

Dark-state cooling of a trapped ion using microwave coupling

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We propose a dark-state cooling method of trapped ion systems in the Lamb-Dicke limit. By microwave dressing the ion, we can obtain two electromagnetically induced transparency structures. The heating effects caused by the carrier and the blue sideband transition vanish due to electromagnetically induced transparency effects and the final mean phonon numbers can be much lower than the recoil limit. Our scheme is robust to fluctuations of microwave power and laser intensities, which provides a broad cooling bandwidth to cool motional modes of a linear ion chain. Moreover, it is more suitable to cool four-level ions on a large-scale ion chip.

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I. INTRODUCTION

Cooling a single trapped ion to the ground state has been a fundamental subject in quantum metrology, quantum simulation, and quantum computation [1–4]. Sideband cooling is the first method to reach the vibrational ground state of a single trapped ion and has been implemented experimentally [5–7]. Two requirements should be satisfied for sideband cooling: (i) the ion should be localized to dimensions much smaller than the optical wavelength [Lamb-Dicke (LD) limit], and (ii) the absorption sidebands should be well resolved (strong confinement), i.e., the vibrational frequency should be much larger than the natural line width Γ of the optical transition for laser cooling. Although sideband cooling can cool the ion close to the ground state, the final mean phonon number is still higher than the recoil limit due to the off-resonant carrier and blue sideband transitions.

To prevent the off-resonant carrier transition, quantum coherent effects have been exploited in several schemes, e.g., the cavity-assisted cooling scheme [8–10] and dark-state cooling schemes [11–18]. And cooling a nanomechanical resonator to its ground state has been proposed based on the concept of quantum interference [19–22]. Electromagnetically induced transparency (EIT) cooling is the first dark-state cooling, proposed by Morigi *et al.* [11], which has been a standard experimental scheme for cooling trapped ions [16,23]. EIT cooling uses a three-level trapped-ion system coupled by two lasers with the same blue detuning, a Fano-like absorption spectrum with a 0 at the carrier transition, and a peak at the first-order red sideband transition. Therefore the EIT cooling scheme can achieve a lower temperature than sideband cooling. Moreover, no strong confinement is required due to the three-level nature of the system.

Further improvements in approaches to reach the ground state are achieved with double-dark-state concepts by eliminating all heating transitions. According to the EIT cooling scheme [11], double-EIT cooling [12] results in a double-dark state for the two EIT structures, thus eliminating both the carrier and the blue sideband transition. An alternative approach that combines the methods of EIT and Stark-shift cooling [13] is the robust and fast cooling scheme [14], which cancels both off-resonant excitations due to the combination of EIT and Stark-shift Hamiltonian contributions. Dark-state cooling using standing waves [17] localizes the ion at the node of the laser to eliminate the carrier transition, and blue sideband heating is prevented by the EIT effect [24]. Ground-state cooling by quantum interference pathways [17] and fast cooling through magnetic gradients [25] can also cancel all the dominating heating effects. In the above schemes, all leading-order heating effects are suppressed, and the whole system can be cooled to a steady state, namely, the double-dark state.

$$|\Psi\rangle = |a_0\rangle|0\rangle + \beta|a_1\rangle|1\rangle + O(\eta^2), \quad (1)$$

where $|a_0\rangle$ and $|a_1\rangle$ are the atomic internal states, $|0\rangle$ and $|1\rangle$ are the vibrational ground state and lowest excitation, respectively, and β is a coefficient of the order $O(\eta)$. Thus, the final phonon number is 0 in the leading order of the expansion in the LD parameter η , i.e.,

$$\langle n \rangle_{ss} = 0 + O(\eta^2). \quad (2)$$

The scheme presented here also eliminates the carrier and blue sideband terms completely (up to the first order in the LD parameter), using an atomic four-level system, a combination of two laser beams and a microwave field. The application of a microwave field is used to dress two bare atomic ground states. Upon tuning the Rabi frequency of the microwave field to the condition $\Omega = -\nu/2$, two EIT structures show up to cancel both the heating transitions.

Our scheme has several advantages in experimental implementation: (i) Similarly to double-EIT cooling [12], EIT

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cooling [11,16], and sympathetic EIT cooling [23], our cooling scheme requires relatively low laser intensities and provides a broad cooling bandwidth such that multiple motional modes of a linear ion chain can be cooled simultaneously compared to previous implementations of conventional Raman sideband cooling. However, the scheme here uses a microwave instead of one of three laser beams in double-EIT cooling, and microwave manipulation components are inherently simpler, more stable, and more scalable compared to lasers [26–30]. (ii) Many cooling setups [14,17,18,25], based on a Λ configuration, require additional repumping fields because the excitation state decays to more than two lower states, while the repumping process can disturb corresponding double-dark states and worsen cooling performance. Instead of repumping the system arbitrarily, we use a suitable microwave to improve the cooling efficiency. Therefore with our scheme the unwanted additional decay pathways can become an advantage. In addition, unwanted upper levels can also affect the cooling result seriously [16,23], while they have a limited affect on cooling the atoms with nuclear spins using our cooling scheme.

