online: <https://doi.org/10.1063/1.5024798>

View Table of Contents: <http://aip.scitation.org/toc/apl/112/17>

Published by the [American Institute of Physics](#)

Articles you may be interested in

[Generating structured light with phase helix and intensity helix using reflection-enhanced plasmonic metasurface at 2 \$\mu\text{m}\$](#)

Applied Physics Letters **112**, 171103 (2018); 10.1063/1.5024433

[Multi-object investigation using two-wavelength phase-shift interferometry guided by an optical frequency comb](#)

Applied Physics Letters **112**, 171101 (2018); 10.1063/1.5024244

[Nonlinear frequency doubling characteristics of asymmetric vortices of tunable, broad orbital angular momentum spectrum](#)

Applied Physics Letters **112**, 171102 (2018); 10.1063/1.5024445

[Finite-key analysis for the 1-decoy state QKD protocol](#)

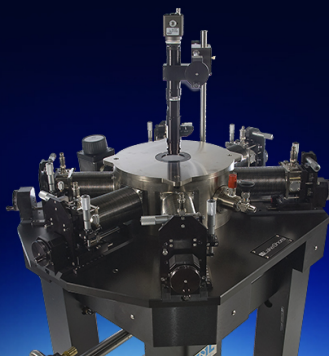
Applied Physics Letters **112**, 171104 (2018); 10.1063/1.5023340

[Surface plasmon dispersion in a mid-infrared Ge/Si quantum dot photodetector coupled with a perforated gold metasurface](#)

Applied Physics Letters **112**, 171107 (2018); 10.1063/1.5029289

[Experimental demonstration of the optical Helmholtz resonance](#)

Applied Physics Letters **112**, 171110 (2018); 10.1063/1.5028256



Cryogenic probe stations
for accurate, repeatable
material measurements

LEARN MORE

Polarization nondegenerate fiber Fabry-Perot cavities with large tunable splittings

Jin-Ming Cui,^{1,2} Kun Zhou,^{1,2} Ming-Shu Zhao,^{1,2} Ming-Zhong Ai,^{1,2} Chang-Kang Hu,^{1,2} Qiang Li,^{1,2} Bi-Heng Liu,^{1,2} Jin-Lan Peng,³ Yun-Feng Huang,^{1,2,a)} Chuan-Feng Li,^{1,2,b)} and Guang-Can Guo^{1,2}

¹Key Laboratory of Quantum Information, Chinese Academy of Sciences, University of Science and Technology of China, Hefei 230026, People's Republic of China

²Synergetic Innovation Center of Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China

³Center for Micro and Nanoscale Research and Fabrication, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China

(Received 4 February 2018; accepted 11 April 2018; published online 23 April 2018)

We demonstrate a type of microcavity with large tunable splitting of polarization modes. This polarization nondegenerate cavity consists of two ellipsoidal concave mirrors with controllable eccentricity by CO₂ laser machining on fiber end facets. The experiment shows that the cavities can combine the advantages of high finesse above 10⁴ and large tunable polarization mode splitting to the GHz range. As the splitting of the cavity can be finely controlled to match atom hyperfine levels or optomechanics phonons, it will blaze a way in experiments on cavity quantum electrodynamics and cavity optomechanics. *Published by AIP Publishing.* <https://doi.org/10.1063/1.5024798>

As an import degree of light, the polarization of photons has been widely applied in quantum information research, including entangled photons,^{1,2} atom physics,³ solid-state quantum memory,⁴ and nitrogen-vacancy centers.⁵ Because of the robust coherence and high-fidelity operation of polarized photons, they can be used as the ideal fly qubits in a quantum network.⁶ To improve the interface between photons and other quantum systems in a quantum network, small mode volume or high-finesse optical cavities are required.^{7,8} The fiber Fabry-Perot cavity (FFPC) based on CO₂ laser-machined mirrors is an ideal device for this purpose by combining the virtues of small mode volume, high finesse, and easy integration with other systems.^{9–15} To date, researchers focus on fabricating perfect spherical surface mirrors to support polarization degenerate cavities and improve the cavity performance,^{16–20} and much progress in cavity quantum electrodynamics (CQED) has been made based on this type of cavity.^{21–24} Although Flatten *et al.* realized ellipsoidal cavity mirrors, the polarization degree is not studied in detail.²⁵

