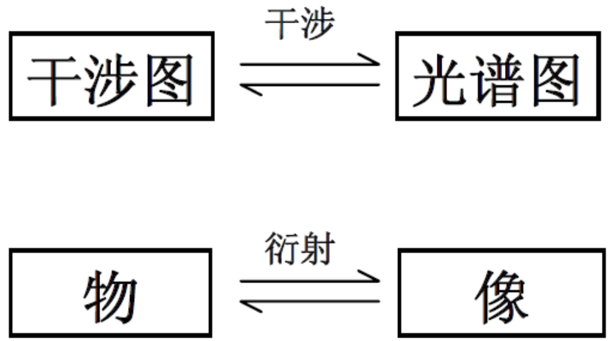
第五章：信息光学简介

5-01衍射系统的屏函数和相因子判断法

1. 衍射系统及屏函数
2. 相因子判断法
3. 透镜的作用及位相变换函数

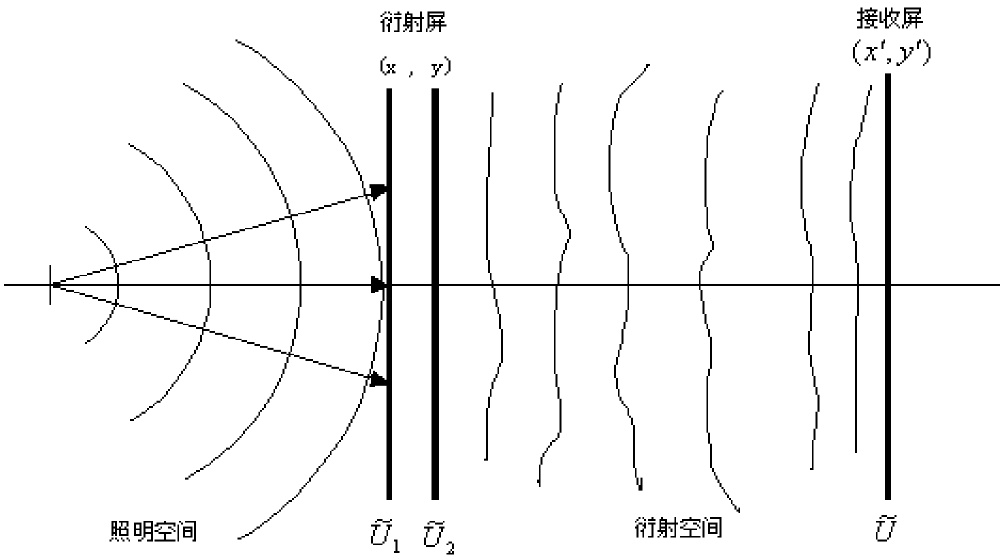
用变换的观点看成像和光谱



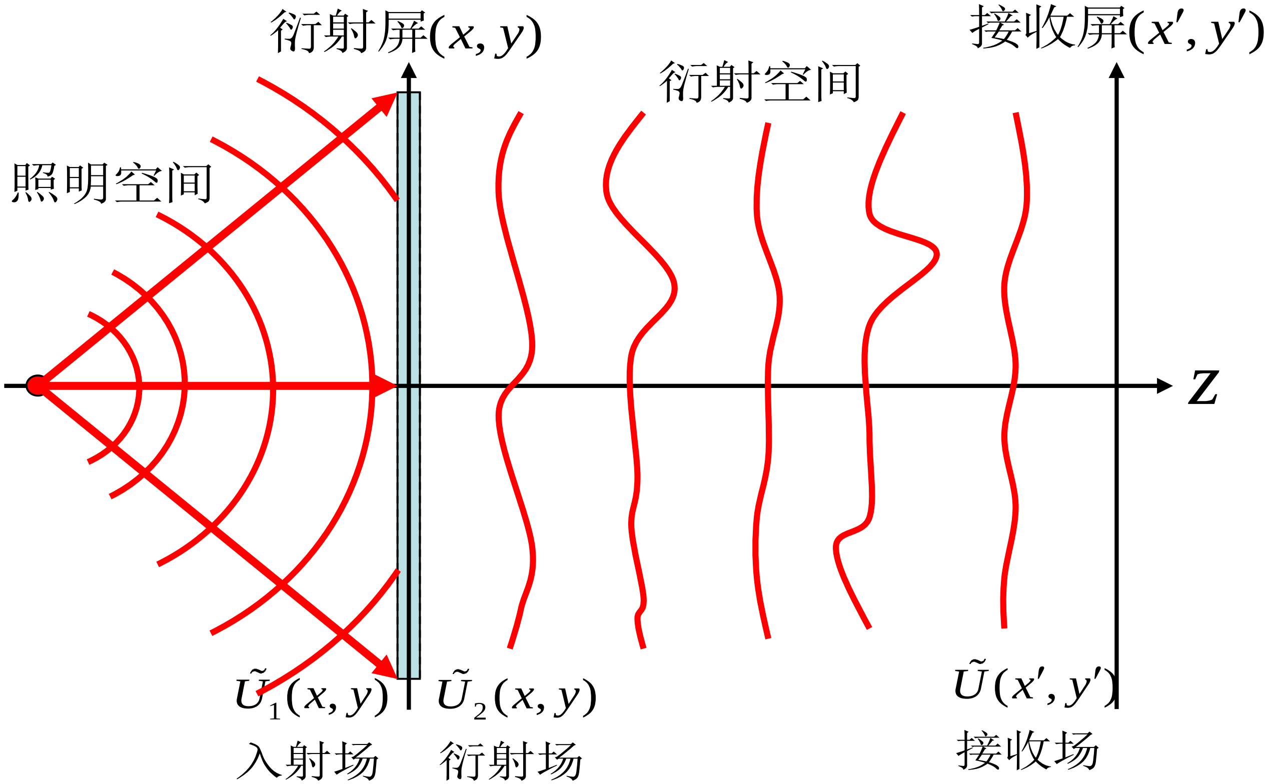
- 光的衍射和干涉最基本的方法: 光的相干叠加。
- 另外一个角度: 入射波场，遇到障碍物之后，波场中各种物理量重新分布，相当于“波前(函数)重构”。衍射障碍物将简单的入射场变换成了复杂的衍射场。
- →可以从障碍物对波场的（数学）变换作用，来分析衍射。
- →从更广义的角度，不仅仅是相干波场的障碍物，非相干系统中的一切使波场或者波面产生改变的因素，它们的作用都可以应用变换的方法处理。