The rest of this paper is organized as follows. In Sec. II, the cooling scheme and corresponding Hamiltonian are presented. Under an appropriately chosen microwave Rabi frequency and laser detuning, the system can stay in a double-dark steady state and all the main heating effects are canceled. In Sec. III, the analytical formula for the cooling rate is obtained by the method of adiabatic elimination and the optimum condition for the cooling process is derived. Section IV provides numerical simulations of the cooling dynamics and the robustness versus the laser and microwave intensity fluctuation. Finally, a brief conclusion is given.

II. COOLING SCHEME AND HAMILTONIAN

Our scheme is shown in Fig. 1. A single ion with mass M is trapped in a one-dimensional harmonic trap with vibrational frequency ν . $|n\rangle$ represents the n th phonon state; $b^\dagger$ and b are the phonon's creation and annihilation operators. The internal states of the ion consist of three nondissipation states, $|d\rangle$, $|r\rangle$, and $|g\rangle$, and one dissipative state, $|e\rangle$, and the corresponding energies are $\hbar\omega_0$, $\hbar\omega_r$, $\hbar\omega_g$, and $\hbar\omega_e$, respectively. The line

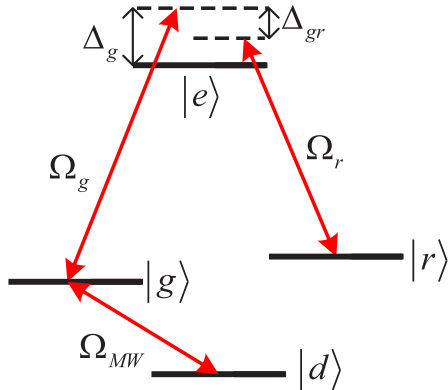


FIG. 1. (Color online) The four-level atom has an excited state $|e\rangle$ and three ground states $|g\rangle$, $|r\rangle$, and $|d\rangle$. $|e\rangle \leftrightarrow |g\rangle$ and $|e\rangle \leftrightarrow |r\rangle$ are coupled by the two laser beams, while $|g\rangle \leftrightarrow |d\rangle$ is driven by a microwave field.

width of state $|e\rangle$ is 2γ . The transitions $|e\rangle \leftrightarrow |g\rangle$ and $|e\rangle \leftrightarrow |r\rangle$ are driven by two lasers with frequencies ω_{L_1} and ω_{L_2} , wave vectors k_{L_1} and k_{L_2} , and Rabi frequencies Ω_g and Ω_r , respectively. States $|d\rangle$ and $|g\rangle$ are coupled to a resonant microwave field with frequency ω_{MW} and Rabi frequency Ω_{MW} . The Hamiltonian of the whole system reads ($\hbar = 0$)

$$H = \nu b^\dagger b - \Delta_g |e\rangle\langle e| - (\Delta_g - \Delta_r) |r\rangle\langle r| + \Omega_g |e\rangle\langle g| e^{ik_{L_1}x} + \Omega_r |e\rangle\langle r| e^{ik_{L_2}x} + \Omega_{MW} |g\rangle\langle d| + \text{H.c.}, \quad (3)$$

where Δ_g, Δ_r are the detunings of the two lasers, respectively, which are

$$\begin{aligned} \Delta_g &= \omega_{L_1} - (\omega_e - \omega_g), \\ \Delta_r &= \omega_{L_2} - (\omega_e - \omega_r), \\ \Delta_{gr} &= \Delta_g - \Delta_r. \end{aligned} \quad (4)$$

The cooling process occurs in the LD limit. Thus, the Hamiltonian, (3), can be expanded by the LD parameters around the trap center,

$$H_{LD} = H_{at} + H_m + V, \quad (5)$$

where

$$H_m = \nu b^\dagger b \quad (6)$$

is the free Hamiltonian for vibrational degrees of freedom (DOF),

$$H_{at} = -\Delta_g |e\rangle\langle e| - \Delta_{gr} |r\rangle\langle r| + (\Omega_g |e\rangle\langle g| + \Omega_r |e\rangle\langle r| + \Omega_{MW} |g\rangle\langle d| + \text{H.c.}) \quad (7)$$

represents the atomic internal DOF, and

$$V = i\eta_1 \Omega_g |e\rangle\langle g| (b + b^\dagger) + i\eta_2 \Omega_r |e\rangle\langle r| (b + b^\dagger) + \text{H.c.} \quad (8)$$

describes the sideband transitions due to the two lasers. Here, the LD parameters are defined by

$$\eta_1 = k_{L_1} \sqrt{\frac{1}{2M\nu}}, \quad \eta_2 = k_{L_2} \sqrt{\frac{1}{2M\nu}}. \quad (9)$$

If $\Omega_{MW} = \Delta_{gr}$, we can rewrite the Hamiltonian of the atomic internal DOF, (7), in a new representation $\{|D\rangle, |B\rangle, |+\rangle, |e\rangle\}$,

$$H_{at} = -(\Delta_g - \Delta_{gr}) |e\rangle\langle e| + 2\Delta_{gr} |+\rangle\langle +| + (\Omega_+ |+\rangle\langle e| + \Omega_B |B\rangle\langle e| + \text{H.c.}), \quad (10)$$

where

$$\begin{aligned} \Omega_B &= \frac{1}{\sqrt{2}} \sqrt{2\Omega_r^2 + \Omega_g^2}, \quad \Omega_+ = \frac{1}{\sqrt{2}} \Omega_g, \\ |+\rangle &= \frac{1}{\sqrt{2}} (|g\rangle + |d\rangle), \\ |B\rangle &= \frac{1}{\sqrt{2\Omega_r^2 + \Omega_g^2}} (\Omega_g |-\rangle + \sqrt{2}\Omega_r |r\rangle). \end{aligned} \quad (11)$$