However, quantum information is encoded in the spin levels in most quantum systems, particularly the nuclear spin levels with long coherent time. To obtain a polarized photon interface with spin levels, polarization nondegenerate cavities or cavity modes with controllable splittings are required in many cases. For example, by off-resonantly coupling two hyperfine states through a single-photon Raman interaction, deterministic swapping of photonic and atomic states can be performed;²⁶ an efficient cavity-cooling scheme can also be demonstrated with nuclear spin levels.²⁷ The splitting of hyperfine levels of atoms is in the GHz range.^{28–31} To support these schemes, optical cavities with large polarization mode splitting to match the spin transition of atoms are required.

Recently, Bouwmeester provided a scheme to entangle phonon states with photon polarization states in optomechanical systems,³² which also requires a cavity with well controlled polarization splitting to match the mechanical frequency.

In this work, we demonstrate polarization nondegenerate fiber Fabry-Perot cavities, with both high-finesse and a fine controllable tuning feature on polarization modes. We first built a tip/tilt scanning stage to operate multiple laser pulses, and arranging a line on the fiber end facet, the controllable eccentricity of 0.1–0.8 can be obtained. To test the performance of the ellipsoidal mirrors, we assembled Fabry-Perot cavities and studied the characteristics of the splitting of the TEM₀₀ mode. The cavity finesse can reach 40 000, while the ratio between splitting and the line width of polarization modes can reach 17.8. We further show that the frequency splitting of the polarization modes can be finely adjusted by rotating the fiber mirror around the fiber axis. In the summary section, further applications of this type of cavity are discussed.

A schematic diagram of the laser machining setup is shown in Fig. 1. The CO₂ laser (Synrad Firestar v40) works in the continuous wave operation mode, and an optical beam shutter (Thorlabs SH05/M) is used to control the duration of the laser pulses. Part of the laser beam is split into a power meter using a beamsplitter to monitor the laser power during the machining process. A spatial filter, which consists of a long focus convex lens (lens 1) (Thorlabs LB7887-F) and a pinhole (Thorlabs P800S), is used to improve the beam quality with circular symmetry. The tip/tilt scanning stage (PI S-330.2SD) can precisely operate the propagation direction of the laser beam. Lens 2 (Thorlabs AL72525-G) focuses the beam onto the fiber-end tip. A fiber clamp, which is mounted on the five-axis adjustment stage, provides fine control of the fiber position, and two microscopes are used to monitor the fibers and fix the positions. Because of the precise positioning accuracy of the motorized linear stage (PI M-521.DD1), the

^{a)}Electronic mail: hyf@ustc.edu.cn

^{b)}Electronic mail: cfli@ustc.edu.cn

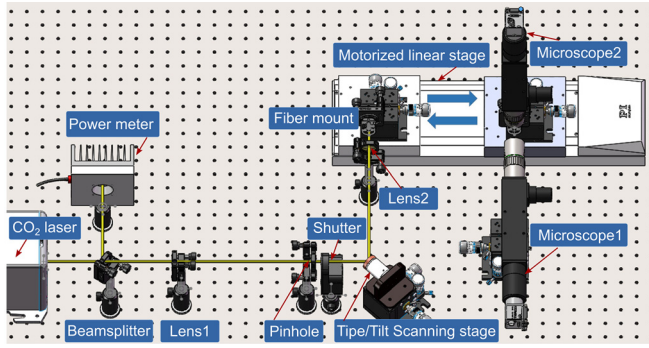


FIG. 1. Machining setup to fabricate the fiber concave mirrors. First, a fiber is clamped using a fiber mount and positioned using the microscopes. Then, the motorized linear stage transmits the mounted fiber to the laser beam focus with high precision and repeatability under $0.1\ \mu\text{m}$. The yellow line represents the trajectory of the laser during the machining. The positions of multiple focused pulses on the fiber tips can be precisely controlled by the tip/tilt scanning stage.

travel between the two locations ensures a typical repeatability down to $100\ \text{nm}$. The entire system can be automatically controlled with a computer in a machining process. As the spot position can move in a two-dimensional plane with the tip/tilt scanning stage, the machining process is flexible to fabricate different surface topographies on the fiber facets and other material substrates.