1. 衍射系统及屏函数

- 能使波前的复振幅（**波前函数**）发生改变的物，统称为**衍射屏。**
- 衍射屏将波的空间分为前场和后场两部分。前场为**照明空间**，后场为**衍射空间。**



- 波在衍射屏的前后表面处的**复振幅**或**波前函数**分别称为**入射场**、**透射场**（或**反射场**），接收屏上的复振幅为**接收场**。



$\tilde{U}_1(x, y)$

入射场

$\tilde{U}_2(x, y)$

透射或反射场

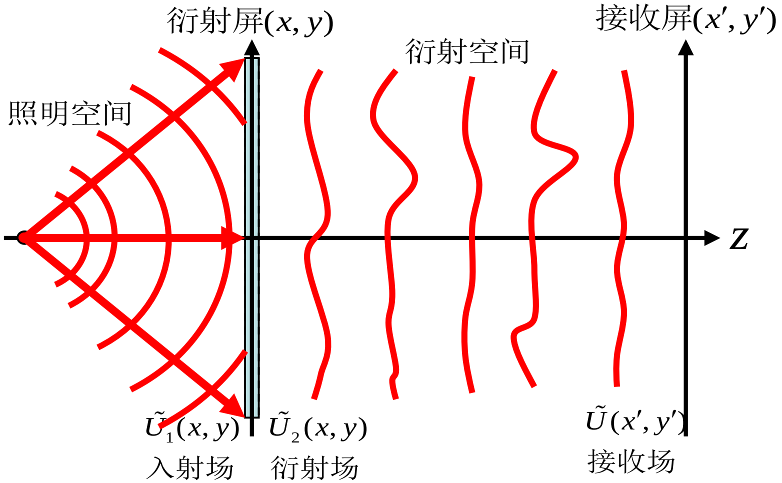
$\tilde{U}(x', y')$

接收场

- 衍射屏的作用是使入射场转换为透射场（或反射场）。用函数表示，就是衍射屏的**透过率**或**反射率**函数，统称**屏函数。**

衍射屏函数

$$\tilde{t}(x,y)=\frac{\tilde{U}_2(x,y)}{\tilde{U}_1(x,y)}$$



$$\tilde{t}(x,y)=t(x,y)\exp[i\varphi_t(x,y)]$$

$t(x,y)$ **屏函数的模。**

模为常数的衍射屏称为相位型的，如透镜、棱镜等。

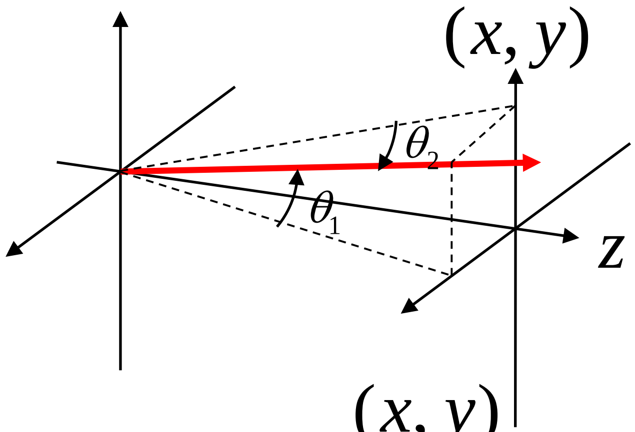
$\varphi_t(x,y)$ **屏函数的幅角即相位。**

幅角为常数的衍射屏称为振幅型的，如单缝、圆孔等。

2.相因子判断法

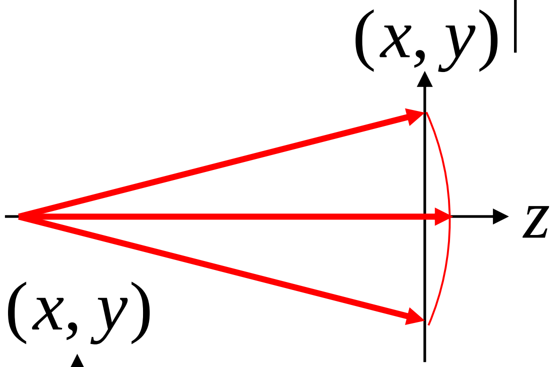
- 知道了衍射屏的屏函数，就可以确定衍射场，进而完全确定接收场。
- 但由于衍射屏的复杂性以及衍射积分求解的困难，完全确定屏函数几乎是不可能的。
- 只能采取一定的近似方法获取衍射场的主要特征。
- 如果知道了屏函数的**相位**，则能通过研究波的**相位**改变来确定波场的变化。这种方法称为**相因子判断法**。
- 一般都是在**傍轴近似**下进行判断。