It can be found that for the dark state $|D\rangle$, which is

$$|D\rangle = \frac{1}{\sqrt{2\Omega_r^2 + \Omega_g^2}}(\sqrt{2}\Omega_r|-\rangle - \Omega_g|r\rangle), \quad (12)$$

where $|-\rangle = \frac{1}{\sqrt{2}}(|g\rangle - |d\rangle)$, state $|D\rangle$ decouples from the Hamiltonian, (10), which is the internal steady state, and the carrier transition vanishes due to the EIT effect. Then we can rewrite the sideband transition V with the representation $\{|D\rangle, |B\rangle, |+\rangle, |e\rangle\}$ as

$$\begin{aligned} V = & i\eta_D\Omega_D|D\rangle\langle e|(b+b^\dagger) + \text{H.c.} \\ & + i\eta_1\Omega_+|+\rangle\langle e|(b+b^\dagger) + \text{H.c.} \\ & + i\eta_B\Omega_B|B\rangle\langle e|(b+b^\dagger) + \text{H.c.}, \end{aligned} \quad (13)$$

with

$$\begin{aligned} \eta_D &= \eta_1 - \eta_2, \\ \eta_B &= \frac{\Omega_g^2\eta_1 + 2\Omega_r^2\eta_2}{2\Omega_r^2 + \Omega_g^2}, \\ \Omega_D &= \frac{\Omega_g\Omega_r}{\sqrt{2\Omega_r^2 + \Omega_g^2}}. \end{aligned} \quad (14)$$

To understand the cooling dynamics, it is convenient to transfer the Hamiltonian to a dressed representation $\{|+\rangle, |-\rangle, |r\rangle, |e\rangle\}$, which is

$$\begin{aligned} H_{\text{at}} = & \Delta_r|r\rangle\langle r| + \Delta_-|-\rangle\langle -| + \Delta_+|+\rangle\langle +| \\ & + \frac{\Omega_g}{\sqrt{2}}(|+\rangle\langle e| + |e\rangle\langle +|) + \frac{\Omega_g}{\sqrt{2}}(|-\rangle\langle e| + |e\rangle\langle -|) \\ & + \Omega_r(|r\rangle\langle e| + |e\rangle\langle r|), \end{aligned} \quad (15)$$

with

$$\Delta_- = \Delta_r, \quad \Delta_+ = \Delta_r + 2\Delta_g, \quad (16)$$

where Δ_r , Δ_+ , and Δ_- are corresponding detunings for transitions $|r\rangle \leftrightarrow |e\rangle$, $|+\rangle \leftrightarrow |e\rangle$, and $|-\rangle \leftrightarrow |e\rangle$, respectively. The transitions $|-\rangle \leftrightarrow |e\rangle$ and $|r\rangle \leftrightarrow |e\rangle$ have the same detunings to produce the dark state $|D\rangle$. Hence the carrier transition is eliminated, and the transitions $|D\rangle|n\rangle \leftrightarrow |e\rangle|n+1\rangle$ and $|+\rangle|n+1\rangle \leftrightarrow |e\rangle|n+1\rangle$ remain. If detuning Δ_{gr} satisfies

$$\Delta_{gr} = -\frac{\nu}{2}, \quad (17)$$

that is, $\Delta_+ = \Delta_r - \nu$, then EIT will arise between $|D\rangle|n\rangle \leftrightarrow |e\rangle|n+1\rangle$ and $|+\rangle|n+1\rangle \leftrightarrow |e\rangle|n+1\rangle$, and the blue sideband heating is coherently canceled while only the cooling transition is left. As expected, Fig. 2 shows a typical scattering spectrum of the cooling laser (the cooling transition $|r\rangle \leftrightarrow |e\rangle$). The spectrum can be understood as consisting of two Fano-like structures, which is characteristic of EIT, and is controlled by the strong-coupling laser field. Under the conditions Eqs. (17) and (34) all the heating transitions vanish, while the maximal red sideband cooling is obtained. Therefore, the cooling cycle will start from the n th vibrational steady state $|D\rangle|n\rangle$, scatter over the excited state $|e\rangle|n-1\rangle$ via the red sideband transition, and dissipate back to the lower vibrational steady state $|D\rangle|n-1\rangle$, and the final double-dark state is

$$|\psi\rangle = \Omega_+|D\rangle|0\rangle - i\eta_D\Omega_D|+\rangle|1\rangle + O(\eta^2), \quad (18)$$

