Transfer matrix

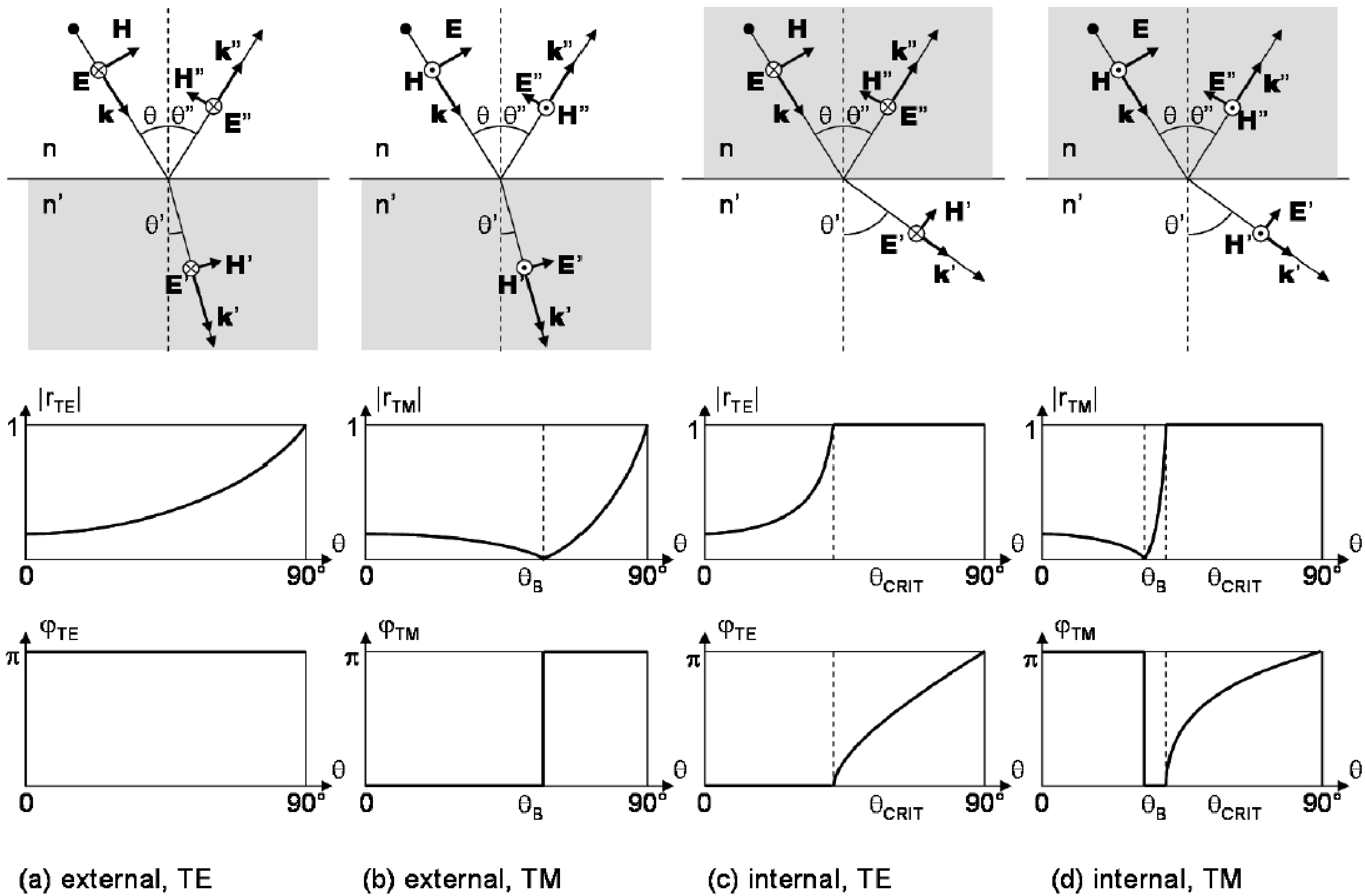
P. Yeh. *Optical Waves in Layered Media*. John Wiley and Sons, 1988.