近轴条件下一些典型的光波在平面波前（ *x,y* ）上的相因子



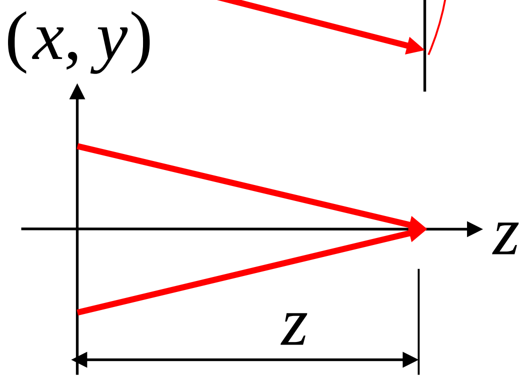
平面波

$$\exp[ik(\sin\theta_1 x + \sin\theta_2 y)]$$



轴上物点发散球面波

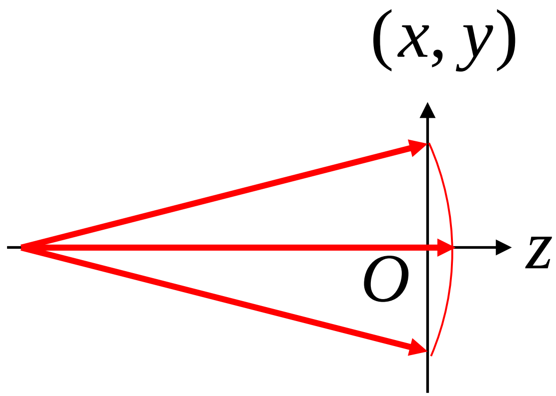
$$\exp[ik\frac{x^2+y^2}{2z}]$$



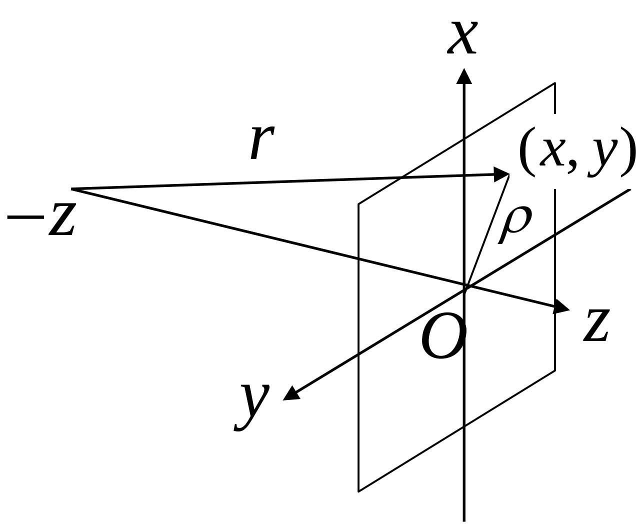
轴上物点汇聚球面波

$$\exp[-ik\frac{x^2+y^2}{2z}]$$

轴上物点球面波的相因子



$$r = \sqrt{z^2 + \rho^2} = z\sqrt{1 + \frac{\rho^2}{z^2}} = z(1 + \frac{1}{2}\frac{\rho^2}{z^2})$$



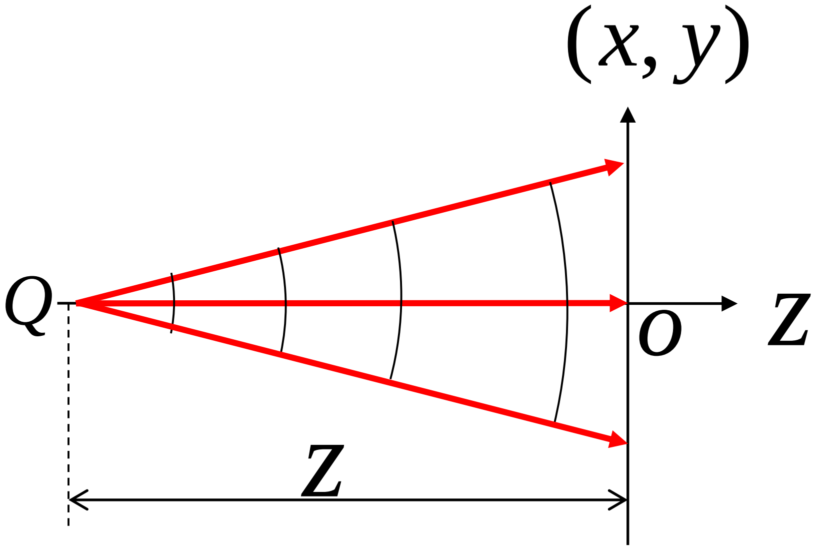
$$\exp[ikr] = \exp[ikz]\exp[ik\frac{x^2 + y^2}{2z}]$$