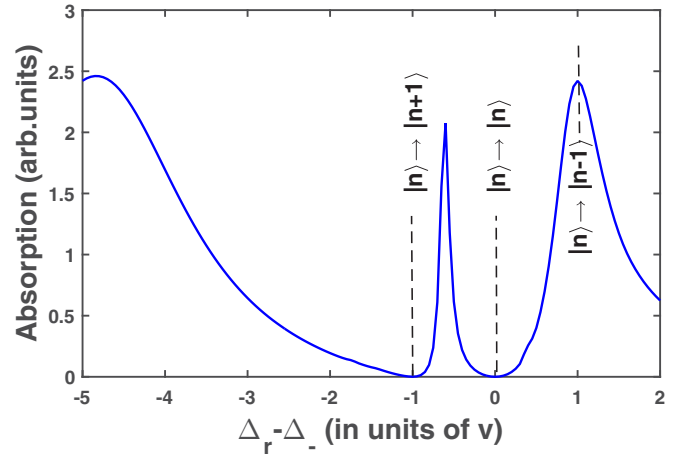


FIG. 2. (Color online) Absorption of cooling laser. As there are two EIT structures, the absorption vanishes at two values for the detuning Δ_r of the cooling laser. Dashed lines mark the positions of these 0's and the absorption maximum. For appropriately chosen parameters, both the carrier transition and the blue-sideband heating are completely canceled, while the red-sideband cooling is maximized. Here, the parameters are $\Omega_g = \gamma/\sqrt{2}$, $\Omega_r = \gamma/20$, and $\Delta_- = \gamma$, which represent the detuning for the transition $|-\rangle \leftrightarrow |e\rangle$. The other parameters are chosen to satisfy Eqs. (17) and (34).

which satisfies $H_{LD}|\psi\rangle = 0$. The final temperature can be evaluated by the mean phonon number of the double-dark state, yielding

$$\langle n \rangle_{ss} = \frac{\eta_D^2\Omega_D^2}{\eta_D^2\Omega_D^2 + \Omega_+^2} + O(\eta^4). \quad (19)$$

According to (11) and (14),

$$\langle n \rangle_{ss} = \frac{2\Omega_g^2\Omega_r^2}{\Omega_g^4 + 2(\eta_D^2 + 1)\Omega_g^2\Omega_r^2}\eta_D^2 + O(\eta^4). \quad (20)$$

If we choose $\Omega_g \gg \Omega_r$, the mean phonon number $\langle n \rangle_{ss}$ will be much lower than the phonon recoil limit $O(\eta^2)$.

III. ANALYTICAL TREATMENT

The master equation is useful for analytically characterizing the cooling dynamics. Both the coherent dynamics and the dissipative nature of the excited level can be accounted for as

$$\frac{d}{dt}\rho = \mathcal{L}\rho = -i[H, \rho] + \mathcal{L}_\gamma\rho, \quad (21)$$

where the Liouvillian $\mathcal{L}_\gamma$ is for three dissipative channels, and

$$\begin{aligned} \mathcal{L}_\gamma\rho = & \sum_{j=g,r,d} \gamma_j \left(2 \int_{-1}^1 ds W(s) e^{ik_j xs} |j\rangle\langle e|\rho|e\rangle\langle j| e^{-ik_j xs} \right. \\ & \left. - |e\rangle\langle e|\rho - \rho|e\rangle\langle e| \right), \end{aligned} \quad (22)$$

where $2\gamma_j$ is the spontaneous decay rate from $|e\rangle$ to $|j\rangle$ ($j = g, r, d$), $W(s)$ is the angular distribution of a spontaneous emission for a dipole transition, and $k_j = (\omega_e - \omega_j)/c$.

In view of the small magnitude of η , the coupling between the internal and the external DOF (i.e., the laser cooling) is slow

compared with the time scale of the internal atomic dynamics in the LD limit. This situation allows adiabatic elimination of the internal DOF, and thus we can obtain the rate equation of the motional state,

$$\frac{d}{dt}p_n = (n+1)A_-p_{n+1} + nA_+p_{n-1} - [(n+1)A_+ + nA_-]p_n, \quad (23)$$

where $p_n = \langle n|\rho|n\rangle$ is the population of the n th vibrational state and $A_\pm$ are the cooling and heating transition rates. Then the rate equation for the mean phonon number $\langle n \rangle$ is

$$\frac{d}{dt}\langle n \rangle = -(A_- - A_+)\langle n \rangle + A_+. \quad (24)$$

If $A_+ < A_-$, the mean phonon number varies with time, as

$$\langle n \rangle = (\langle n \rangle_0 - \langle n \rangle_{ss})e^{-Wt} + \langle n \rangle_{ss}, \quad (25)$$

where $\langle n \rangle_0$ is the initial mean phonon number, $\langle n \rangle_{ss}$ is the final mean phonon number, and W is the rate of the cooling process:

$$\langle n \rangle_{ss} = \frac{A_+}{A_- - A_+} + O(\eta^2), \quad (26)$$

$$W = A_- - A_+. \quad (27)$$

To obtain the cooling transition rates $A_\pm$, we can define $\Gamma_{n \rightarrow n \pm 1}$ as the cooling and heat scattering amplitudes,

$$\Gamma_{n \rightarrow n \pm 1} = 2\gamma \langle n \pm 1 | \langle D | W_\gamma G(E_n) V | D \rangle | n \rangle |^2, \quad (28)$$

where $\Gamma_{n \rightarrow n-1}$ and $\Gamma_{n \rightarrow n+1}$ describe the cooling and heating scattering process from the n th-vibration steady state $|D\rangle|n\rangle$ to the $(n \pm 1)$ th-vibration steady state $|D\rangle|n \pm 1\rangle$ via the sideband transition V . E_n is the energy of state $|D\rangle|n\rangle$, the operator $W_\gamma = |D\rangle\langle e|$ describes the dissipation back to the internal dark state $|D\rangle$ from the excited state $|e\rangle$, and the function $G(z)$ is defined by

$$G(z) = \frac{1}{z - H_{\text{eff}}}, \quad (29)$$

where the effective Hamiltonian $H_{\text{eff}} = H_{LD} - i\gamma|e\rangle\langle e|$.

