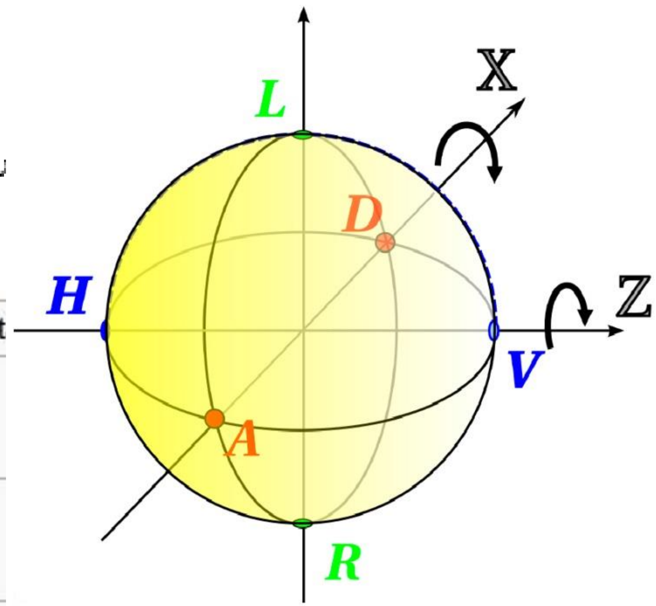
# **Jones vector** by R. C. Jones in 1941

**http://en.wikipedia.org/wiki/Jones_vector**

$$\begin{pmatrix} E_x(t) \\ E_y(t) \end{pmatrix} = \begin{pmatrix} E_{0x}e^{i(kz-\omega t+\phi_x)} \\ E_{0y}e^{i(kz-\omega t+\phi_y)} \end{pmatrix} = \begin{pmatrix} E_{0x}e^{i\phi_x} \\ E_{0y}e^{i\phi_y} \end{pmatrix} e^{i(kz-\omega$$

| Polarization | Corresponding Jones vector | Typical ket Not |
|---|---|---|
| Linear polarized in the x-direction<br>Typically called 'Horizontal' | $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ | $\lvert H\rangle$ |
| Linear polarized in the y-direction<br>Typically called 'Vertical' | $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ | $\lvert V\rangle$ |
| Linear polarized at 45° from the x-axis<br>Typically called 'Diagonal' L+45 | $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ | $\lvert D\rangle = \frac{1}{\sqrt{2}}(\lvert H\rangle + \lvert V\rangle)$ |
| Linear polarized at −45° from the x-axis<br>Typically called 'Anti-Diagonal' L-45 | $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ | $\lvert A\rangle = \frac{1}{\sqrt{2}}(\lvert H\rangle - \lvert V\rangle)$ |
| Right Hand Circular Polarized<br>Typically called RCP or RHCP | $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$ | $\lvert R\rangle = \frac{1}{\sqrt{2}}(\lvert H\rangle - i\lvert V\rangle)$ |
| Left Hand Circular Polarized<br>Typically called LCP or LHCP | $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ +i \end{pmatrix}$ | $\lvert L\rangle = \frac{1}{\sqrt{2}}(\lvert H\rangle + i\lvert V\rangle)$ |

## **Bloch sphere**



# Jones matrices

| Optical element | Corresponding Jones matrix |
|---|---|
| Linear polarizer with axis of transmission horizontal[1] | $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ |
| Linear polarizer with axis of transmission vertical[1] | $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ |
| Linear polarizer with axis of transmission at ±45° with the horizontal[1] | $\frac{1}{2}\begin{pmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{pmatrix}$ |
| Right circular polarizer[1] | $\frac{1}{2}\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$ |
| Left circular polarizer[1] | $\frac{1}{2}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$ |
| Linear polarizer with axis of transmission at angle $\theta$ with the horizontal. (Shown construction from rotating up from the horizontal into the polarizing element, the polarizing element, and then rotating back down into the horizontal.) | $\begin{pmatrix} \cos^2(\theta) & \cos(\theta)\sin(\theta) \\ \sin(\theta)\cos(\theta) & \sin^2(\theta) \end{pmatrix} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix}$ |

| Phase retarders | Corresponding Jones matrix |
|---|---|
| Quarter-wave plate with fast axis vertical[2][note 1] | $e^{i\pi/4}\begin{pmatrix}1 & 0\\ 0 & -i\end{pmatrix}$ |
| Quarter-wave plate with fast axis horizontal[2] | $e^{i\pi/4}\begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix}$ |
| Half-wave plate with fast axis at angle $\theta$ w.r.t the horizontal axis[3] | $\begin{pmatrix}\cos 2\theta & \sin 2\theta\\ \sin 2\theta & -\cos 2\theta\end{pmatrix}$ |
| Any birefringent material (phase retarder)[4] | $\begin{pmatrix}e^{i\phi_x}\cos^2\theta + e^{i\phi_y}\sin^2\theta & (e^{i\phi_x} - e^{i\phi_y})\cos\theta\sin\theta\\ (e^{i\phi_x} - e^{i\phi_y})\cos\theta\sin\theta & e^{i\phi_x}\sin^2\theta + e^{i\phi_y}\cos^2\theta\end{pmatrix}$ |