As shown in a previous study, the polarization and symmetry of the laser beam are two detrimental factors, which result in fiber mirrors with uncontrolled eccentricity.³³ With better laser beam characteristics, designing the machining process with fewer pulses can be easier. When we design the following process to obtain the controllable eccentricity with a continuous change, a nearly perfect laser beam must be used to minimize the uncontrolled eccentricity caused by a single-shot pulse. The laser beam can be qualified by the eccentricity of the fiber mirror created by a single-shot laser pulse, which is defined as $\varepsilon = \sqrt{1 - R_{\min}/R_{\max}}$, where $R_{\min}$ and $R_{\max}$ are the minimum and maximum radii of curvatures (ROCs) among different sections of fiber mirror. In our experiment, using a spatial filter with a pinhole smaller than the diffraction spot, the eccentricity can be reduced to 0.10 ± 0.02 .

We machined hundreds of single mode fibers and empirically found a simple and effective process to fabricate ellipsoidal mirrors by applying multiple laser-shot pulses. A typical temporal shape of the laser pulses is shown in Fig. 2(a), the laser pulse length is around $30\ \text{ms}$, while the interval between pulses is $1\ \text{s}$. Ten laser pulses were successively applied onto five laser-shot dots, which were uniformly arranged in the middle of the fiber tips, as shown in Fig. 2(b). The eccentricity can be tuned by several laser parameters and the machining process. Here, we selected a fixed laser power of $1.3\ \text{W}$ and a laser beam waist of $50\ \mu\text{m}$. By changing the relative spatial intervals of the dots and the pulse length of the laser, different eccentricities can be obtained.

A typical ellipsoidal fiber mirror is characterized by two perpendicular principle axes with an ROC along the major axis (R_2) larger than that of the minor axis (R_1) in Fig. 2(c). We determined the degree of eccentricity as $\varepsilon = \sqrt{1 - R_1/R_2}$, which accounts for the dominant reason for the splitting of

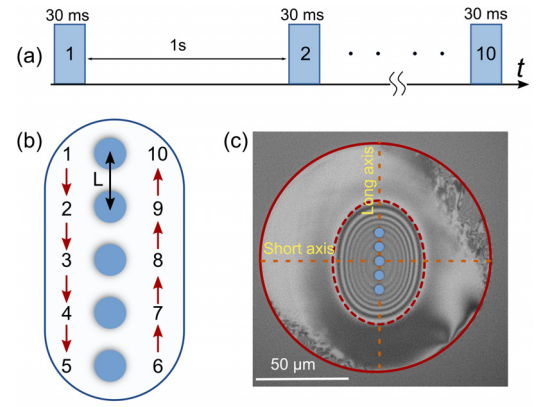


FIG. 2. Machining process to fabricate ellipsoidal mirrors. (a) Temporal shape of the laser pulses: ten laser pulses were used, each laser pulse length is around $30\ \text{ms}$, and the interval between pulses is $1\ \text{s}$. (b) The ten laser pulses were applied back and forth on five shot positions, the blue dots mark the laser-shot positions, and L is the space between two adjacent dots. (c) Interferogram of a typical machined ellipsoidal fiber mirror taken from a home-built white light interferometer: the fiber has a $125\ \mu\text{m}$ cladding diameter, and five shot positions marked as blue spots are arranged along the long axis. The ellipsoidal fiber mirror has a smaller ROC along the short axis than that along the long axis.