光在介质表面的反射和折射

$$\begin{cases} t_s = \dfrac{2\sin i_2 \cos i_1}{\sin(i_1 + i_2)} \\ t_p = \dfrac{2\sin i_2 \cos i_1}{\sin(i_1 + i_2)\cos(i_1 - i_2)} \end{cases}$$

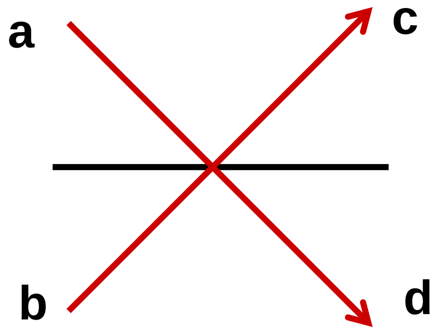
$$\begin{cases} r_s = -\dfrac{\sin(i_1 - i_2)}{\sin(i_1 + i_2)} \\ r_p = \dfrac{\tan(i_1 - i_2)}{\tan(i_1 + i_2)} \end{cases}$$

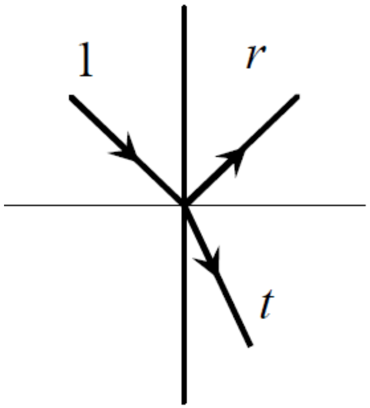
n/n'=1.5 n'/n=1.5

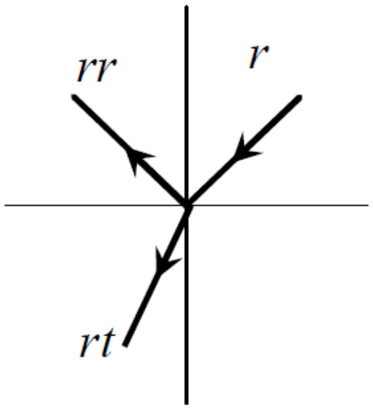


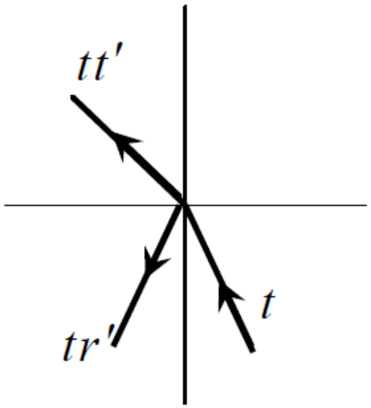
n
n'
H
E
k
k"
H"
E"
θ
θ"
θ'
H'
E'
k'
|r_TE|
|r_TM|
1
0
90°
θ
θ_B
θ_CRIT
φ_TE
φ_TM
π
(a) external, TE
(b) external, TM
(c) internal, TE
(d) internal, TM

