

# High-Q and Unidirectional Emission Whispering Gallery Modes: Principles and Design

Chang-Ling Zou, Fang-Wen Sun, Chun-Hua Dong, Fang-Jie Shu, Xiao-Wei Wu, Jin-Ming Cui, Yong Yang, Zheng-Fu Han, and Guang-Can Guo

**Abstract**—The general principles of the unidirectional emission of a high-Q whispering gallery mode (WGM) in deformed circular microcavities are studied and presented in this paper. In the phase space of deformed microcavities, light in the chaotic sea leaks out from the cavity through specific refraction regions, the positions of which are determined by the stable islands and the unstable manifolds. By changing the cavity shape, we can tune the positions of the refraction regions, as well as fixed points, to achieve unidirectional emission with narrow angular divergence. The microcavity with high-Q and unidirectional emission makes high-efficient free-space excitation and collection possible; thus, it can be applied for far-field detection of nanoparticles and low-threshold lasers with collimated emission.

**Index Terms**—Asymmetric resonant cavities, directional emission, laser resonators, microcavities, ray chaos, whispering gallery mode (WGM).

## I. INTRODUCTION

RECENTLY, dielectric microcavities as high-potential elements for integrated devices have attracted increasing attention. Special attention is paid to the whispering gallery mode (WGM) microresonators, because this rotational symmetrical structure can well confine energy by total internal reflection. The unique features of WGMs, high-quality (Q) factor and low-mode volume, allow them to be applied in wide ranges, from nonlinear optics, low-threshold lasers, sensitive sensors, to cavity quantum electrodynamics [1]. However, the isotropic emission property of WGMs results in the low efficiency of the free-space coupling to cavity. Although the near-field coupling

components can be used to solve this problem, the large footprint and poor capacity of resisting disturbance will also be a big obstacle in practical applications [2]. It is found that the slightly deformed circular microcavity, also called asymmetric resonant cavity (ARC), can radiate light in certain directions. The ARCs, such as the extensively studied quadruple [3]–[7] and stadium microcavities [8]–[10], can be efficiently excited and collected in free space, which make them very useful for strong photon–atom coupling [11], laser [12], and quantum optomechanics [13], [14]. In addition to the easy free-space coupling in optical applications, the deformed microcavities can also serve as open billiards for experimental research on quantum chaos [15], [16].

In order to make free-space excitation and collection of WGMs more efficient in practical application, people are pursuing cavities emitting in single direction with narrow angular divergence, such as spiral-shaped [16]–[19] and rounded isosceles triangle-shaped [20] microcavities. However, their low-Q factors greatly limit the applications in low-threshold laser and cavity quantum electrodynamics. To solve this, several strategies like introducing extra defect or cavity to the resonator have been proposed, and collimated unidirectional lasers have been demonstrated in experiments [21]–[23]. It is still recent that the fabrication-friendly limaçon-shaped microcavities with both unidirectional emission and high Q are theoretically proposed [24] and experimentally realized by several groups [25]–[28]. However, the unidirectional emission of limaçon-shaped microcavity is valid only for high-refractive-index materials ( $n > 2.7$ ). There is still no report of high-Q and unidirectional emission in low-refractive-index microcavities, such as silica ( $n = 1.45$ ).

In this paper, we studied and summarized the principles of unidirectional emission of high-Q WGMs in ARCs. By symmetry analysis, we get the necessary conditions to achieve the high-Q and unidirectional emission in an ARC: the cavity boundary should be smooth and slightly deformed from a circle to support WGMs, and the cavity should have no more than one axis of symmetry and onefold rotational symmetry. With detailed analysis based on ray dynamics, the phase-space structure required for unidirectional emission is given. Thus, the boundary shapes corresponding to such specific phase-space structure can be found by ray simulations. Taking the silica material as an example, we successfully designed a cavity boundary shape. The wave simulation shows that the emission peak with divergence angle less than  $30^\circ$  contains 75% of total radiation power, verifying the unidirectional emission of high-Q ( $3.56 \times 10^5$ ) WGMs.

Manuscript received July 31, 2012; revised September 17, 2012; accepted September 19, 2012. Date of publication September 28, 2012; date of current version May 13, 2013. This work was supported in part by the National Fundamental Research Program of China under Grant 2011CB921200, in part by the Knowledge Innovation Project of Chinese Academy of Sciences under Project 60921091, and in part by the National Natural Science Foundation of China under Grant 11004184. (Corresponding author: F.-W. Sun.)

C.-L. Zou, F.-W. Sun, X.-W. Wu, J.-M. Cui, Z.-F. Han, and G.-C. Guo are with the Key Lab of Quantum Information, University of Science and Technology of China, Hefei 230026, China (e-mail: senew@mail.ustc.edu.cn; fwsun@ustc.edu.cn; xiaowei@mail.ustc.edu.cn; jmcui@mail.ustc.edu.cn; zfhan@ustc.edu.cn; gcguo@ustc.edu.cn).

C.-H. Dong is with Oregon University, Eugene, OR 97403 USA (e-mail: dchcyf@mail.ustc.edu.cn).

F.-J. Shu is with the Department of Physics, Shangqiu Normal University, Shangqiu 476000, China (e-mail: shufangjie@gmail.com).

Y. Yang is with the National Engineering Laboratory for Optical Fiber Sensing Technologies, Wuhan University of Technology, Wuhan 430070, China (e-mail: yangy79@gmail.com).