polarization modes. Figure 3(a) illustrates the surface cut along the two principle axes, which were measured using an atomic force microscope (AFM, Bruker Dimension Icon). A sixth-order polynomial was used to fit the section curves,³⁴ and the radii of curvature at the center calculated from the fits are $R_2 = 137\ \mu\text{m}$ and $R_1 = 66\ \mu\text{m}$, corresponding to the eccentricity up to 0.72 . The residuals of the fits are plotted in Fig. 3(b). To investigate how the laser pulse length and spot space impact the mirror profile, a series of concave mirrors with different machining parameters were made. The eccentricities of these mirrors under different pulse lengths, along with the spot space, are plotted in Fig. 3(c). The relationship between the eccentricity and space spot is linear under certain pulse lengths, which indicates that the eccentricity can be controlled. The eccentricity between 0.1 and 0.8 can be realized in our experiment, and the larger eccentricity is limited to the diameter of the fiber cladding and the size of the focused laser spot with this machining technique. To further show the relationship between eccentricity and the laser pulse length under certain spot space, the fitted linear slopes in Fig. 3(c) are plotted in Fig. 3(d). When the laser pulse time is $25\ \text{ms}$ to $40\ \text{ms}$, the relationship between eccentricity per unit of spot space (μm) and the pulse length is approximately linear.

An accurate vector cavity theory^{33,35,36} notes that the frequency splitting of the fundamental orthogonal polarization modes in a cavity that consists of two ellipsoidal concave mirrors (a, b) is given by³³

$$\Delta\nu = \frac{\nu_{\text{FRS}}}{2\pi} \sqrt{(\Delta\varphi_a)^2 + (\Delta\varphi_b)^2 + 2\Delta\varphi_a\Delta\varphi_b \cos\theta}, \quad (1)$$

where $\nu_{\text{FSR}} = c/2L$ is the free spectral range of the cavity with cavity length L and $\Delta\varphi_{a,b} = \varepsilon_{a,b}^2/kR_{a1,b1}$ is the differential phase between polarization modes on mirror a or mirror b . The relative angle between the two major axes of the ellipsoidal mirrors is described by θ . The full-width at half-maximum (FWHM) frequency of a single peak is given by

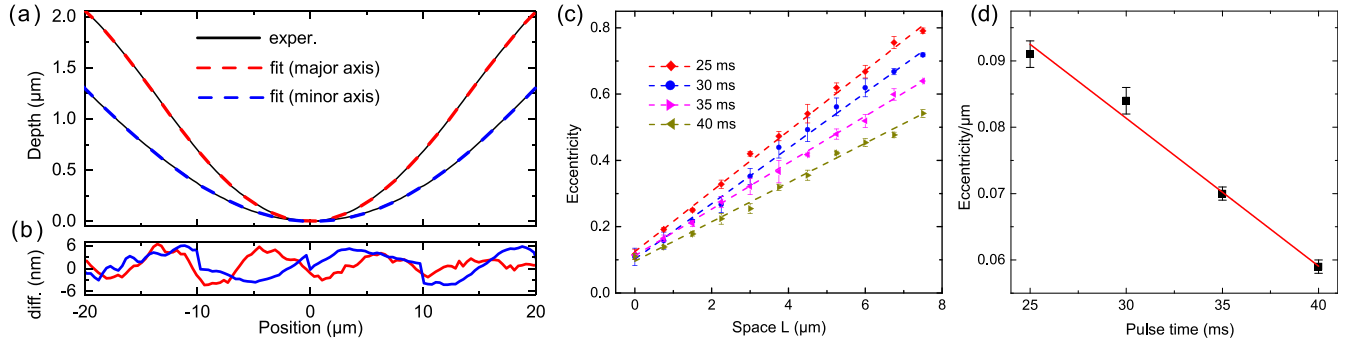


FIG. 3. (a) Profile sections of two perpendicular principle axes obtained from the AFM measurement (black solid lines). The red and blue dashed lines represent the fitted polynomial of the curve along the major axis and minor axes, respectively. (b) Residuals of the fits in (a). (c) Controllable eccentricity under different pulse lengths and spot spaces. (d) Linear relationship between eccentricity per unit of spot space (μm) and pulse length: the slope of the red fitted line is $(2.2 \pm 0.1) \times 10^{-3} \mu\text{m}^{-1} \cdot \text{ms}^{-1}$.

$$\delta\nu = \frac{\nu_{\text{FSR}}}{F}, \quad (2)$$

where F is the finesse of the cavity. To compare the splitting of the fundamental modes to FWHM, it is convenient to introduce the dimensionless factor

$$M = \frac{\Delta\nu}{\delta\nu} = \frac{F}{2\pi} \sqrt{(\Delta\varphi_a)^2 + (\Delta\varphi_b)^2 + 2\Delta\varphi_a\Delta\varphi_b \cos \theta} \quad (3)$$

to define the degree of the fundamental mode splitting. The M factor is not related to the cavity length but is notably sensitive to the mirror structure, which includes the ε square ($0 \leq \varepsilon \leq 1$) and the minor R_1 . It is convenient to obtain different splittings only by rotating one of the mirrors.