光学分束器







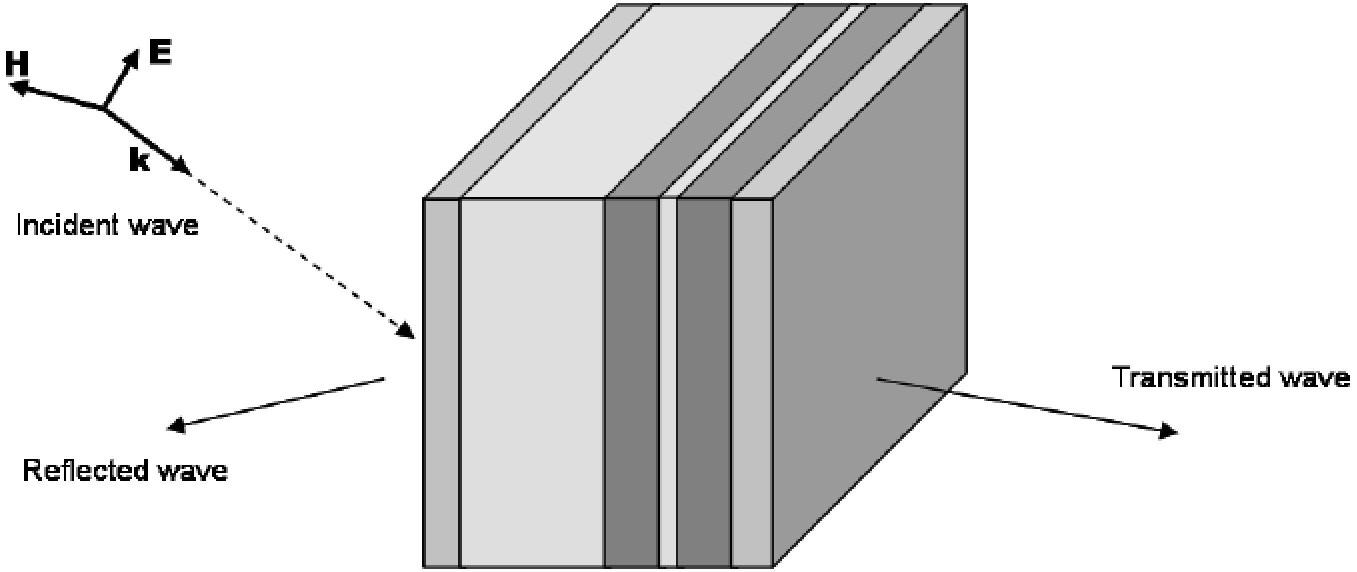


$$\begin{cases} r^2 + tt' = 1 \\ r' = -r \end{cases}$$

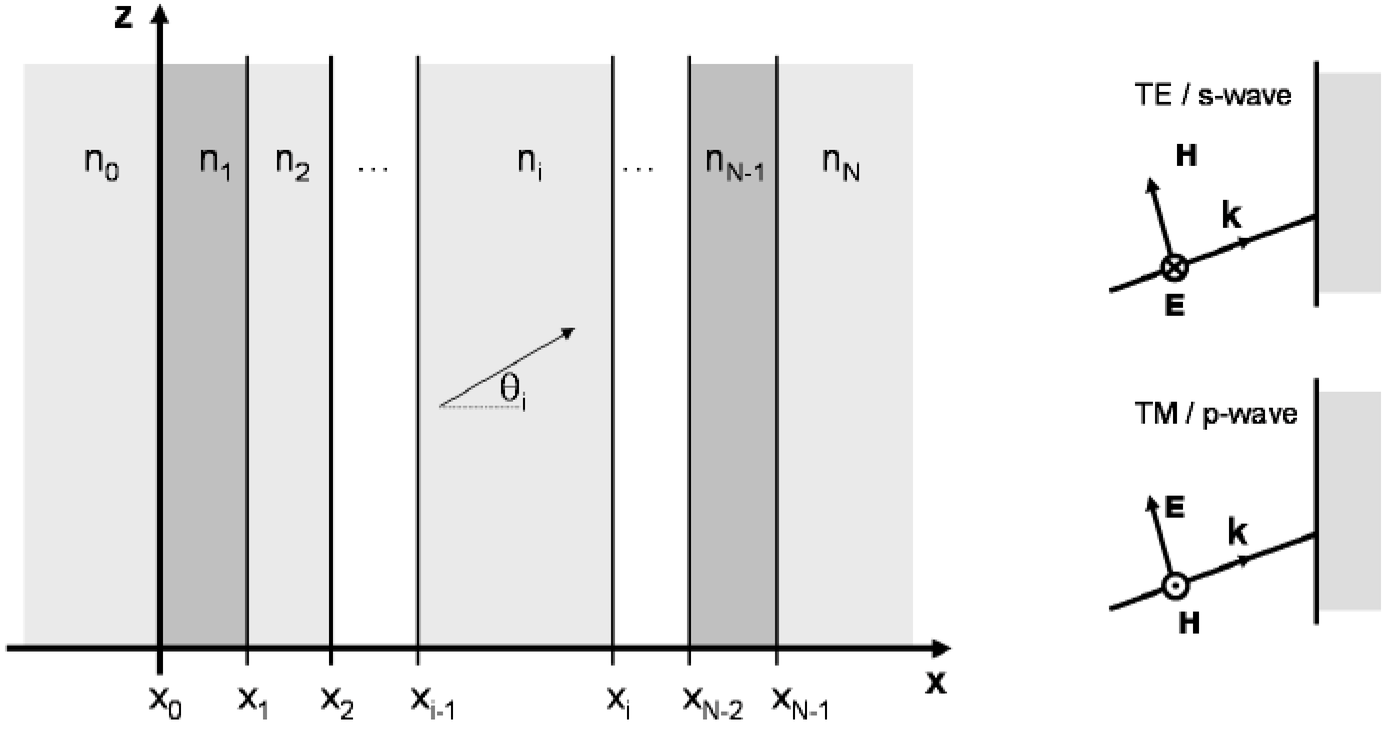
$$c = a \cdot r$$
$$d = a \cdot t$$
$$c = b \cdot t'$$
$$d = b \cdot r'$$