Calculating Eq. (28), we can get

$$\Gamma_{n \rightarrow n \pm 1} = 2\gamma\Omega_D^2\eta_D^2(n + \delta_\pm)v^2 \frac{(\mp v - 2\Delta_{gr})^2}{|f(\mp v)|^2}, \quad (30)$$

where $\delta_+ = 1, \delta_- = 0$, and

$$\begin{aligned} f(x) = & x^3 + (\Delta_g - \Delta_{gr})x^2 \\ & + (\Omega_B^2 + \Omega_+^2 + 2\Delta_g\Delta_{gr} + 2\Delta_{gr}^2)x \\ & + 2\Omega_B^2\Delta_{gr} + i\gamma(x - 2\Delta_{gr})x. \end{aligned} \quad (31)$$

Because $\Gamma_{n \rightarrow (n \pm 1)} = (n \pm \delta_\pm)A_\pm$, the following holds:

$$A_\pm = 2\gamma\Omega_D^2\eta_D^2v^2 \frac{(\mp v - 2\Delta_{gr})^2}{f(\mp v)}. \quad (32)$$

As expected, A_+ vanishes at $\Delta_{gr} = -\frac{v}{2}$, which means that the carrier and the blue sideband heating are inhibited, and the final mean phonon number n_{ss} will be 0 in the leading order of the expansion in the LD parameter η . Then the corresponding

cooling transition rate is

$$A_{n \rightarrow n-1} = 2\gamma\Omega_D^2\eta_D^2 \frac{4v^2}{(3v^2 + 2v\Delta_g - 2\Omega_B^2 - \Omega_+^2)^2 + 4\gamma^2v^2}. \quad (33)$$

We can obtain the maximal cooling rate at

$$3v^2 + 2v\Delta_g - 2\Omega_B^2 - \Omega_+^2 = 0. \quad (34)$$

The maximum W is

$$W = \frac{2\Omega_D^2\eta_D^2}{\gamma}. \quad (35)$$

According to Eq. (26), the final average phonon number $\langle n \rangle_{ss}$ is

$$\langle n \rangle_{ss} = 0 + O(\eta^2), \quad (36)$$

which is the same as Eq. (2).

IV. NUMERICAL RESULTS AND DISCUSSION

We have described the model and obtained the analytical result of the cooling process. In this section, we use the quantum toolbox [31] for numerical simulation of the master equation Eq. (3). A $^{171}\text{Yb}^+$ ion is regarded as an excellent candidate for larger scale quantum computation with qubits encoded in two hyperfine states [28,29,32–36]. Our scheme can be experimentally implemented in the $^{171}\text{Yb}^+$ ion system, where the energy levels of $|r\rangle$, $|g\rangle$, $|d\rangle$, and $|e\rangle$ can be realized by choosing $|^2S_{1/2}, F=1, m_F=-1\rangle$, $|^2S_{1/2}, F=1, m_F=0\rangle$, $|^2S_{1/2}, F=1, m_F=1\rangle$, and $|^2P_{1/2}, F=0, m_F=0\rangle$, respectively. We can use two 369-nm laser beams to couple the transitions $|^2S_{1/2}, F=1, m_F=-1\rangle \leftrightarrow |^2P_{1/2}, F=0, m_F=0\rangle$ and $|^2S_{1/2}, F=1, m_F=0\rangle \leftrightarrow |^2P_{1/2}, F=0, m_F=0\rangle$ and use an acousto-optical modulator driver to tune the frequency difference between these two transitions. To drive the transition $|^2S_{1/2}, F=1, m_F=0\rangle \leftrightarrow |^2S_{1/2}, F=1, m_F=1\rangle$, a magnetic field can be used to split these two degenerate levels, and then the transition can be driven with a microwave field. The branching ratios out of the excitation state $|^2P_{1/2}, F=0, m_F=0\rangle$ to the three hyperfine states, $|^2S_{1/2}, F=1, m_F=-1\rangle$, $|^2S_{1/2}, F=1, m_F=0\rangle$, and $|^2S_{1/2}, F=1, m_F=1\rangle$ are the same; that is, $\gamma_g = \gamma_r = \gamma_d$.

As shown in Fig. 3, the analytical prediction of the cooling dynamics in Eq. (25) is consistent with the numerical simulation, and the final phonon number of our scheme can reach the order of $O(10^{-4})$. As $\eta_D = 0.1$, the corresponding recoil energy is approximately $O(\eta_D^2) - O(10^{-2})$, and then the final temperature in our scheme can be much lower than the recoil energy for $\Omega_g \gg \Omega_r$.