We have machined many ellipsoidal mirrors with different eccentricities using single mode fibers with a $125 \mu\text{m}$ cladding diameter. Then, the fiber mirrors were coated with a highly reflective broadband coating, where a reflectivity of 99.992% was obtained at 672 nm . The two fibers were mounted and aligned face to face to form a cavity, and one fiber was clamped on the shear piezo to scan several free spectral ranges (FSRs). The other one was clamped in a continuous rotation mount. We measured the transmission spectrum with a 672 nm laser coupled into one of the fibers and detected at the exit port of the other fiber using a photodetector while rotating the fiber in a cycle and maintaining the two mirrors at a fixed distance. To calibrate the frequency, the laser was modulated at a frequency of 3448.8 MHz using an electro-optical modulator to obtain two side-bands in the transmission spectrum.

We studied the frequency splitting of the fundamental modes with an ellipsoidal fiber cavity consisting of two ellipsoidal fiber mirrors. The two fiber mirrors were measured to have $\varepsilon_a = 0.72$, $R_{a1} = 48.7 \mu\text{m}$, and $\varepsilon_b = 0.68$, $R_{b1} = 35.6 \mu\text{m}$ by AFM. The finesse of the cavity, which was determined by calculating the ratio of the FSR and the linewidth, is $40\,000 \pm 1000$.

A linear-polarization laser at a wavelength of 672 nm was coupled into the cavity, and we measure the transmission of the cavity to characterize the cavity parameters. A typical transmission spectrum with several FSRs is present in Fig. 4(a), and the third peak from the left is zoomed in in Fig. 4(b). All the six peaks described in Fig. 4(b) are simulated by fitting multiple Lorentzian functions. The calculated linewidths are $\delta\nu_1 = 186.6 \text{ MHz}$ and $\delta\nu_2 = 173.4 \text{ MHz}$; the