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/JSTQE.2012.2220896

## II. PRINCIPLE

In the beginning, we analyze the symmetry of cavity to deduce two necessary conditions for high-Q and unidirectional emission in an ARC. First, the deformation of an ARC from a circular cavity should be slight, since high-Q WGMs are supported under continuous and smooth boundary. In such a deformed microcavity, light can be well confined in the cavity when the incident angle is close to  $\pi/2$ , corresponding to the rays localized high above the critical line in phase space. Light of the high-Q WGMs can only leak out from the cavity through tunneling to the lower part of phase space. Second, the boundary shape should have no more than one axis of symmetry. Because for planar deformed cavities, two or more lines of symmetry or  $m$ -fold rotational symmetry with  $m > 1$  will lead to at least two emission directions in far field [3]–[10]. Therefore, one way to achieve unidirectional emission is to design an appropriate cavity shape with only one axis of symmetry and make the pair of emission beams from symmetric clockwise (CW) and counterclockwise (CCW) traveling modes parallel to the axis of symmetry.

According to the aforementioned guidelines, the cavity boundary can be expressed in a general form

$$R(\phi) = \begin{cases} R_0(1 - \varepsilon \sum a_i \cos^i \phi), & -\pi/2 < \phi \leq \pi/2 \\ R_0(1 - \varepsilon \sum b_i \cos^i \phi), & \pi/2 < \phi \leq 3\pi/2 \end{cases} \quad (1)$$

where  $\phi$  is the polar angle about the symmetric  $x$ -axis,  $\varepsilon$  is the general deformation, and  $a_i$  and  $b_i$  are coefficients with  $i \geq 2$ . For the cavity described by this expression, the boundary is continuous and smooth everywhere. For simplicity, we cut off the high-order terms, i.e.,  $a_i = b_i = 0$  for  $i \geq 5$ , to keep the boundary varying slowly. In addition, to ensure no more than one axis of symmetry, at least one inequality needs to be hold among  $a_2 \neq b_2$ ,  $a_3 \neq -b_3$ , and  $a_4 \neq b_4$ .

Fig. 1(a) shows a general boundary shape for unidirectional emission, which satisfies the two necessary conditions analyzed earlier. Rays whose incident angles  $\chi$  are larger than the critical angle  $\chi_c$  ( $\sin \chi_c = 1/n$ , and  $n$  is the refractive index of the cavity) will continuously reflect on the boundary, forming WGMs, and leak out from cavity through barrier tunneling or refraction when  $\chi < \chi_c$ . In both cases, the leaked rays are nearly tangent to the cavity boundary. Therefore, for unidirectional emission, we assume that the CW waves mainly leak at the lower pole while the CCW waves at the upper pole of the cavity as shown by the thick green arrows in Fig. 1(a).

In order to study the underlying mechanism in detail and find the further requirement of unidirectional emission, we take the simple half-quadrupole-half-circle (HQHC) cavity [29] as an example [see Fig. 1(b)]

$$R(\phi) = R_0 \left[ 1 - \frac{1 + \text{sign}(\cos \phi)}{2} \varepsilon \cos^2 \phi \right]. \quad (2)$$

In the limit of  $R_0 \gg \lambda$ , light in cavity can be treated as rays. Although the wave nature of light is important in a microcavity, the ray dynamics is still an efficient tool to analyze and understand the emission properties of ARCs [30]–[34]. The sequence of rays reflected inside the cavity in real space can be

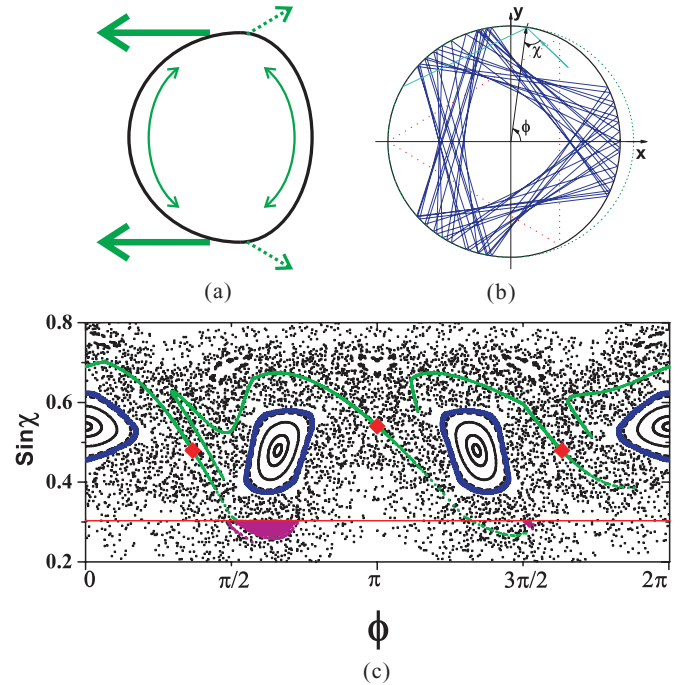


Fig. 1. (a) Schematic illustration of the unidirectional emission from the one-axis symmetric microcavities. (b) Ray trajectories inside a HQHC microcavity with  $\varepsilon = 0.11$ . The blue lines and red dashed lines are the stable and unstable period-3 orbits, respectively. (c) Phase-space structure of the HQHC cavity. The blue circles and red diamonds show the stable islands and fixed points of the unstable period-3 orbits, respectively. The red line corresponds to the critical angle, and the green points show the unstable manifolds. Magenta points are the collection of the locations where the refractions occur for the rays initially distributed uniformly in the region  $\sin \chi > 0.7$ .

represented as the Poincaré surface of section (SOS) in phase space. Each ray reflection on the boundary is recorded with Birkhoff coordinates [35]  $(\phi, \sin \chi)$ , where  $\phi$  and  $\chi$  denote the polar angle of reflection position and incident angle, respectively. By simulating the consequent reflections of rays with random initial positions and incident angles, we can get the SOS of the HQHC cavity by treating the cavity as a billiard [15], which is shown in Fig. 1(c). Here, we only consider CCW propagating rays ( $\sin \chi > 0$ ), while the CW SOS can be obtained by symmetry transformation.