如果取$z = 0$处相位为0，则有

$$\exp[ik\frac{x^2 + y^2}{2z}]$$

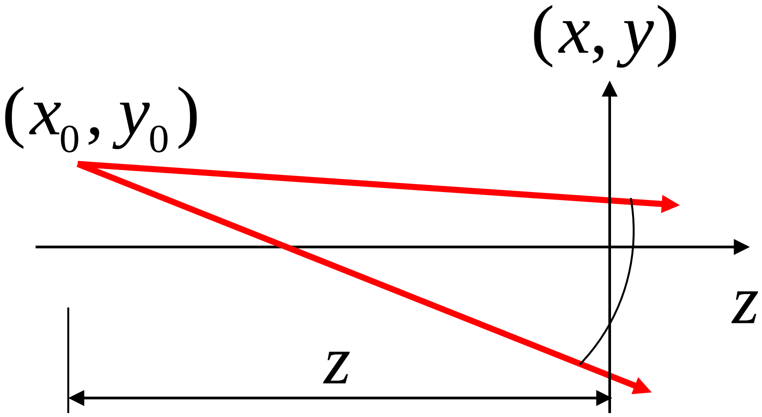
- **以原点相位为0，xoy平面上点（*x*，*y*）的相位因子**

$$\exp[ik\frac{x^2+y^2}{2z}]$$

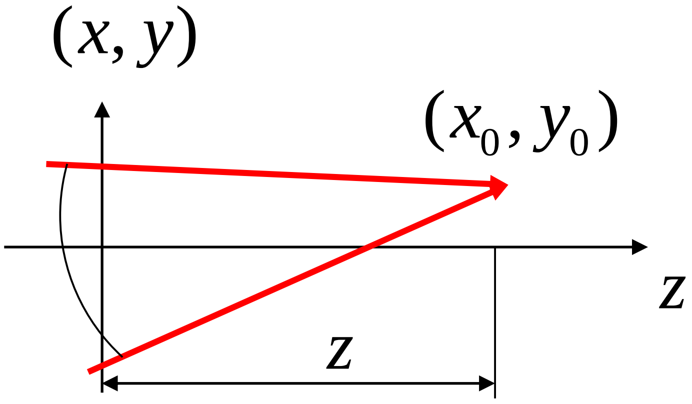


- **以物点相位为0，xoy平面上点（*x*，*y*）的相位因子**

$$\exp[ikz+ik\frac{x^2+y^2}{2z}]$$

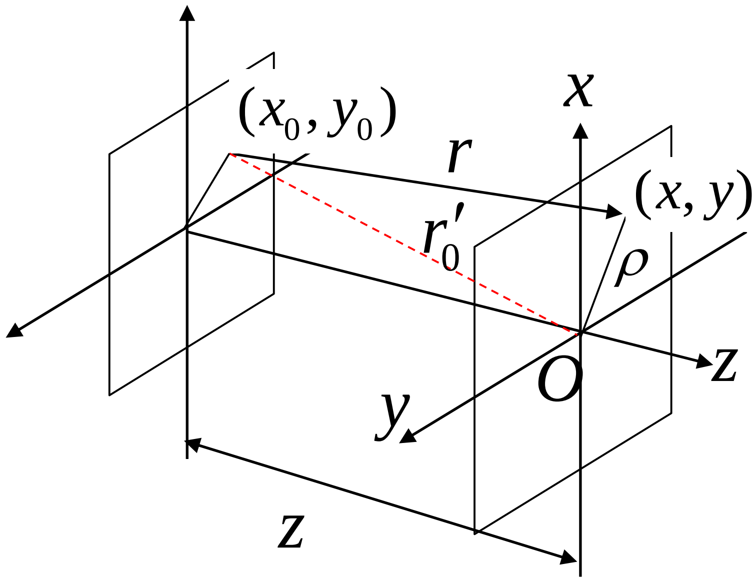


$$\exp[ik(\frac{x^2+y^2}{2z}-\frac{xx_0+yy_0}{z})]$$



$$\exp[-ik(\frac{x^2+y^2}{2z}-\frac{xx_0+yy_0}{z})]$$

轴外物点球面波的相因子



$$r_0' \approx z$$

$$r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + z^2}$$

$$= \sqrt{z^2 + x_0^2 + y_0^2 + x^2 + y^2 - 2(xx_0 + yy_0)}$$

$$= r_0' \sqrt{1 + \frac{x^2+y^2}{r_0'^2} - \frac{2(xx_0+yy_0)}{r_0'^2}}$$

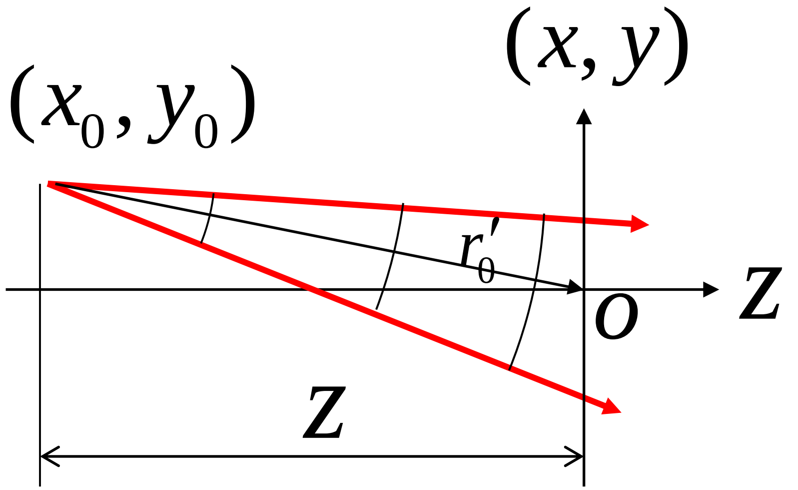
$$\approx r_0'(1 + \frac{x^2+y^2}{2r_0'^2} - \frac{xx_0+yy_0}{r_0'^2})$$

$$\approx r_0' + \frac{x^2+y^2}{2z} - \frac{xx_0+yy_0}{z}$$

$$\exp[ikr] = \exp[ikr_0']\exp[ik(\frac{x^2+y^2}{2z} - \frac{xx_0+yy_0}{z})]$$

- 以原点相位为0，xoy平面上点（*x*，*y*）的位相因子

$$\exp[ik(\frac{x^2+y^2}{2z}-\frac{xx_0+yy_0}{z})]$$



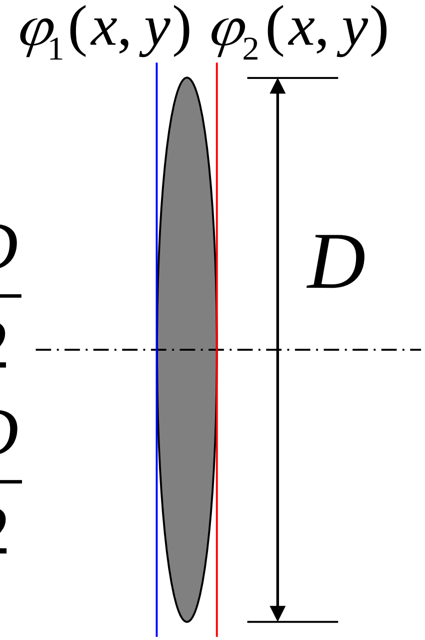
- 以物点相位为0，xoy平面上点（*x*，*y*）的相位因子

$$\exp[ikr_0'+ik(\frac{x^2+y^2}{2z}-\frac{xx_0+yy_0}{z})]$$

3.透镜的作用及位相变换函数

- 设透镜的有效口径为*D*

$$\tilde{t}_L = \frac{A_2}{A_1}\exp[i(\varphi_2-\varphi_1)] = \begin{cases} a(x,y)\mathrm{e}^{i\varphi_L(x,y)}, & r<\dfrac{D}{2} \\ 0, & r>\dfrac{D}{2} \end{cases}$$