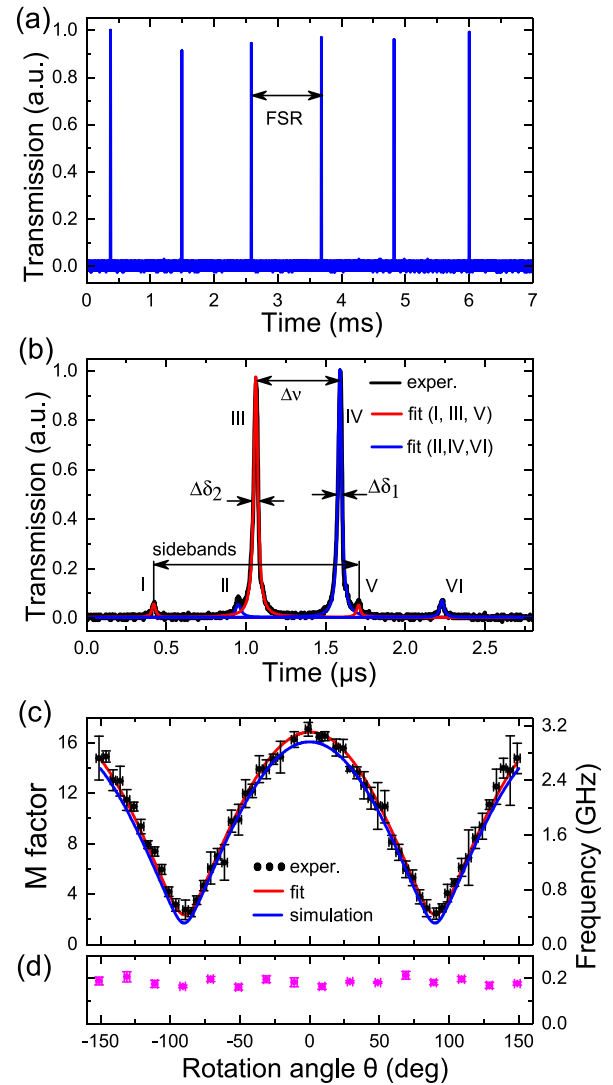


FIG. 4. (a) Transmission spectrum of the FFPC at 672 nm by scanning the cavity length in several FSRs. (b) A close-up view of the third peak from the left. The black line shows the original data of the splitting peaks of the fundamental mode. Peaks I and V represent the sidebands of the polarization splitting peak III. Peaks II and VI represent the sidebands of the polarization splitting peak IV. The laser is modulated with sidebands at $\pm 3448.8 \text{ MHz}$. (c) Tunable splitting with the change in the rotation angle. The left ordinate is the M factor, and the right ordinate is the corresponding splitting frequency of the two polarization modes. The splitting in (c) and the linewidth in (d), which depend on the rotation angle, have identical units and magnitudes in the right ordinate.

splitting is $\Delta\nu = 3.19$ GHz, and the rotation angle θ is 0, and so, the corresponding splitting degree is $M = 17.8$. The difference in the linewidth of two peaks originates from the nonlinear sweeping of the shear piezo. Furthermore, by changing the polarization direction of the input laser with a half-wave plate, we find that the transmission of any polarization peaks can be tuned to disappear or the two peaks are tuned to have equal transmissions. This result proves that the two polarization modes are totally independent, and this feature is meaningful as the cavity can be considered a narrow-band polarization filter.

The frequency splitting and linewidths of the peaks were measured with different rotating angles θ . The cavity length was kept around 20 ± 1 μm using a microscope during the rotation. The M factor changing with the rotating angles, which is not related to the cavity length, is shown in Fig. 4(c). As both the controlled cavity length and the nonlinear sweeping of the shear piezo contribute to the cavity length uncertainty, the measured linewidths in Fig. 4(d) also have some uncertainty, which are statistically calculated as $\delta\nu = 184.2 \pm 12.3$ MHz. Applying the M factor and the averaged linewidth, the splitting can be tuned from 0.4 to 3.2 GHz, as shown in Fig. 4(c). The splitting can be further enlarged by fabricating mirrors with larger eccentricity, a smaller radius of curvature, and a shorter cavity length. We have performed theoretical simulation with Eq. (3), by using the measured eccentricities $\varepsilon_{a,b}$ and ROCs $R_{a1,b1}$ of two mirrors. As the profiles of the mirrors have other higher order profiles than the ellipsoidal model, they will introduce other phase differences between polarization modes, which cause the slight difference of the simulation curve and the fitted curve of data.

In conclusion, we have developed a method to fabricate ellipsoidal fiber mirrors with controllable eccentricity and made high-finesse micro-cavities with tunable polarization mode splitting. At a cavity length of 20 μm , the finesse of the cavity can reach 40 000, which can be further improved by optimizing the reflection of the mirrors; the splitting of fundamental polarization modes can be finely tuned to 3.2 GHz, which can be further improved to tens of GHz by decreasing the cavity length to several micrometers. These features make this type of cavity a valuable device for many potential and practical applications in research and technology. For example, the nondegenerate cavity modes can be applied to synchronously couple two different emitted photons in CQED experiments to spin states, e.g., the hyperfine splitting of atoms ^{87}Rb ,²⁴ trapped ions $^{43}\text{Ca}^+$,³¹ $^{171}\text{Yb}^+$,²⁸ spin states of NV centers,³⁷ or nuclear-spin quantum memory.³⁰ Cavity-based quantum networks with single atoms and optical photons can be flexibly performed based on this technology.⁷ In cavity optomechanics, accurate side-band thermometry and phonon-photon entanglement can be supported by the cavity.³² Furthermore, ellipsoidal cavities can be applied in various techniques, ranging from cavity-enhanced polarimetry,³⁸ cavity ring-down spectroscopy³⁹ to laser stabilization⁴⁰ and sensing applications.⁴¹

This work was supported by the National Key Research and Development Program of China (Grant Nos. 2017YFA0304100 and 2016YFA0302700), the National Natural Science Foundation of China (Grant Nos. 11325419,

11404319, 11474268, and 61327901), the Fundamental Research Funds for the Central Universities (Grant Nos. WK2470000018 and WK2470000026), the Key Research Program of Frontier Sciences (CAS No. QYZDY-SSW-SLH003), the National Youth Top Talent Support Program of National High-level Personnel of Special Support Program (Grant No. BB2470000005), the Youth Innovation Promotion Association of Chinese Academy of Sciences (Grant No. 2017492), and Anhui Provincial Natural Science Foundation (Grant No. 1608085QA22).