In the cavity with continuous, smooth, and slightly deformed circular-like boundary, there usually exist periodic, quasi-periodic, and chaotic ray trajectories, corresponding to the stable islands, Kolmogorov–Arnold–Moser [35] curves and chaotic sea in phase space, respectively. In addition, in the phase space of 2-D ARC, these different structures are mutually isolated. In real space, rays in the chaotic sea [see the black points in Fig. 1(c)] show chaotic motions, while the conserved rays in stable islands [see the blue curves in Fig. 1(c)] correspond to stable period-3 orbits [see the blue solid lines in Fig. 1(b)] [35]. Schwefel *et al.* [31] have shown that the rays in the chaotic sea will evolve following the unstable manifolds [see the green curves in Fig. 1(c)] and leak out when they cross the critical line ( $\sin \chi = 1/3.3$ ) [see the red line in Fig. 1(c)].

Now, we can generalize the mechanism of the directional emission in the HQHC cavity. In a slightly deformed circular

cavity, there exist WGMs whose energy can be localized high upon the critical line in phase space. A portion of energy will directly leak out through barrier tunneling at curved dielectric interface [36] when the incident angle of ray  $\chi > \chi_c$ . Another portion of energy will enter into the chaotic sea through dynamical tunneling [15], then follows the manifolds and refract out quickly after few reflections. The special regions where the nonuniform manifolds and critical refraction line cross correspond to the place where most rays refract out on the boundary and lead to directional emission. By setting initial rays uniformly distributed in the region of  $\sin\chi > 0.7$ , we get the areas where those rays refract out [see the magenta points in Fig. 1(c)], which are well consistent with the unstable manifolds.

The unstable manifolds cross the unstable fixed points [see the red diamonds in Fig. 1(c)] and surround the stable islands [35]; thus, the relative positions of fixed points and islands determine the shapes of manifolds and the regions on the boundary where rays refract out (we call them “*refraction regions*”). In the case of HQHC cavity in Fig. 1(c), the two stable islands in the center are lower than the third island. Thus, there are two main refraction regions at  $\phi \approx \pi/2$  and  $3\pi/2$ , corresponding to two directional emissions with far-field angles of  $180^\circ$  and  $360^\circ$ . In addition, the fixed point before the refraction region at  $\phi \approx \pi/2$  is lower than that before the other refraction region at  $\phi \approx 3\pi/2$ , making the manifolds here shorter and steeper. As a result, the light has higher probability to enter the refraction region and leak out of the cavity at  $\phi \approx \pi/2$ . Likewise, the CW propagating rays leak out from the boundary at  $\phi \approx 3\pi/2$  and emit to the same direction with CCW propagating rays. Therefore, we can expect unidirectional emission in the HQHC cavity. By the modified Fresnel law [36] weighted ray simulation, we can get the far-field distribution by summing up leaked energy of all refraction events. The unidirectional emission is clearly shown by the peak in Fig. 2(b).

By applying the boundary element method to solve the Maxwell equations numerically [37]–[39], we can observe the high-Q WGMs in this HQHC microcavity. Fig. 2(a) shows the electric field intensity distribution of the quasi-fundamental (radiation quantum number  $q = 1$ ) transverse magnetic (TM) polarized WGM, and Fig. 2(b) shows the corresponding far-field distribution of odd and even parity WGMs. Both the tangent light emission from  $\phi \approx \pi/2$  and  $3\pi/2$  in the near-field pattern and the unidirectional far-field distribution are consistent with our prediction by ray models. The transverse electric polarized modes have similar optical properties, and the results are not shown here. By projecting the wavefunction of the WGM into the phase space by the Husimi-Q function [40], [41], we can further study the correspondence between the ray model and wave simulation. The Husimi distribution under the critical line is concentrated around  $\phi = \pi/2$  and  $3\pi/2$ , which corresponds to the refraction region predicted by ray model in Fig. 1(c).

### III. DESIGN THE ARC FOR LOW REFRACTIVE INDEX MATERIAL

As discussed earlier, we can tune the boundary shape to adjust the phase-space structure and control the emission direction of