The dependence of the final cooling result on the Rabi frequencies Ω_g and Ω_r is shown in Figs. 4 and 5, respectively. In Fig. 4, with a fixed Ω_r and other parameters chosen under optimal conditions, Eqs. (17) and (34), the final mean phonon number decreases with increasing Ω_g . In contrast, as the Rabi frequency Ω_g is fixed, the final cooling result worsens with increasing Ω_r , which is shown in Fig. 5. In the two figures, the numerical simulation agrees with the analytical prediction, Eq. (20). The final phonon number increases with increasing

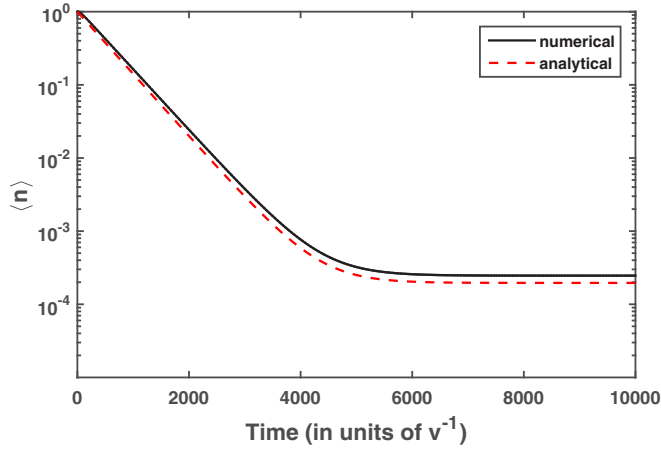


FIG. 3. (Color online) Numerical simulation of cooling a $^{171}\text{Yb}^+$ ion. The parameters are $\eta_g = -\eta_r = 0.05$, $2\gamma = 20\nu$, $\gamma_g = \gamma_r = \gamma_d = 10\nu/3$, $\Omega_g = 10\nu$, $\Omega_r = 1\nu$, $\Omega_{\text{MW}} = \Delta_{gr} = -0.5\nu$, $\Delta_g = 74.5\nu$. Analytical predictions are shown as dashed lines. Solid lines correspond to numerical results. The final phonon number is much lower than the recoil limit $O(10^{-2})$.

strength of Ω_g , while it decreases with increasing Ω_r , because they affect the final double-dark state according to Eq. (18).

Our cooling scheme is robust with respect to the fluctuation of the Rabi frequencies. As depicted in Fig. 6 or 7, when the fluctuation of the Rabi frequency Ω_{MW} or Ω_g is approximately 20%, the final mean phonon number varies by the order of 10^{-3} . Therefore, the cooling scheme is insensitive to the fluctuation of the microwave power and the laser optical intensity.

The validity of Eq. (35) is constrained to the conditions of the adiabatic elimination of the internal DOF. The external

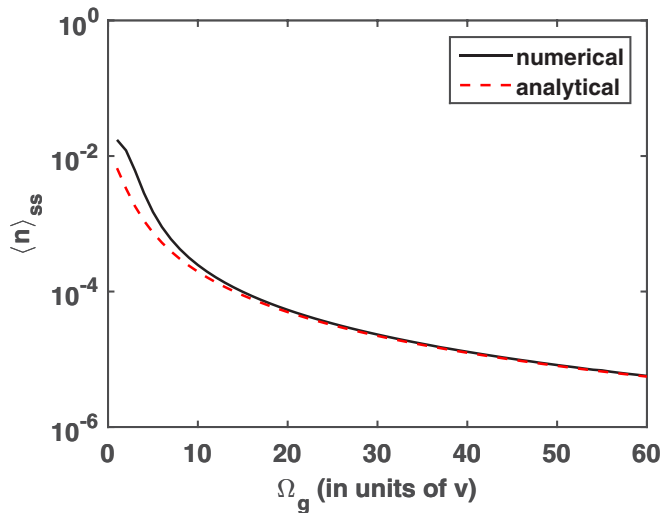


FIG. 4. (Color online) Numerical simulation of the final phonon numbers as a function of Ω_g . The parameters are $\eta_g = -\eta_r = 0.05$, $2\gamma = 20\nu$, $\gamma_g = \gamma_r = \gamma_d = 10\nu/3$, $\Omega_r = 1\nu$, $\Omega_{\text{MW}} = \Delta_{gr} = -0.5\nu$; Δ_g is chosen under the optimal condition, Eq. (34). The dashed line is obtained by analytical prediction, while the solid line corresponds to numerical results. The cooling result gets better with increasing Ω_g .

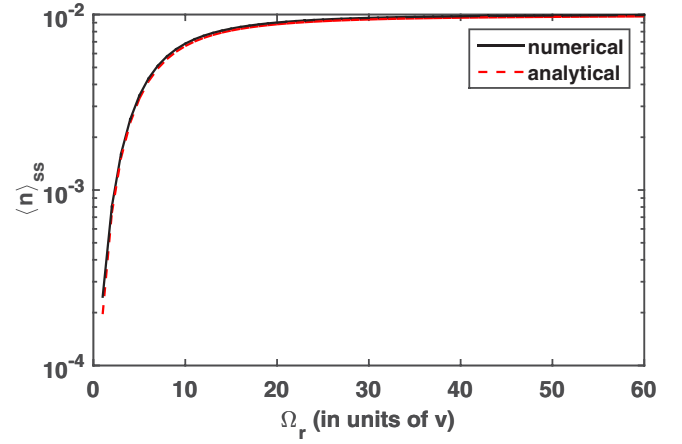


FIG. 5. (Color online) Numerical simulation of the final phonon numbers as a function of Ω_r . The parameters are $\eta_g = -\eta_r = 0.05$, $2\gamma = 20\nu$, $\gamma_g = \gamma_r = \gamma_d = 10\nu/3$, $\Omega_g = 10\nu$, $\Omega_{\text{MW}} = \Delta_{gr} = -0.5\nu$; Δ_g are chosen under the optimal condition, Eq. (34). The dashed line is obtained with the analytical prediction, while the solid line corresponds to numerical results. Curves show that the final phonon number decreases with increasing Ω_r .