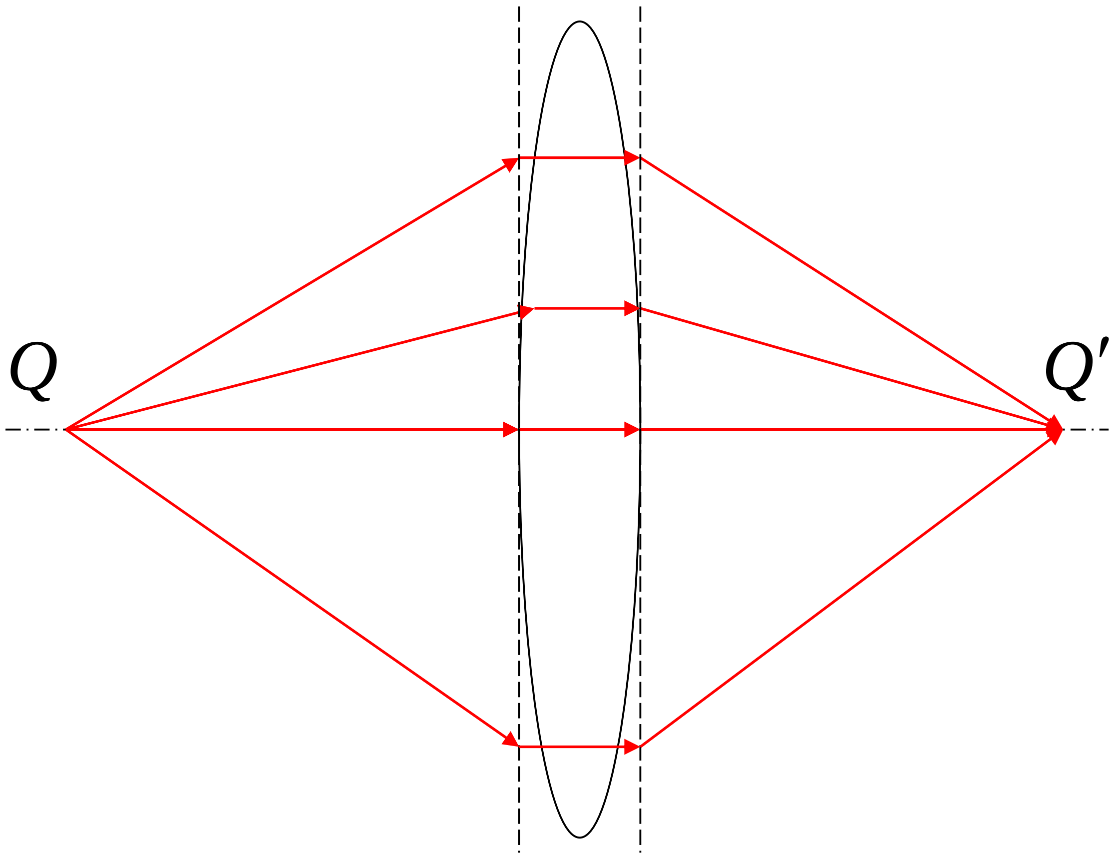


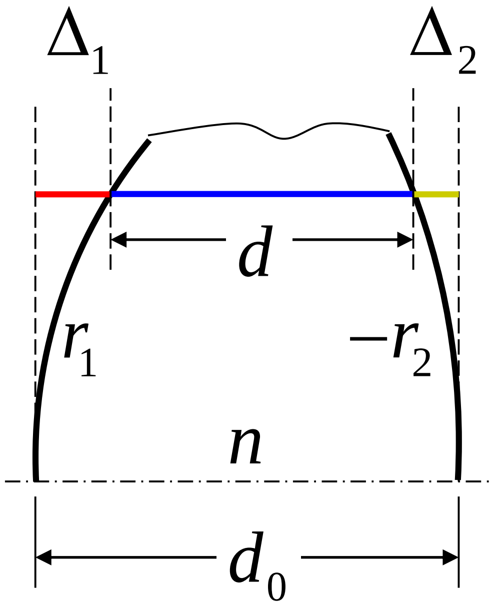
- 忽略透镜的吸收 $a(x,y) = A_2 / A_1 = 1$

$$\tilde{t}_L(x,y) = \exp[i\varphi_L(x,y)] = \exp\{i[\varphi_2(x,y)-\varphi_1(x,y)]\}$$