# Stokes parameters

by George Gabriel Stokes in 1852

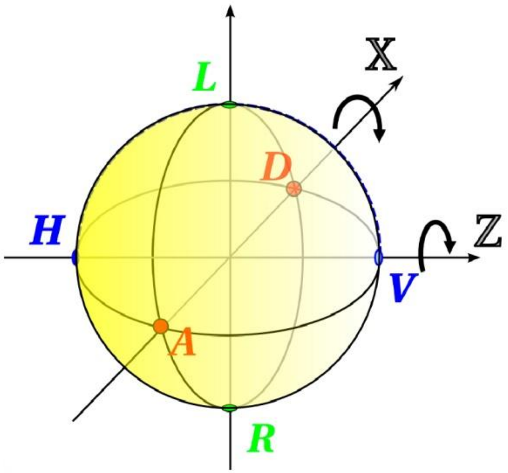
**http://en.wikipedia.org/wiki/Stokes_parameters**

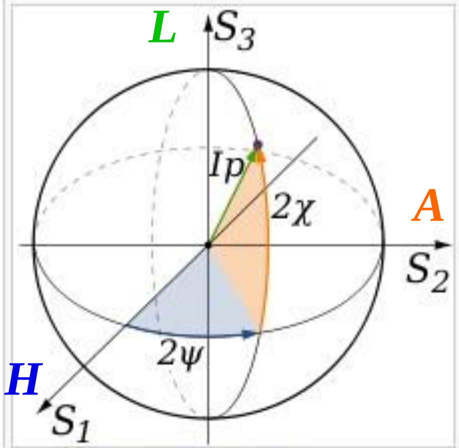
$$S_0 = I$$
$$S_1 = pI\cos 2\psi \cos 2\chi$$
$$S_2 = pI\sin 2\psi \cos 2\chi$$
$$S_3 = pI\sin 2\chi$$

**Stokes vectors**

$$\vec{S} = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

## Bloch (Poincaré) sphere



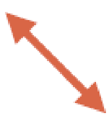



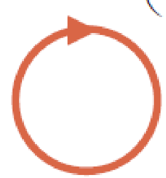

The Poincaré sphere is the parametrisation of the last three Stokes' parameters in spherical coordinates

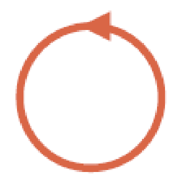
# 各种偏振态

$$S_{\mathrm{LHP}} = I_0 \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad S_{\mathrm{LVP}} = I_0 \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \quad S_{\mathrm{L+45P}} = I_0 \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix},$$

$$S_{\mathrm{L-45P}} = I_0 \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \quad S_{\mathrm{RCP}} = I_0 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad S_{\mathrm{LCP}} = I_0 \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix},$$







## Stokes parameters

$$\begin{aligned} I &\equiv \langle E_x\rangle^2 + \langle E_y\rangle^2, \\ I &= \langle E_a\rangle^2 + \langle E_b\rangle^2, \\ I &= \langle E_l\rangle^2 + \langle E_r\rangle^2, \\ Q &\equiv \langle E_x\rangle^2 - \langle E_y\rangle^2, \\ U &\equiv \langle E_a\rangle^2 - \langle E_b\rangle^2, \\ V &\equiv \langle E_l\rangle^2 - \langle E_r\rangle^2, \end{aligned}$$

$$\vec{S} = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

**x, y 水平垂直线偏振（H,V）**
**a, b +/-45$^o$偏振基 （A,D）**
**l, r左右旋圆偏振基 （L,R）**

# Degree of polarization （DOP）

The DOP $\mathscr{P}$ is defined by the equation

$$\mathscr{P} = \frac{I_{\text{pol}}}{I_{\text{tot}}} = \frac{\sqrt{S_1^{\,2} + S_2^{\,2} + S_3^{\,2}}}{S_0}, \qquad 0 \le \mathscr{P} \le 1,$$

where $I_{tot}$ is the total intensity. These results show that the relation between the Stokes parameters must be broadened to

$$S_0^{\,2} \ge S_1^{\,2} + S_2^{\,2} + S_3^{\,2},$$

where the = and > sign indicate completely and unpolarized/partially polarized light, respectively.