dynamics governed by $\eta_D\Omega_D$ must always be much slower than the internal dynamics, which is limited by the dissipation rate 2γ . Figure 8 shows the cooling rate obtained by numerical simulations and by calculation using Eq. (35). As expected, the rate formula describes the rate quite well in the limit of small Ω_D values, whereas it fails completely in the opposite limit because the adiabatic elimination condition is no longer fulfilled. The maximum cooling rate, about $3 \times 10^{-3}\nu$, appears at the point of $\Omega_D = 1.2\nu$.

Compared with the previous methods [11,14], although the cooling rate of our scheme is slower than these two, the final

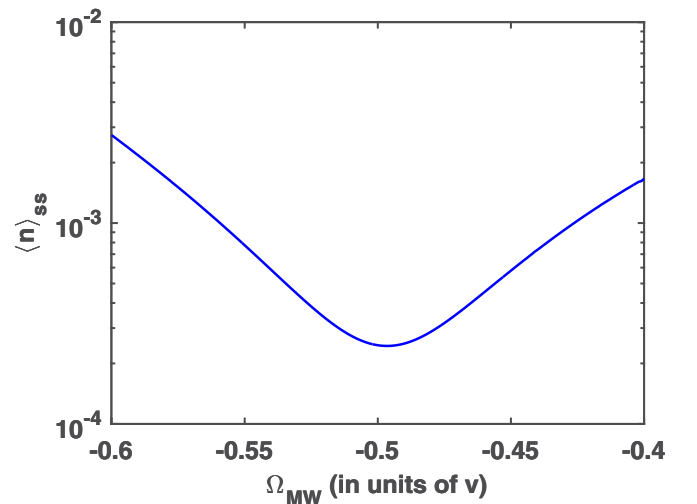


FIG. 6. (Color online) Numerical simulation of the final phonon numbers as a function of Ω_{MW} . The parameters are $\eta_g = -\eta_r = 0.05$, $2\gamma = 20\nu$, $\gamma_g = \gamma_r = \gamma_d = 10\nu/3$, $\Omega_g = 10\nu$, $\Omega_r = 1\nu$, $\Delta_{gr} = -0.5\nu$, $\Delta_g = 74.5\nu$. The corresponding optimal value of Ω_{MW} from the analytical prediction is at $\Omega_{\text{MW}0} = -0.5\nu$. The cooling result is robust against the fluctuation of the microwave driver's power Ω_{MW} .

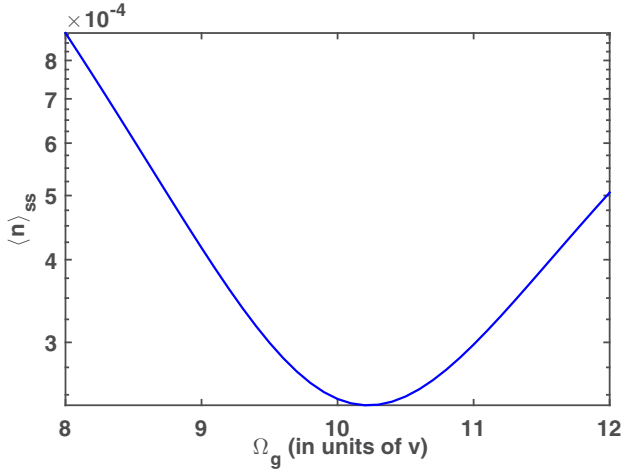


FIG. 7. (Color online) Numerical simulation of the final phonon numbers as a function of Ω_g . The parameters are $\eta_g = -\eta_r = 0.05$, $2\gamma = 20\nu$, $\gamma_g = \gamma_r = \gamma_d = 10\nu/3$, $\Omega_r = 1\nu$, $\Omega_{\text{MW}} = \Delta_{gr} = -0.5\nu$, $\Delta_g = 74.5\nu$. The corresponding optimal value of Ω_g from the analytical prediction is at $\Omega_g = 10\nu$. The cooling result is robust against the fluctuation of Ω_g , which depends on the optical laser intensity.

mean phonon number here is much lower. In Ref. [11], EIT cooling can only reach the order of $O(10^{-2})$, which is two orders of magnitude higher than our result, $O(10^{-4})$, due to the first-order blue sideband heating existing in their scheme. Meanwhile, for the same LD parameters, in our scheme, the final mean phonon number can be much lower than η_D^2 if $\Omega_g \gg \Omega_r$ is chosen. However, in Ref. [14], to eliminate the heating transitions, the Rabi frequencies of the two EIT lasers have to be tuned to the same frequency, and there $\Omega_g = \Omega_r$, so the final phonon number is just $\langle n \rangle_{ss} = \frac{\eta_D^2}{4}$, which is higher than our result. The dynamics of four-level systems, which