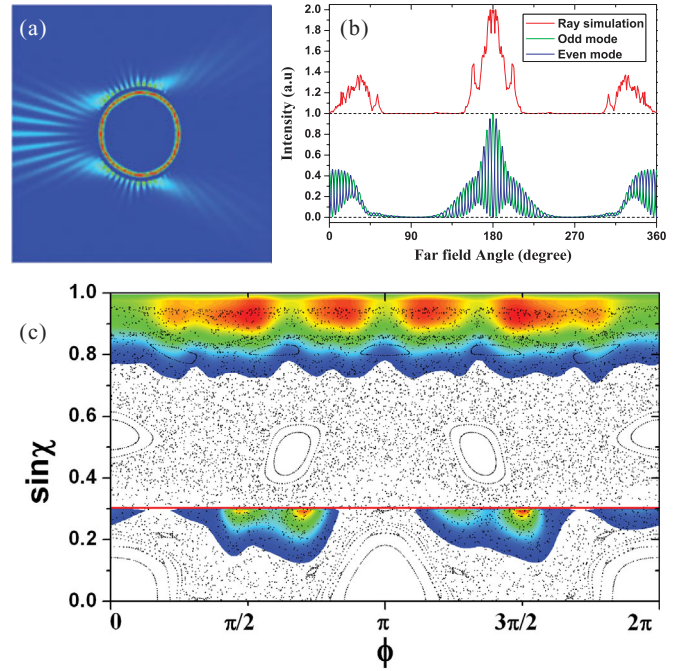


Fig. 2. (a) Spatial distribution of the TM polarized quasi-fundamental WGM in HQHC, with  $kr \approx 26.15$  in vacuum and  $Q = 3.3 \times 10^5$ . The field outside the cavity is magnified for clear illustration. (b) Far-field patterns for the TM polarized mode from ray (red) and wave (blue and green lines for even and odd parity modes) simulations. (c) Husimi distribution of the mode and the SOS in phase space. The Husimi function below the critical line is magnified.

ARCs. Here, we take the extensively studied silica microcavities ( $n = 1.45$ ) as an example, and design a boundary shape for high-Q and unidirectional emission by simple ray simulations.

Similar to the role period-3 orbits in high-index cavities ( $n = 3.3$ ), the period-4 orbits mainly determine the silica cavity's directional emission since they are close to the critical line. In the  $x$ -axis symmetric cavities, there are rectangle- and diamond-shaped period-4 orbits. According to the Poincaré–Birkhoff fixed point theorem, one of them is stable while another should be unstable. Thus, there are two different types of the structure. The first type of the cavities supports stable diamond and unstable rectangle orbits. For example, the HQHC cavity with  $\epsilon = 0.10$  is shown in Fig. 3(c). The detailed SOS [see Fig. 3(a)] shows that the stable period-4 islands near the critical line prevent the emission around  $\phi = \pm\pi/2$ , and allow the directional emission from the refraction regions around  $\phi = \pm\pi/4$ . By the ray simulation with modified Fresnel's law, the multidirectional far-field distribution is calculated in Fig. 3(e), which is consistent with the former experimental study on HQHC microspheres [29].

Therefore, the unidirectional emission can only be realized in the other type of cavities which supports stable rectangle and unstable diamond orbits. In addition, the stable islands near  $\phi = 3\pi/4$  and  $5\pi/4$  and the fixed point (the red triangle in SOS) near  $\phi = \pi/2$  should be lower. To solve the problem, we design the cavity with the assistance of a computer by randomly choosing parameters ( $a_i$  and  $b_i$ ,  $i = 2, 3, 4$ ) and analyzing the phase structure to check whether it satisfies the requirements.

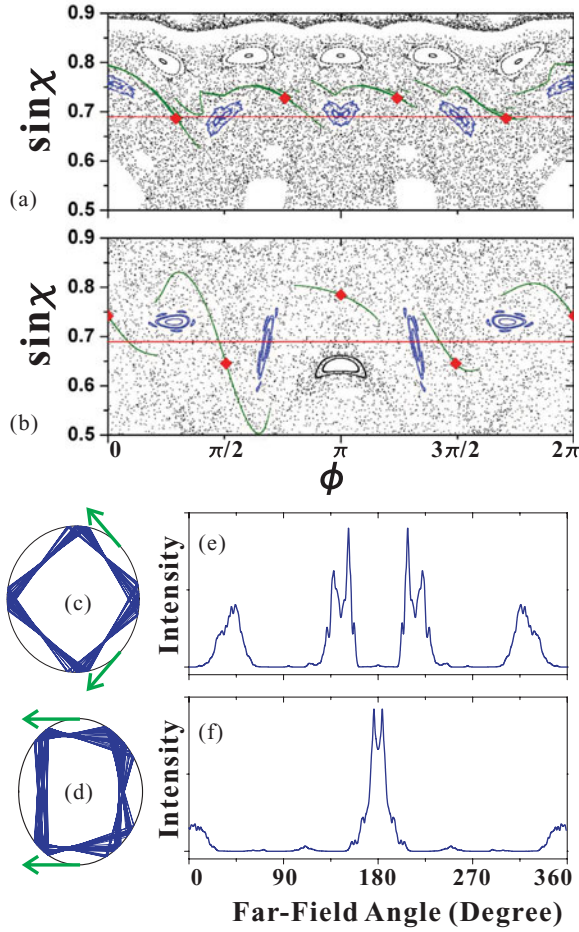


Fig. 3. Ray dynamics of two ARCs made of silica. (a) and (b) are the phase space structures. The black dots are SOS, while the blue dots and red diamonds are the period-4 stable islands and unstable fixed points, respectively. The red line is the critical refraction line, and the green points show the unstable manifolds. Correspondingly, (c) and (d) show the cavity boundaries and the ray trajectories of stable period-4 orbits, while (e) and (f) are the far-field patterns obtained by ray simulations with modified Fresnel's law.

Afterward, the ray simulations are performed on selected cavity shapes to obtain the far-field distributions.