- ¹J.-W. Pan, Z.-B. Chen, C.-Y. Lu, H. Weinfurter, A. Zeilinger, and M. Żukowski, *Rev. Mod. Phys.* **84**, 777 (2012).
- ²P. G. Kwiat, E. Waks, A. G. White, I. Appelbaum, and P. H. Eberhard, *Phys. Rev. A* **60**, R773 (1999).
- ³B. B. Blinov, D. L. Moehring, L.-M. Duan, and C. Monroe, *Nature* **428**, 153 (2004).
- ⁴Z.-Q. Zhou, W.-B. Lin, M. Yang, C.-F. Li, and G.-C. Guo, *Phys. Rev. Lett.* **108**, 190505 (2012).
- ⁵B. Hensen, H. Bernien, A. E. Dréau, A. Reiserer, N. Kalb, M. S. Blok, J. Ruitenbergh, R. F. L. Vermeulen, R. N. Schouten, C. Abellán, W. Amaya, V. Pruneri, M. W. Mitchell, M. Markham, D. J. Twitchen, D. Elkouss, S. Wehner, T. H. Taminiau, and R. Hanson, *Nature* **526**, 682 (2015).
- ⁶H. J. Kimble, *Nature* **453**, 1023 (2008).
- ⁷A. Reiserer and G. Rempe, *Rev. Mod. Phys.* **87**, 1379 (2015).
- ⁸A. Mundt, A. Kreuter, C. Becher, D. Leibfried, J. Eschner, F. Schmidt-Kaler, and R. Blatt, *Phys. Rev. Lett.* **89**, 103001 (2002).
- ⁹Y. Colombe, T. Steinmetz, G. Dubois, F. Linke, D. Hunger, and J. Reichel, *Nature* **450**, 272 (2007).
- ¹⁰R. Albrecht, A. Bommer, C. Deutsch, J. Reichel, and C. Becher, *Phys. Rev. Lett.* **110**, 243602 (2013).
- ¹¹A. Jeantet, Y. Chassagneux, C. Raynaud, P. Roussignol, J.-S. Lauret, B. Besga, J. Estève, J. Reichel, and C. Voisin, *Phys. Rev. Lett.* **116**, 247402 (2016).
- ¹²A. Shkarin, N. Flowers-Jacobs, S. Hoch, A. Kashkanova, C. Deutsch, J. Reichel, and J. Harris, *Phys. Rev. Lett.* **112**, 013602 (2014).
- ¹³M. Mader, J. Reichel, T. W. Hänsch, and D. Hunger, *Nat. Commun.* **6**, 7249 (2015).
- ¹⁴T. Hümmer, J. Noe, M. S. Hofmann, T. W. Hänsch, A. Högele, and D. Hunger, *Nat. Commun.* **7**, 12155 (2016).
- ¹⁵J. Benedikter, H. Kaupp, T. Hümmer, Y. Liang, A. Bommer, C. Becher, A. Krueger, J. M. Smith, T. W. Hänsch, and D. Hunger, *Phys. Rev. Appl.* **7**, 024031 (2017).
- ¹⁶D. Hunger, T. Steinmetz, Y. Colombe, C. Deutsch, T. W. Hänsch, and J. Reichel, *New J. Phys.* **12**, 065038 (2010).
- ¹⁷D. Hunger, C. Deutsch, R. J. Barbour, R. J. Warburton, and J. Reichel, *AIP Adv.* **2**, 012119 (2012).
- ¹⁸L. Greuter, S. Starsieles, D. Najer, A. Ludwig, L. Duempelmann, D. Rohner, and R. J. Warburton, *Appl. Phys. Lett.* **105**, 121105 (2014).
- ¹⁹H. Takahashi, J. Morphew, F. Oručević, A. Noguchi, E. Kassa, and M. Keller, *Opt. Express* **22**, 31317 (2014).
- ²⁰K. Ott, S. Garcia, R. Kohlhaas, K. Schüppert, P. Rosenbusch, R. Long, and J. Reichel, *Opt. Express* **24**, 9839 (2016).
- ²¹S. Kuhr, S. Gleyzes, C. Guerlin, J. Bernu, U. B. Hoff, S. Deléglise, S. Osnaghi, M. Brune, J.-M. Raimond, S. Haroche *et al.*, *Appl. Phys. Lett.* **90**, 164101 (2007).
- ²²J. Volz, R. Gehr, G. Dubois, J. Estève, and J. Reichel, *Nature* **475**, 210 (2011).
- ²³G. Barontini, L. Hohmann, F. Haas, J. Estève, and J. Reichel, *Science* **349**, 1317 (2015).
- ²⁴B. Hacker, S. Welte, G. Rempe, and S. Ritter, *Nature* **536**, 193 (2016).
- ²⁵L. C. Flatten, A. A. P. Trichet, and J. M. Smith, *Laser Photonics Rev.* **10**, 257 (2016).
- ²⁶S. Rosenblum, A. Borne, and B. Dayan, *Phys. Rev. A* **95**, 033814 (2017).
- ²⁷V. Vuletić, H. W. Chan, and A. T. Black, *Phys. Rev. A* **64**, 033405 (2001).
- ²⁸S. Olmschenk, K. Younge, D. Moehring, D. Matsukevich, P. Maunz, and C. Monroe, *Phys. Rev. A* **76**, 052314 (2007).
- ²⁹M. Harris, C. Adams, S. Cornish, I. McLeod, E. Tarleton, and I. Hughes, *Phys. Rev. A* **73**, 062509 (2006).
- ³⁰J. J. L. Morton, A. M. Tyryshkin, R. M. Brown, S. Shankar, B. W. Lovett, A. Ardavan, T. Schenkel, E. E. Haller, J. W. Ager, and S. A. Lyon, *Nature* **455**, 1085 (2008).