$$\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} t' & r \\ r' & t \end{pmatrix} \begin{pmatrix} b \\ a \end{pmatrix} = \begin{pmatrix} t & r \\ -r & t \end{pmatrix} \begin{pmatrix} b \\ a \end{pmatrix}$$

多层光学薄膜



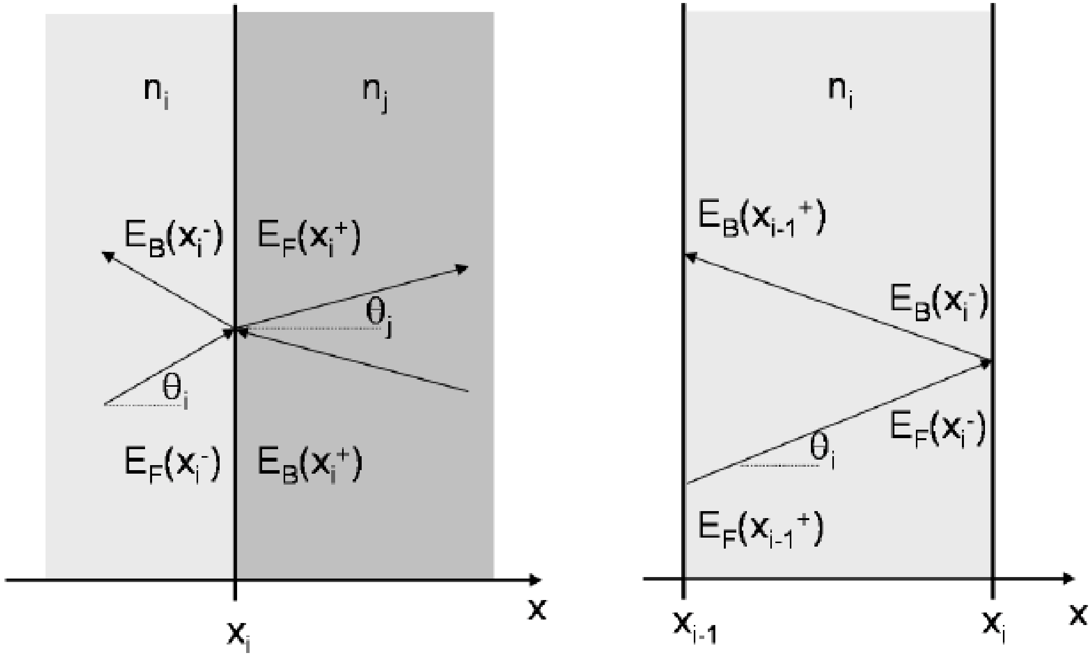
$$n(x) = \begin{cases} n_0, & x < x_0, \\ n_1, & x_0 < x < x_1, \\ n_2, & x_1 < x < x_2, \\ \vdots & \\ n_N, & x_{N-1} < x, \end{cases}$$



(a) (b)

$$\begin{aligned} E(x,y,z) &= A_F e^{-j(k_{x,i}x + k_z z)} + A_B e^{-j(-k_{x,i}x + k_z z)} \\ &= A_F e^{-jk_{x,i}x} e^{-jk_z z} + A_B e^{+jk_{x,i}x} e^{-jk_z z} \\ &= E_F(x) e^{-jk_z z} + E_B(x) e^{-jk_z z} \end{aligned}$$

$$k_{x,i} = \left[(n_i k_0)^2 - k_z^2 \right]^{1/2}, i = 0, 1, 2, ..., N, \qquad k_{x,i} = n_i k_0 \cos\theta_i.$$