We formulate the directional emission with the function of directionality [25]

$$U = -\frac{\int I(\theta)\cos\theta d\theta}{\int I(\theta)d\theta} \quad (3)$$

where  $I(\theta)$  is the far-field intensity distribution. Finally, we get the cavity of highest directionality ( $U = 0.53$ ) in an ensemble of 1000 cavity shapes, with the parameters  $\varepsilon = 1.0$ ,  $a_2 = 0.2491$ ,  $a_3 = -0.0520$ ,  $a_4 = -0.0783$ ,  $b_2 = 0.2538$ ,  $b_3 = 0.0446$ , and  $b_4 = -0.0214$ . The SOS and the stable rectangle period-4 orbit are shown in Fig. 3(b) and (d). It satisfies the aforementioned conditions and gives good unidirectional far-field patterns as shown in Fig. 3(f). Because the SOS of this cavity assembles a monster face (two upper islands and the island in the center assemble a face with two eyes and a mouth on it, and two long islands around the “mouth” assemble the beard), we call the particular cavity for unidirectional emission the “Face” cavity.

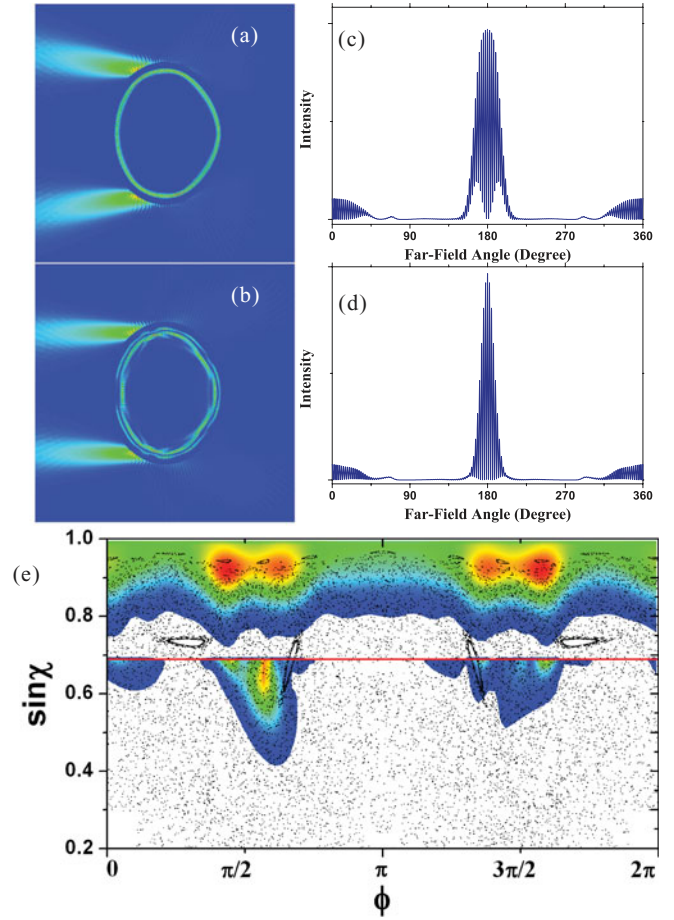


Fig. 4. Spatial electric field intensity distributions of (a) quasi-fundamental and (b) quasi-second-order WGMs in Face cavity with  $\varepsilon = 1.2$ ,  $kr \approx 80.85$ ,  $Q = 3.56 \times 10^5$  for the quasi-fundamental mode, and  $kr \approx 81.14$ ,  $Q = 2.77 \times 10^3$  for the quasi-second-order mode. The field outside the cavity is magnified for clear illustration. (c) and (d) are the corresponding far-field distributions. (e) Husimi distribution of the quasi-fundamental mode and the SOS in phase space.

With further study of the Face cavity against the deformation (vary  $\varepsilon$  but fix  $a_i$  and  $b_i$ ) by ray simulation, we find that the optimal deformation for unidirectional emission is  $\varepsilon = 1.2$ . Then, taking these parameters, we verify the unidirectional emission of the Face cavity by the wave simulations. Fig. 4(a) and (b) shows the near-field patterns of quasi-fundamental and quasi-second-order WGMs, with Q-factor up to  $3.56 \times 10^5$  and  $2.77 \times 10^3$ , respectively. The corresponding far-field distributions, plotted in Fig. 4(c) and (d), indicate a narrow divergence unidirectional emission, where more than 75% of total radiation emission can be collected within  $30^\circ$  and  $20^\circ$ , respectively. Fig. 4(e) is the corresponding Husimi distribution of the quasi-fundamental mode [see Fig. 4(a)], which clearly shows the leakage of energy around  $\phi = \pi/2$ .

In Fig. 5, the dependences of directionality and Q-factor on the deformation  $\varepsilon$  of the Face cavity are studied by wave simulation and compared with the results of ray simulation. With the increase of the deformation, the Q-factor exponentially decreases. This is because the area of chaotic region increases with the deformation, and the larger chaotic region will lead to

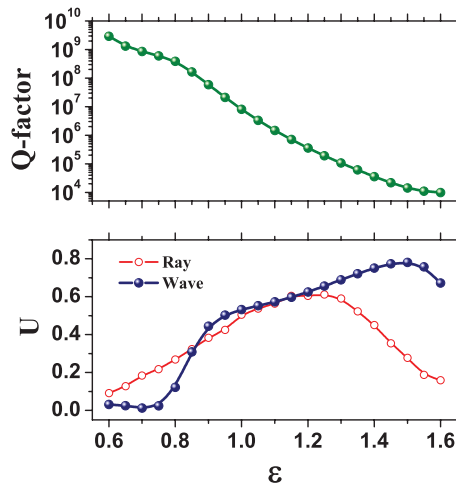


Fig. 5. Q-factor and the directionality  $U$  of the quasi-fundamental WGM in the Face-cavity vary with the deformation  $\varepsilon$ . The solid dots and circles are results of wave and ray simulations, respectively.