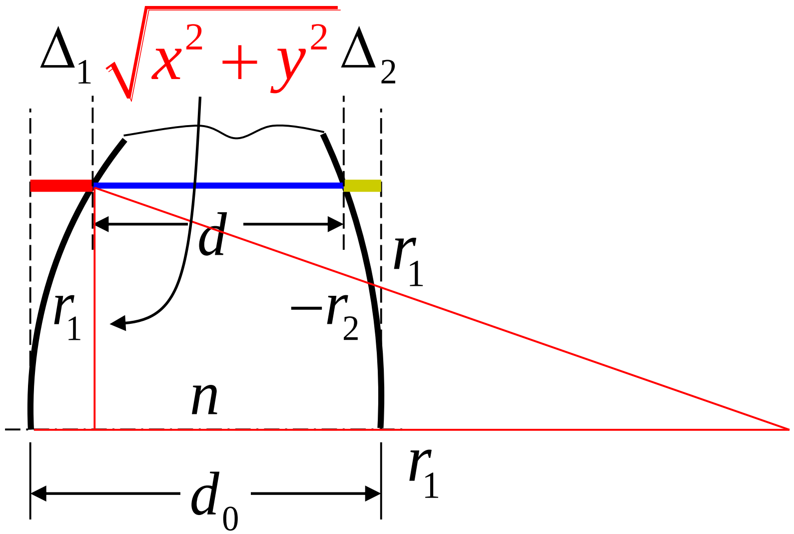
透镜的透射屏函数

- 傍轴近似，入射波前、出射波前取平面
- 认为透镜中光波波矢平行于光轴





光波经透镜后的相位差



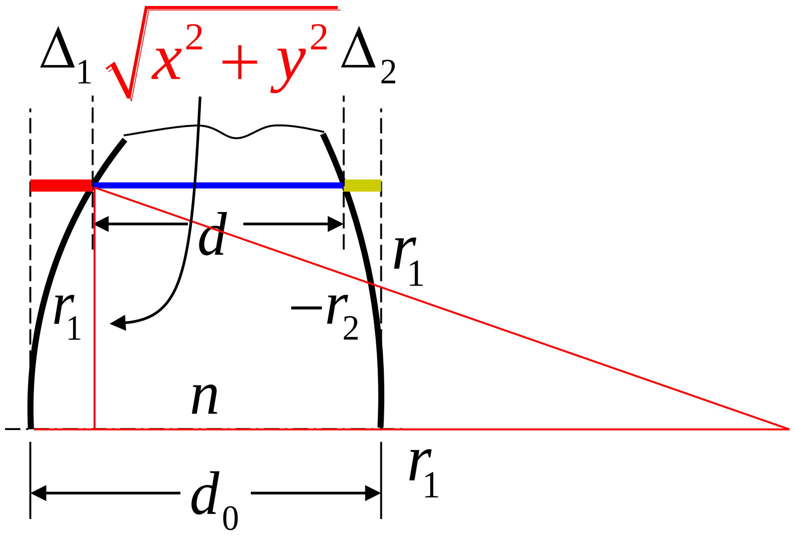
$$\varphi_L(x,y)=\frac{2\pi}{\lambda}[\Delta_1+\Delta_2+nd(x,y)]$$

$$=\frac{2\pi}{\lambda}[\Delta_1+\Delta_2+n(d_0-\Delta_1-\Delta_2)]$$

$$=\varphi_0-\frac{2\pi}{\lambda}(n-1)(\Delta_1+\Delta_2)$$

$$\varphi_0=\frac{2\pi}{\lambda}nd_0$$

傍轴条件下



$$\Delta_1(x,y)=r_1-\sqrt{r_1^2-(x^2+y^2)}$$

$$=r_1\left(1-\sqrt{1-\frac{x^2+y^2}{r_1^2}}\right)$$

$$\approx\frac{x^2+y^2}{2r_1}$$

$$\Delta_2(x,y)=-r_2-\sqrt{r_2^2-(x^2+y^2)}\approx-\frac{x^2+y^2}{2r_2}$$

$$\varphi_L(x,y)=\varphi_0-\frac{2\pi}{\lambda}\frac{n-1}{2}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)(x^2+y^2)=\varphi_0-k\frac{x^2+y^2}{2F}$$

其中 $F=\dfrac{1}{(n-1)\left(\dfrac{1}{r_1}-\dfrac{1}{r_2}\right)}$

透镜的相位变换函数（屏函数）

$$\tilde{t}_L(x,y)=\exp(i\varphi_0)\exp[-ik\frac{x^2+y^2}{2F}]=\exp(i\varphi_0)\exp[i\varphi_L]$$

$$F=\frac{1}{(n-1)(\frac{1}{r_1}-\frac{1}{r_2})}$$

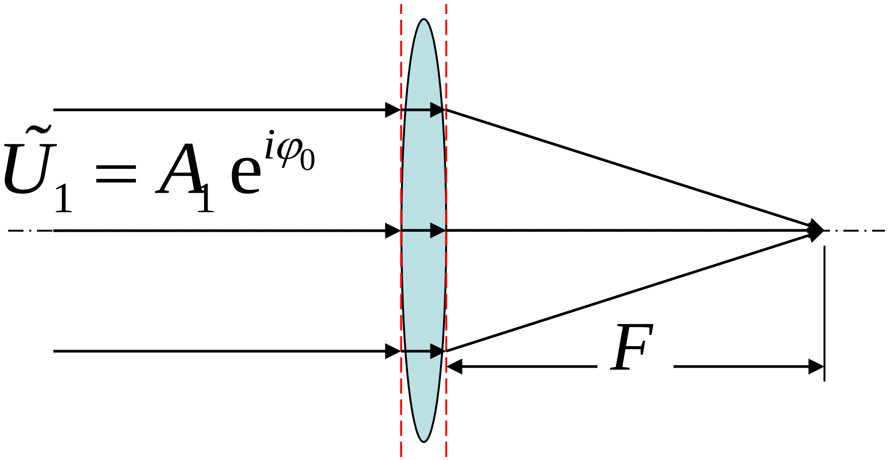
相因子 $\varphi_L=-k\frac{x^2+y^2}{2F}$

常数相因子不起作用，可略去

透镜对波面的变换: **平面波入射**

$$\varphi_L = -k\frac{x^2+y^2}{2F}$$

$$\tilde{U}_2 = \tilde{U}_1 \tilde{t}_L(x,y) = A_1 e^{i\varphi_0} e^{-ik\frac{x^2+y^2}{2F}} = A_1 e^{-ik\frac{x^2+y^2}{2F}+i\varphi_0}$$