(a) (b)

$$\begin{bmatrix} E_F(x_i^+) \\ E_B(x_i^-) \end{bmatrix} = \begin{bmatrix} t_{ij} & r_{ji} \\ r_{ij} & t_{ji} \end{bmatrix} \begin{bmatrix} E_F(x_i^-) \\ E_B(x_i^+) \end{bmatrix}$$

$$r_{ij} = -r_{ji}$$

$$t_{ij}t_{ji} - r_{ij}r_{ji} = 1$$

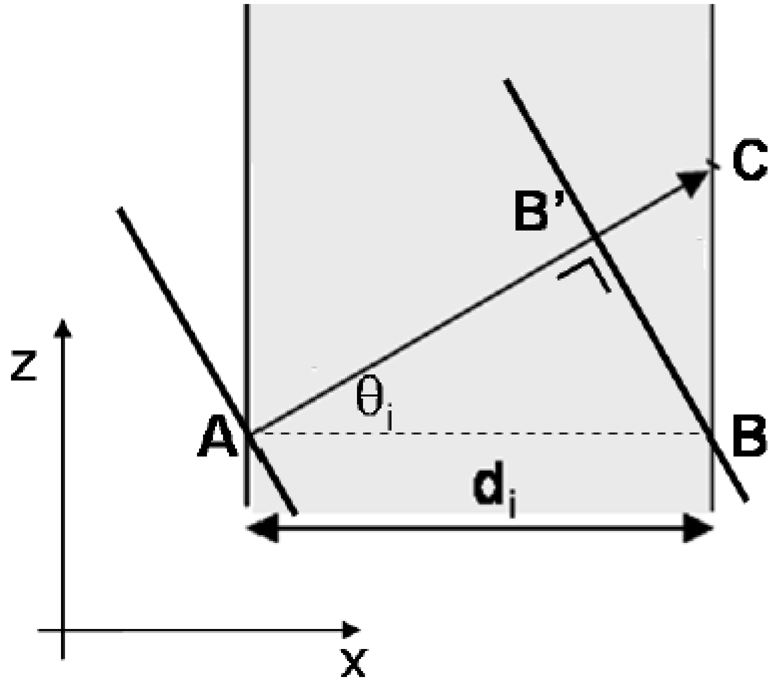
界面变换矩阵

$$\begin{bmatrix} E_F(x_i^-) \\ E_B(x_i^-) \end{bmatrix} = \begin{bmatrix} \frac{1}{t_{ij}} & -\frac{r_{ji}}{t_{ij}} \\ \frac{r_{ij}}{t_{ij}} & t_{ji} - \frac{r_{ij}}{t_{ij}} r_{ji} \end{bmatrix} \begin{bmatrix} E_F(x_i^+) \\ E_B(x_i^+) \end{bmatrix}$$

$$\begin{bmatrix} E_F(x_i^-) \\ E_B(x_i^-) \end{bmatrix} = \frac{1}{t_{ij}} \begin{bmatrix} 1 & r_{ij} \\ r_{ij} & 1 \end{bmatrix} \begin{bmatrix} E_F(x_i^+) \\ E_B(x_i^+) \end{bmatrix}$$

$$\mathbf{T}_{ij} = \frac{1}{t_{ij}} \begin{bmatrix} 1 & r_{ij} \\ r_{ij} & 1 \end{bmatrix}$$