easier escape from WGMs to the chaotic region by dynamical tunneling. In addition, the barrier tunneling rate is enhanced at some positions on the boundary when the radius of curvature is reduced. In Fig. 5(b), the ray simulations show good agreement with the wave simulations that the directionality first increases and then decreases with increasing deformation. The wave simulation shows the directionality  $U > 0.5$  when  $\varepsilon = 0.9$ – $1.6$ , with the quality factor ranges from  $10^8$  to  $10^4$ . The tradeoff between  $U$  and  $Q$  when  $\varepsilon$  is smaller than 1.5 can be explained as that the increase of dynamical tunneling rate will lead more light to refract out at the favorite point ( $\phi = \pi/2$ ) to increase the directionality, which clearly indicates the important role the dynamical tunneling played in the directional emission. When the deformation further increases, the phase space structure is greatly changed, and the leaking region is shifted away from  $\phi = \pi/2$ ; hence, the directionality reduces.

#### IV. CONCLUSION

In conclusion, we proposed and examined the principles of unidirectional emission of a high-Q WGM in deformed microcavities. Based on the necessary condition of a continuous and single-line symmetric boundary shape, we can well control the directional emission from an ARC by setting boundary shape to adjust the stable islands and fixed points of unstable orbits in phase space. Actually, to some extent, the design of the dielectric ARC for directional emission is the control of chaotic transport in phase space [42]. Thanks to the ray-wave correspondence, the cavity for high-Q and unidirectional emission can be obtained by simple ray simulations. A simple fabrication-friendly boundary shape is designed for silica ARCs. The wave simulations show that the single major emission peak in far field from high-Q ( $3.56 \times 10^5$ ) WGMs in it has a divergence angle less than  $30^\circ$ , but contains power as large as 75% of total radiation. We expect such ARCs can be applied in high sensitive detections in far field [43] and low-threshold lasers with collimated emission [44].

#### REFERENCES

- [1] K. J. Vahala, *Optical Microcavities*. Singapore: World Scientific, 2004.
- [2] Y.-Z. Yan, C.-L. Zou, S.-B. Yan, F.-W. Sun, Z. Ji, J. Liu, Y.-G. Zhang, L. Wang, C.-Y. Xue, W.-D. Zhang, Z.-F. Han, and J.-J. Xiong, "Packaged silica microsphere-taper coupling system for robust thermal sensing application," *Opt. Express*, vol. 19, pp. 5753–5759, Mar. 2011.
- [3] A. Mekis, J. U. Nöckel, G. Chen, A. D. Stone, and R. K. Chang, "Ray chaos and Q spoiling in lasing droplets," *Phys. Rev. Lett.*, vol. 75, pp. 2682–2685, Oct. 1995.
- [4] C. Gmachl, F. Capasso, E. E. Narimanov, J. U. Nöckel, A. D. Stone, J. Faist, D. L. Sivco, and A. Y. Cho, "High-power directional emission from microlasers with chaotic resonators," *Science*, vol. 280, pp. 1556–1564, Jun. 1998.
- [5] S.-B. Lee, J.-H. Lee, J.-S. Chang, H.-J. Moon, S. W. Kim, and K. An, "Observation of scarred modes in asymmetrically deformed micro-cylinder lasers," *Phys. Rev. Lett.*, vol. 88, pp. 033903-1–033903-4, Jan. 2002.
- [6] S. Lacey, H. Wang, D. Foster, and J. Nöckel, "Directional tunneling escape from nearly spherical optical resonators," *Phys. Rev. Lett.*, vol. 91, pp. 033902-1–033902-4, Jul. 2003.
- [7] Y.-F. Xiao, C.-H. Dong, Z.-F. Han, G.-C. Guo, and Y.-S. Park, "Directional escape from a high-Q deformed microsphere induced by short CO<sub>2</sub> laser pulses," *Opt. Lett.*, vol. 32, pp. 644–646, Mar. 2007.
- [8] T. Harayama, T. Fukushima, P. Davis, P. O. Vaccaro, T. Miyasaka, T. Nishimura, and T. Aida, "Lasing on scar modes in fully chaotic microcavities," *Phys. Rev. E*, vol. 67, pp. 015207-1–015207-4, Mar. 2003.
- [9] W. Fang, A. Yamilov, and H. Cao, "Analysis of high-quality modes in open chaotic microcavities," *Phys. Rev. A*, vol. 72, pp. 023815-1–023815-7, Jan. 2005.
- [10] M. Lebental, J. S. Lauret, R. Hierle, and J. Zyss, "Highly directional stadium-shaped polymer micro-lasers," *Appl. Phys. Lett.*, vol. 88, pp. 031108-1–031108-3, 2006.
- [11] Y. S. Park, A. Cook, and H. Wang, "Cavity QED with diamond nanocrystals and silica microspheres," *Nano Lett.*, vol. 6, pp. 2075–2079, Aug. 2006.
- [12] J. Yang, S.-B. Lee, J.-B. Shim, S. Moon, S.-Y. Lee, S. W. Kim, J.-H. Lee, and K. An, "Enhanced nonresonant optical pumping based on turnstile transport in a chaotic microcavity laser," *Appl. Phys. Lett.*, vol. 93, pp. 061101-1–061101-3, Aug. 2008.
- [13] V. Fiore, Y. Yang, M. Kuzyk, R. Barbour, L. Tian, and H. Wang, "Storing optical information as a mechanical excitation in a silica optomechanical resonator," *Phys. Rev. Lett.*, vol. 107, pp. 133601-1–133601-4, Aug. 2011.
- [14] Y.-S. Park and H. Wang, "Resolved-sideband and cryogenic cooling of an optomechanical resonator," *Nat. Phys.*, vol. 5, pp. 489–493, Jun. 2009.
- [15] H.-J. Stöckmann, *Quantum Chaos*. Cambridge, U.K.: Cambridge Univ. Press, 2000.
- [16] S.-Y. Lee, S. Rim, J.-W. Ryu, T.-Y. Kwon, M. Choi, and C.-M. Kim, "Quasiscattered resonances in a spiral-shaped microcavity," *Phys. Rev. Lett.*, vol. 93, pp. 164102-1–164102-4, Oct. 2004.
- [17] J. Wiersig and J. Main, "Fractal Weyl law for chaotic microcavities: Fresnel's laws imply multifractal scattering," *Phys. Rev. E*, vol. 77, pp. 036205-1–036205-8, Mar. 2008.
- [18] G. D. Chern, H. E. Tureci, A. D. Stone, R. K. Chang, M. Kneissl, and N. M. Johnson, "Unidirectional lasing from InGaN multiple-quantum-well spiral-shaped micropillars," *Appl. Phys. Lett.*, vol. 83, pp. 1710–1712, Sep. 2003.
- [19] M. Hentschel, T.-Y. Kwon, M. A. Belkin, R. Audet, and F. Capasso, "Angular emission characteristics of quantum cascade spiral microlasers," *Opt. Express*, vol. 17, pp. 10335–10343, Jun. 2009.
- [20] M. S. Kurdonglyan, S.-Y. Lee, S. Rim, and C.-M. Kim, "Unidirectional lasing from a microcavity with a rounded isosceles triangle shape," *Opt. Lett.*, vol. 29, pp. 2758–2760, Dec. 2004.
- [21] J. Wiersig and M. Hentschel, "Unidirectional light emission from high-Q modes in optical microcavities," *Phys. Rev. A*, vol. 73, pp. 031802-1–031802-4, Mar. 2006.
- [22] J. W. Ryu, S. Y. Lee, and S. W. Kim, "Coupled nonidentical microdisks: Avoided crossing of energy levels and unidirectional far-field emission," *Phys. Rev. A*, vol. 79, pp. 053858-1–053858-7, May 2009.
- [23] Q. J. Wang, C. Yan, N. Yu, J. Unterhinninghofen, J. Wiersig, C. Pflugl, L. Diehl, T. Edamura, M. Yamanishi, H. Kan, and F. Capasso, "Whispering-gallery mode resonators for highly unidirectional laser action," *Proc. Natl. Acad. Sci. USA*, vol. 107, pp. 22407–22412, Dec. 2010.
- [24] J. Wiersig and M. Hentschel, "Combining directional light output and ultralow loss in deformed microdisks," *Phys. Rev. Lett.*, vol. 100, pp. 033901-1–033901-4, Jan. 2008.