汇聚到轴上F处的球面波

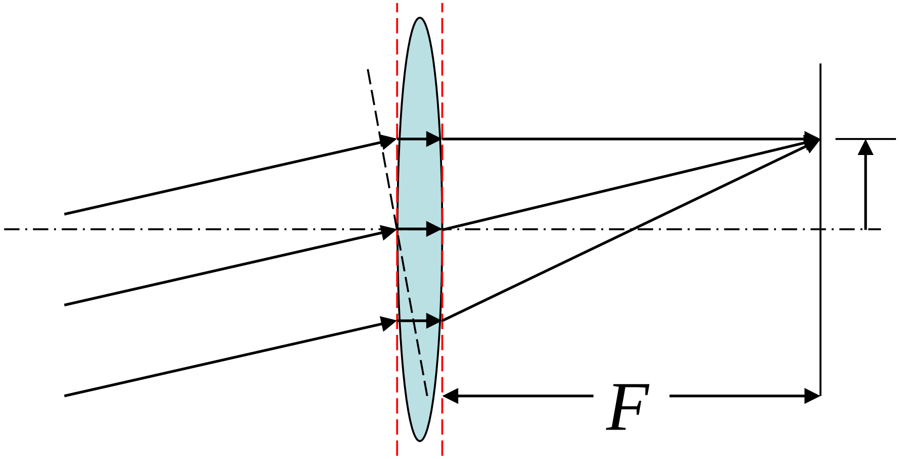
焦距 $f = F$

透镜对波面的变换: **平面波入射**

$$\tilde{U}_1 = A_1 \mathrm{e}^{ik(x\sin\theta_1 + y\sin\theta_2)}$$

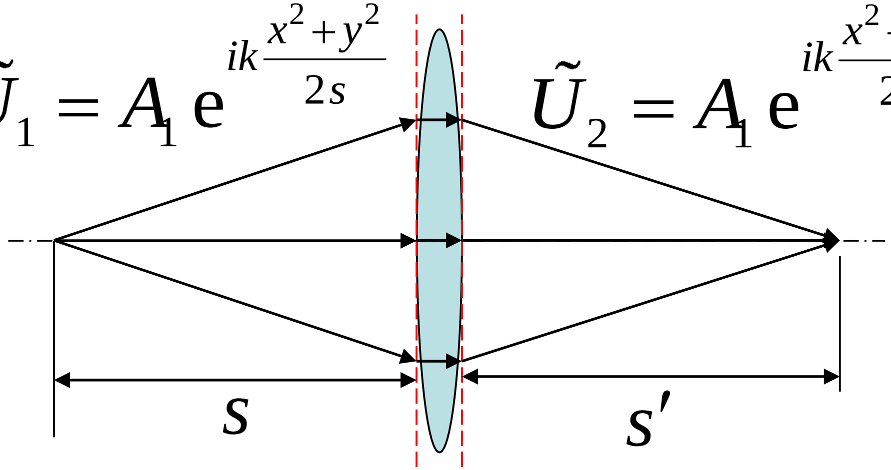
$$\tilde{U}_2 = A_1 \mathrm{e}^{-ik[\frac{x^2+y^2}{2F} - (x\sin\theta_1 + y\sin\theta_2)]}$$

$$= A_1 \mathrm{e}^{-ik[\frac{x^2+y^2}{2F} - \frac{xF\sin\theta_1 + yF\sin\theta_2}{F}]}$$



汇聚到轴上($F\sin\theta_1$, $F\sin\theta_2$, F)处的球面波

透镜对波面的变换: **球面波入射**



$$\tilde{U}_1 = A_1 e^{ik\frac{x^2+y^2}{2s}}$$

$$\tilde{U}_2 = A_1 e^{ik\frac{x^2+y^2}{2s}} e^{-ik\frac{x^2+y^2}{2F}} = A_1 e^{-ik(\frac{x^2+y^2}{2F}-\frac{x^2+y^2}{2s})}$$

$$= A_1 e^{-ik\frac{x^2+y^2}{2}(\frac{1}{F}-\frac{1}{s})}$$

汇聚球面波

球心 $$s' = (\frac{1}{F}-\frac{1}{s})^{-1} = \frac{sF}{s-F}$$

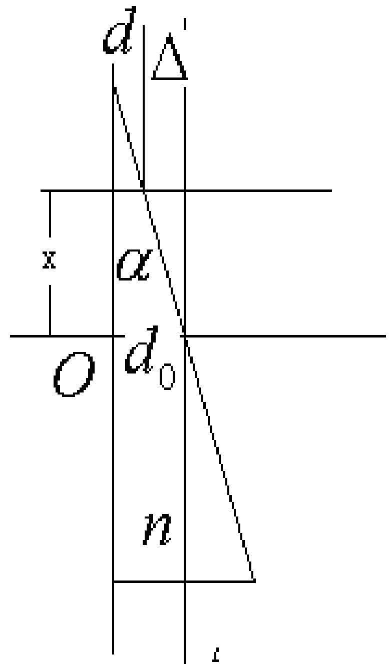
$$\frac{F}{s}+\frac{F}{s'}=1$$ **Gauss 公式**

棱镜的相位变换函数（透过率函数）

- 薄的楔形棱镜，可以得到

$$\varphi_P(x,y)=\frac{2\pi}{\lambda}(\Delta+nd)=\frac{2\pi}{\lambda}(\Delta+nd_0-n\Delta)$$

$$=\varphi_0-\frac{2\pi}{\lambda}(n-1)\Delta$$



$\Delta=x\alpha$ $\quad d_0$ **棱镜中心处的厚度**

$$\varphi_P(x,y)=-k(n-1)\alpha x$$

$$\tilde{t}_P(x,y)=\exp[-ik(n-1)(\alpha_1 x+\alpha_2 y)]$$ **二维情况下**

作业：P52: 1, 2, 3

4. 2 正弦光栅

1. 空间频率的概念
2. 正弦光栅
3. 正弦光栅的衍射图样
4. 正弦光栅的组合
5. 任意光栅的屏函数及其傅里叶级数展开
6. 过高频信息产生衰逝波
7. 对夫琅和费衍射的再认识

1.空间频率的概念

频率的概念：随时间的周期简谐振动 $f(t+T)=f(t)$

频率： $\nu=\dfrac{1}{T}$， 圆频率： $\omega=2\pi\nu$

谐波： $\nu_n=n\dfrac{1}{T}$， $\omega_n=2\pi n\nu$

频谱展开：

$$f(t)=\sum_{n=-\infty}^{\infty}u(\omega_n)e^{i\omega_n t}\quad u(\omega_n)=\frac{1}{T}\int_{-T/2}^{T/2}f(t)e^{-i\omega_n t}dt$$