传输矩阵

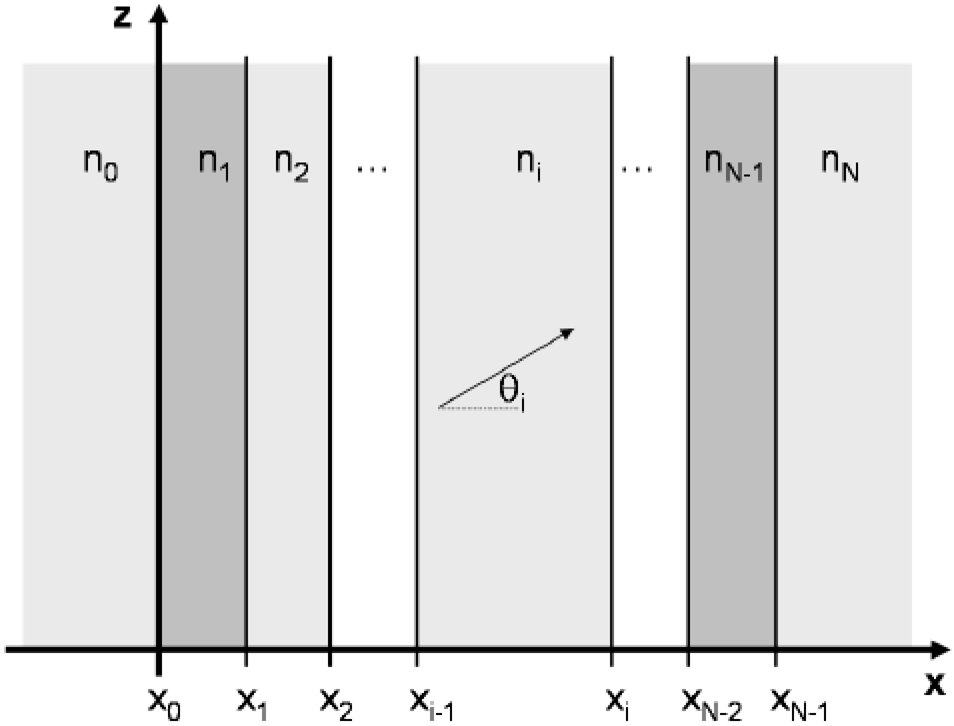


$$\begin{aligned} E_F(x_i^-) &= E_F(x_{i-1}^+)e^{-jk_{x,i}d_i} \\ E_B(x_{i-1}^+) &= E_B(x_i^-)e^{-jk_{x,i}d_i}, \end{aligned}$$

$$\mathbf{T}_i = \begin{bmatrix} e^{j\Phi_i} & 0 \\ 0 & e^{-j\Phi_i} \end{bmatrix}$$

$$\Phi_i = k_{x,i}d_i = \frac{2\pi}{\lambda_0} n_i d_i \cos\theta_i$$

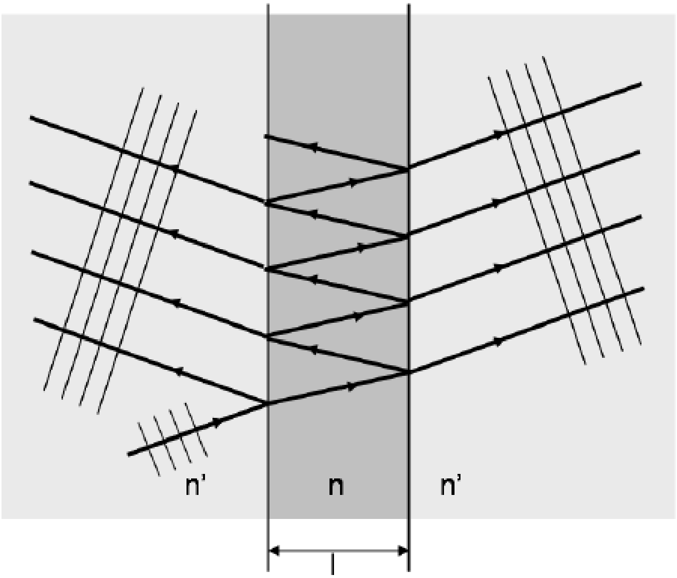
Transfer matrix



$$\begin{bmatrix} E_F(x_0^-) \\ E_B(x_0^-) \end{bmatrix} = \mathbf{T}_{0N} \begin{bmatrix} E_F(x_{N-1}^+) \\ E_B(x_{N-1}^+) \end{bmatrix} = \begin{bmatrix} T_{11}^{0N} & T_{12}^{0N} \\ T_{21}^{0N} & T_{22}^{0N} \end{bmatrix} \begin{bmatrix} E_F(x_{N-1}^+) \\ E_B(x_{N-1}^+) \end{bmatrix}$$

$$\mathbf{T}_{0N} = \mathbf{T}_{01}\mathbf{T}_{1}\mathbf{T}_{12}\mathbf{T}_{2}...\mathbf{T}_{(N-1)}\mathbf{T}_{(N-1)N}$$