- [25] Q. Song, W. Fang, B. Liu, S.-T. Ho, G. S. Solomon, and H. Cao, "Chaotic microcavity laser with high quality factor and unidirectional output," *Phys. Rev. A*, vol. 80, pp. 041807-1–041807-4, Oct. 2009.
- [26] C. Yan, Q. J. Wang, L. Diehl, M. Hentschel, J. Wiersig, N. Yu, C. Pflugl, F. Capasso, M. A. Belkin, T. Edamura, M. Yamanishi, and H. Kan, "Directional emission and universal far-field behavior from semiconductor lasers with limaçon-shaped microcavity," *Appl. Phys. Lett.*, vol. 94, pp. 251101-1–251101-3, Jun. 2009.
- [27] S. Shinohara, M. Hentschel, J. Wiersig, T. Sasaki, and T. Harayama, "Ray-wave correspondence in limaçon-shaped semiconductor microcavities," *Phys. Rev. A*, vol. 80, pp. 031801-1–031801-4, Sep. 2009.
- [28] C.-H. Yi, M.-W. Kim, and C.-M. Kim, "Lasing characteristics of a limaçon-shaped microcavity laser," *Appl. Phys. Lett.*, vol. 95, pp. 141107-1–141107-3, Oct. 2009.
- [29] Y.-F. Xiao, C.-H. Dong, C.-L. Zou, Z.-F. Han, L. Yang, and G.-C. Guo, "Low-threshold microlaser in a high-Q asymmetrical microcavity," *Opt. Lett.*, vol. 34, pp. 509–511, Feb. 2009.
- [30] J. U. Nöckel and A. D. Stone, "Ray and wave chaos in asymmetric resonant optical cavities," *Nature*, vol. 385, pp. 45–47, Jan. 1997.
- [31] H. G. L. Schwefel, N. B. Rex, H. E. Tureci, R. K. Chang, A. D. Stone, T. Ben-Messaoud, and J. Zyss, "Dramatic shape sensitivity of directional emission patterns from similarly deformed cylindrical polymer lasers," *J. Opt. Soc. Amer. B*, vol. 21, pp. 923–934, May. 2004.
- [32] V. A. Podolskiy and E. E. Narimanov, "Chaos-assisted tunneling in dielectric microcavities," *Opt. Lett.*, vol. 30, pp. 474–476, Mar. 2005.
- [33] S. Shinohara, T. Fukushima, and T. Harayama, "Light emission patterns from stadium-shaped semiconductor microcavity lasers," *Phys. Rev. A*, vol. 77, pp. 033807-1–033807-5, Mar. 2008.
- [34] S. B. Lee, J. Yang, S. Moon, J. H. Lee, K. An, J. B. Shim, H. W. Lee, and S. W. Kim, "Universal output directionality of single modes in a deformed microcavity," *Phys. Rev. A*, vol. 75, pp. 001802-1–001802-4, Jan. 2007.
- [35] L. E. Reichl, *The Transition to Chaos in Conservative Classical Systems: Quantum Manifestations*. New York: Springer-Verlag, 1992.
- [36] M. Hentschel and H. Schomerus, "Fresnel laws at curved dielectric interfaces of microresonators," *Phys. Rev. E*, vol. 65, pp. 045603-1–045603-4, Apr. 2002.
- [37] J. Wiersig, "Boundary element method for resonances in dielectric microcavities," *J. Opt. A: Pure Appl. Opt.*, vol. 5, pp. 53–60, Dec. 2003.
- [38] C.-L. Zou, H. G. L. Schwefel, F.-W. Sun, Z.-F. Han, and G.-C. Guo, "Quick root searching method for resonances of dielectric optical microcavities with the boundary element method," *Opt. Express*, vol. 19, pp. 15669–15678, Aug. 2011.
- [39] C.-L. Zou, Y. Yang, Y.-F. Xiao, C.-H. Dong, Z.-F. Han, and G.-C. Guo, "Accurately calculating high quality factor of whispering-gallery modes with boundary element method," *J. Opt. Soc. Amer. B*, vol. 26, pp. 2050–2053, Nov. 2009.
- [40] K. Husimi, "Some formal properties of the density matrix," *Proc. Phys. Math. Soc. Jpn.*, vol. 22, pp. 264–314, Feb. 1940.
- [41] M. Hentschel, H. Schomerus, and R. Schubert, "Husimi functions at dielectric interfaces: Inside–outside duality for optical systems and beyond," *Europhys. Lett.*, vol. 62, pp. 636–642, Jun. 2003.
- [42] J.-B. Shim, S.-B. Lee, S. W. Kim, S.-Y. Lee, J. Yang, S. Moon, J.-H. Lee, and K. An, "Uncertainty-limited turnstile transport in deformed microcavities," *Phys. Rev. Lett.*, vol. 100, pp. 174102-1–174102-4, May. 2008.
- [43] Q. Song and Y. L. Kim, "Perturbation of nanoparticle on deformed microcavity," *J. Lightw. Technol.*, vol. 28, pp. 2818–2821, Oct. 2010.
- [44] X.-F. Jiang, Y.-F. Xiao, C.-L. Zou, L. He, C.-H. Dong, B.-B. Li, Y. Li, F.-W. Sun, L. Yang, and Q. Gong, "Highly unidirectional emission and ultralow-threshold lasing from on-chip ultrahigh-Q microcavities," *Adv. Mater.*, vol. 24, pp. OP260–264, Sep. 2012.