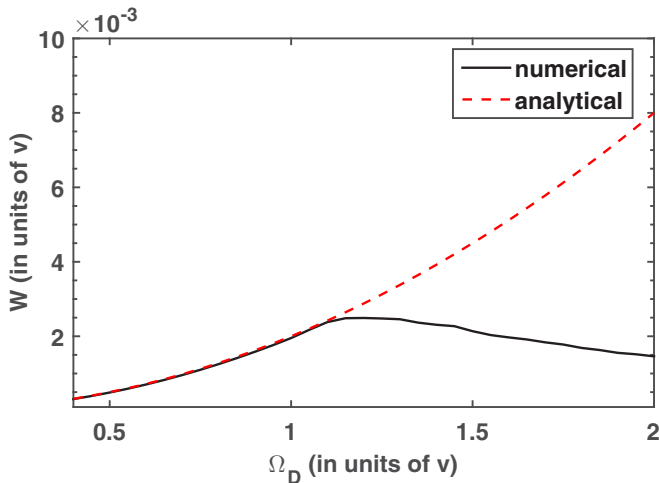


FIG. 8. (Color online) Comparison of the cooling rate W obtained by means of analytical [dashed (red) line] or numerical (solid black line) calculations as a function of the Rabi frequency Ω_D . $\eta_g = -\eta_r = 0.05$, $2\gamma = 20\nu$, $\gamma_g = \gamma_r = \gamma_d = 10\nu/3$, $\Omega_g = 10\nu$, $\Omega_{\text{MW}} = \Delta_{gr} = -0.5\nu$; Δ_g is chosen under the optimal condition, Eq. (34).

is more complex than that of triple systems, could lead to a slower cooling rate.

Our scheme is more suitable for the ions with hyperfine structures such as $^{171}\text{Yb}^+$ and $^{111}\text{Cd}^+$. For previous methods based on the three-level Λ configuration, in experimental implementation with special ions such as $^{40}\text{Ca}^+$ and $^{24}\text{Mg}^+$, unwanted upper levels result in a decreased cooling performance [16,23], whereas for $^{171}\text{Yb}^+$ and other parameters here, the unwanted hyperfine structure $|^2P_{1/2}, F=1\rangle$, which includes three hyperfine excitation states in radiant $2\gamma = 19.7\text{ MHz}$, has a natural 2.1-GHz gap with the level $|^2P_{1/2}, F=0, m_F=0\rangle$, and only $|^2P_{1/2}, F=1, m_F=-1\rangle$ and $|^2P_{1/2}, F=0, m_F=1\rangle$ can lead to a weaker scattering rate, $R \approx \frac{2}{3} \times (\frac{\Omega_g}{\Delta})^2 \times 2\gamma = 0.32\text{ kHz}$, compared to the cooling rate, $W = 2\text{ kHz}$, according to Eq. (35). Hence the cooling scheme here is efficient. Moreover, we only require one laser to implement Doppler cooling and our cooling scheme simultaneously instead of Raman sideband cooling.

Recently, large-scale quantum-computer architectures are more and more attractive [26–30,32–42], and scaling can potentially be achieved by storing ions in multizone arrays where information is moved in the processor by physically transporting the ions [37,38] or teleporting [39]. It is important to recool the qubit ion because ions can be heated by ambient noisy electric fields and/or during ion transport [32]. Experimentally, “refrigerant” ions can be used to cool the qubit ions sympathetically [4,23,37,38,40–42]. Microwaves can be applied via waveguides (structures that can guide radiation) that are part of the chip on which the ion trap is integrated [29], and thus do not require alignment, while laser beams must be carefully aligned to interact with the trapped ions [28]. Moreover, it is easier to generate and control microwave fields, which are more robust to amplitude, phase, frequency, and polarization noise compared to lasers [26,27], and these parameters are difficult to control in integrated optics [30]. Hence, compared to double-EIT cooling, our cooling scheme uses a microwave driver instead of one laser beam to ease the experimental condition for cooling “refrigerant” ions in multizone arrays and make it more feasible to realize double-dark state cooling on the ion chip first.

V. CONCLUSION

In conclusion, we have shown a new efficient double-dark-state cooling scheme for a trapped ion system using the EIT effect combined with microwave Stark shift. Microwave coupling is used to dress the ion levels and obtain two EIT structures, providing a double-dark state. At leading order of the LD expansion, this suppresses all heating excitations. As a consequence, the final temperature in this scheme can be much lower than the recoil energy. It is robust against fluctuations of the microwave power and the laser intensities. Compared with EIT cooling [11] and fast and robust cooling [14], the final mean phonon numbers can be much lower using our method. Moreover, this scheme is better for the experimental realization of cooling trapped ions in a large scalable ion trap in comparison with the double-EIT cooling method [12] by using a microwave component to replace one laser. Importantly, the unwanted additional decay pathways

can become an advantage, and the unwanted upper levels have a limited effect in our method compared with three-level Λ setups [12,14,17,18,25] for trapped ions with $1/2$ nuclear spins.

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