F-P cavity



$$\mathbf{T}_{12} = \frac{1}{t_{12}} \begin{bmatrix} 1 & r_{12} \\ r_{12} & 1 \end{bmatrix} = \frac{1}{t_{12}} \begin{bmatrix} 1 & r \\ r & 1 \end{bmatrix}$$

$$\mathbf{T}_{23} = \frac{1}{t_{23}} \begin{bmatrix} 1 & r_{23} \\ r_{23} & 1 \end{bmatrix} = \frac{1}{t_{23}} \begin{bmatrix} 1 & -r \\ -r & 1 \end{bmatrix}$$

$$r = r_{12} = -r_{23}.$$

$$\mathbf{T}_2 = \begin{bmatrix} e^{j\Phi} & 0 \\ 0 & e^{-j\Phi} \end{bmatrix} = e^{j\Phi} \begin{bmatrix} 1 & 0 \\ 0 & e^{-2j\Phi} \end{bmatrix}$$

$$\Phi = \frac{2\pi}{\lambda_0} nl \cos\theta$$

$$\mathbf{T}_{13} = \mathbf{T} = \mathbf{T}_{12}\mathbf{T}_2\mathbf{T}_{23} = \frac{1}{t_{12}t_{23}} e^{j\Phi} \begin{bmatrix} 1 - r^2 e^{-2j\Phi} & -r(1 - e^{-2j\Phi}) \\ r(1 - e^{-2j\Phi}) & -r^2 + e^{-2j\Phi} \end{bmatrix}$$

$$r_{FP} = \frac{T_{21}}{T_{11}} = \frac{r(1-e^{-2j\Phi})}{1-r^2e^{-2j\Phi}}$$

$$t_{FP} = \frac{1}{T_{11}} = \frac{t_{12}t_{23}e^{-j\Phi}}{1-r^2e^{-2j\Phi}},$$

$$R_{FP} = \left|\frac{T_{21}}{T_{11}}\right|^2 = \frac{4r^2\sin^2\Phi}{(1-r^2)^2+4r^2\sin^2\Phi}$$

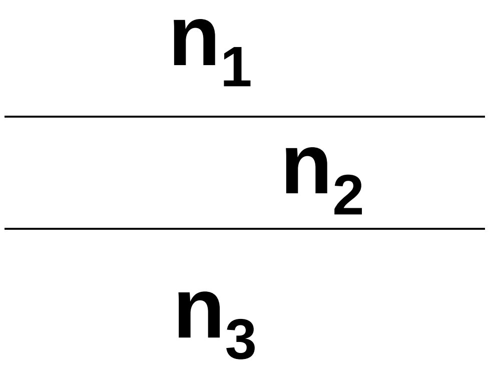
$$T_{FP} = \left|\frac{1}{T_{11}}\right|^2 = \frac{|t_{12}t_{23}|^2}{(1-r^2)^2+4r^2\sin^2\Phi}$$

$$R = r_{12}^2 = r_{23}^2 = r^2$$

$$T = t_{12}t_{23}.$$

$$\delta = 2\Phi = 2\frac{2\pi}{\lambda_0}nl\cos\theta.$$

Anti-reflective coating



$$n_1 < n_2 < n_3$$

$$d = \frac{1}{4}\frac{\lambda_0}{n_2}$$

$$r = \frac{r_{12} + r_{23}e^{-2j\Phi}}{1 + r_{12}r_{23}e^{-2j\Phi}} = \frac{r_{12} - r_{23}}{1 - r_{12}r_{23}}$$

$$\Phi = \frac{2\pi}{\lambda_0}n_2 d = \frac{\pi}{2} \qquad r_{ij} = \frac{n_i - n_j}{n_i + n_j}$$

$$n_2 = \sqrt{n_1 n_3}.$$