**Chang-Ling Zou** received the B.S. degree from the University of Science and Technology of China, Hefei, China, in 2008, where he is currently working toward the Ph.D. degree in optics.

**Fang-Wen Sun** received the B.S. and Ph.D. degrees from the University of Science and Technology of China, Hefei, China (USTC), in 2001 and 2007, respectively.

He is currently an Associate Professor at USTC.

**Chun-Hua Dong** received the B.S. and Ph.D. degrees from the University of Science and Technology of China, Hefei, China, in 2006 and 2011, respectively. He is currently a Postdoctoral Scholar at Oregon University, Eugene, OR.

**Fang-Jie Shu** received the B.S. and Ph.D. degrees in physics from the University of Science and Technology of China, Hefei, China, in 2002 and 2007, respectively.

He is currently an Associate Professor at Shangqiu Normal University, Shangqiu, He'nan, China.

**Xiao-Wei Wu** received the B.S. degree from Tianjin University, Tianjin, China, and the Ph.D. degree from University of Science and Technology of China, Hefei, China, in 2005 and 2010, respectively.

He is currently at Anhui Askay Quantum Technology Co., Ltd., China.

**Jin-Ming Cui** received the B.S. degree from the University of Science and Technology of China, Hefei, China, in 2008, where he is currently working toward the Ph.D. degree in optics.

**Yong Yang** received the B.S. degree from Wuhan University, Wuhan, China, and the Ph.D. degree from the University of Science and Technology of China, Hefei, China, in 2002 and 2008, respectively.

During 2008–2010, he was a Postdoctoral Researcher in the Laboratory of Quantum Information and Computation. In 2010, he joined the University of Oregon as a Postdoctoral Researcher. He is currently a Postdoctoral Scholar in Tyndall National Institute, University College Cork, Cork, Ireland, and is also a Lecturer in the National Engineering Laboratory for Optical Fiber Sensing Technologies, Wuhan University of Technology, Wuhan, China. His research interests include sensing application for nanofibers and whispering gallery mode microcavities, and optomechanics.

**Zheng-Fu Han** received the B.S. degree from the Anhui University, Hefei, China, and the Master's degree from the University of Science and Technology of China, Hefei, in 1983 and 1990, respectively, both in physics.

His research interests include the quantum information and its device design, protocol and realization of quantum cryptography, and its applications.

**Guang-Can Guo** received the B.S. degree from the University of Science and Technology of China (USTC), Hefei, China, in 1965.

He is currently the Chief President of the Key Laboratory of Quantum Information, the Professor of Department of Physics of USTC, the President of Chinese Association of Quantum Optics, and the Chief Scientist of "Quantum Communication and Quantum Information Technology"—National Fundamental Research Program. He is also an Academician of Chinese Academy of Sciences, Beijing, China.