- ³¹T. P. Harty, D. T. C. Allcock, C. J. Ballance, L. Guidoni, H. A. Janacek, N. M. Linke, D. N. Stacey, and D. M. Lucas, [Phys. Rev. Lett.](#) **113**, 220501 (2014).
- ³²F. Buters, M. Weaver, H. Eerens, K. Heeck, S. de Man, and D. Bouwmeester, [Phys. Rev. A](#) **94**, 063813 (2016).
- ³³M. Uphoff, M. Brekenfeld, G. Rempe, and S. Ritter, [New J. Phys.](#) **17**, 013053 (2015).
- ³⁴B. Brandstätter, A. McClung, K. Schüppert, B. Casabone, K. Friebe, A. Stute, P. O. Schmidt, C. Deutsch, J. Reichel, R. Blatt, and T. E. Northup, [Rev. Sci. Instrum.](#) **84**, 123104 (2013).
- ³⁵M. Lax, W. H. Louisell, and W. B. McKnight, [Phys. Rev. A](#) **11**, 1365 (1975).
- ³⁶A. L. Cullen, [IEEE Trans. Microwave Theory Tech.](#) **24**, 534 (1976).
- ³⁷A. Gruber, A. Dräbenstedt, C. Tietz, L. Fleury, J. Wrachtrup, and C. Von Borczyskowski, [Science](#) **276**, 2012 (1997).
- ³⁸M. Durand, J. Morville, and D. Romanini, [Phys. Rev. A](#) **82**, 031803 (2010).
- ³⁹H. Huang and K. K. Lehmann, [Appl. Opt.](#) **47**, 3817 (2008).
- ⁴⁰T. Hansch and B. Couillaud, [Opt. Commun.](#) **35**, 441 (1980).
- ⁴¹T. Le, A. Savchenkov, N. Yu, L. Maleki, and W. Steier, [Appl. Opt.](#) **48**, 458 (2009).