任意函数的频谱展开：$u(\omega)=\int f(t)e^{i\omega t}dt$

$$f(t)=\int u(\omega)e^{i\omega t}d\omega$$

物理意义：任意随时间变化的振动可展开成一系列简谐振动的迭加。

空间频率：某一面内光场随空间位置的周期性变化

$$\tilde{U}(x, y) = Ae^{ik\sin\theta x} = Ae^{iq_x x}$$

等价关系：

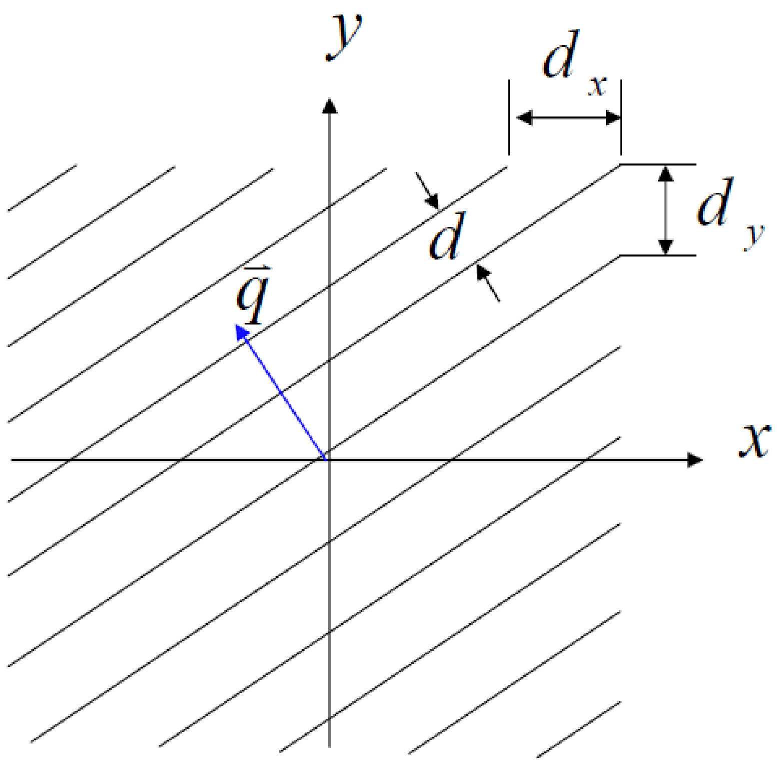
时间周期 T $\Leftrightarrow$ 空间周期 d

时间频率 $\nu = \frac{1}{\mathrm{T}}$ $\Leftrightarrow$ 空间频率 $\mathrm{f} = \frac{1}{\mathrm{d}}$

时间圆频率 $\omega = 2\pi\nu$ $\Leftrightarrow$ 空间圆频率 $q = 2\pi\mathrm{f}$

（波矢）

例



$$\vec{q} = q_x \vec{e}_x + q_y \vec{e}_y$$
$$= q \cos\theta \vec{e}_x + q \sin\theta \vec{e}_y$$

$$d_x = \frac{2\pi}{q_x} = \frac{1}{f_x}$$

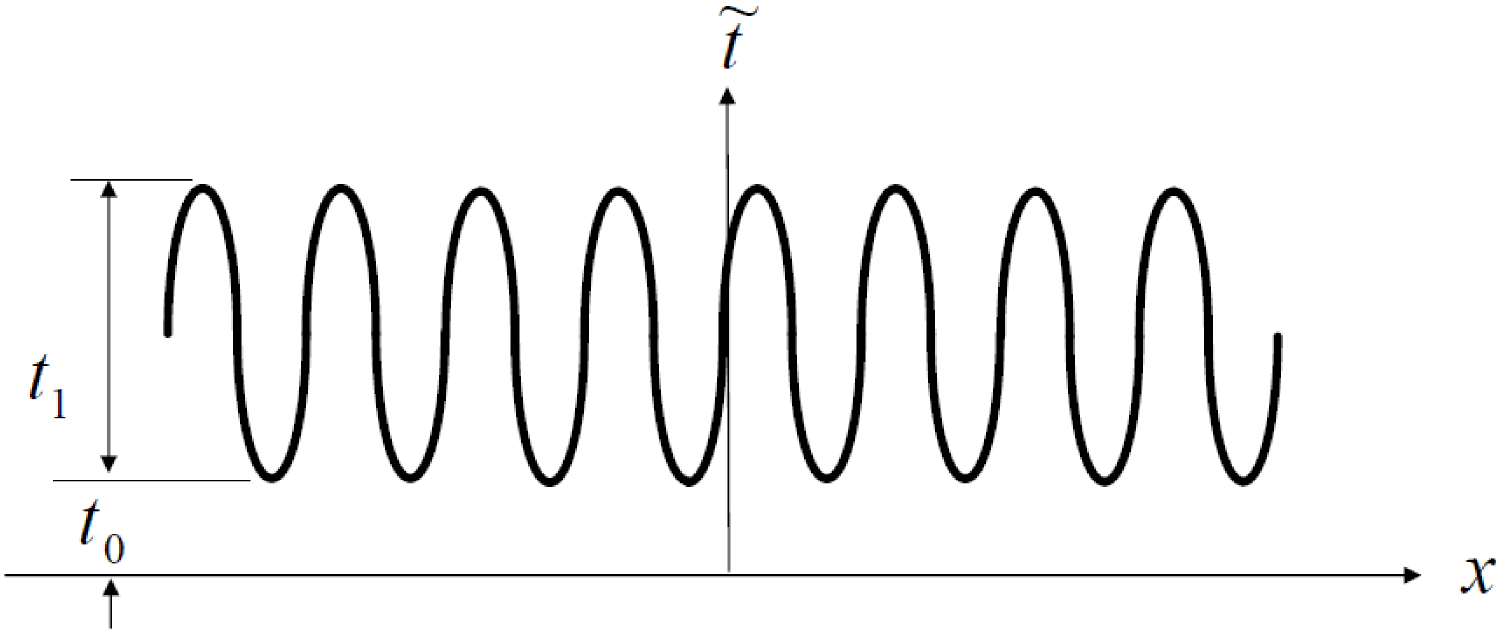
$$d_y = \frac{2\pi}{q_y} = \frac{1}{f_y}$$

$$\frac{1}{d^2} = \frac{1}{d_x^2} + \frac{1}{d_y^2}$$

$$d = \frac{2\pi}{\sqrt{q_x^2 + q_y^2}} = \frac{1}{q}$$

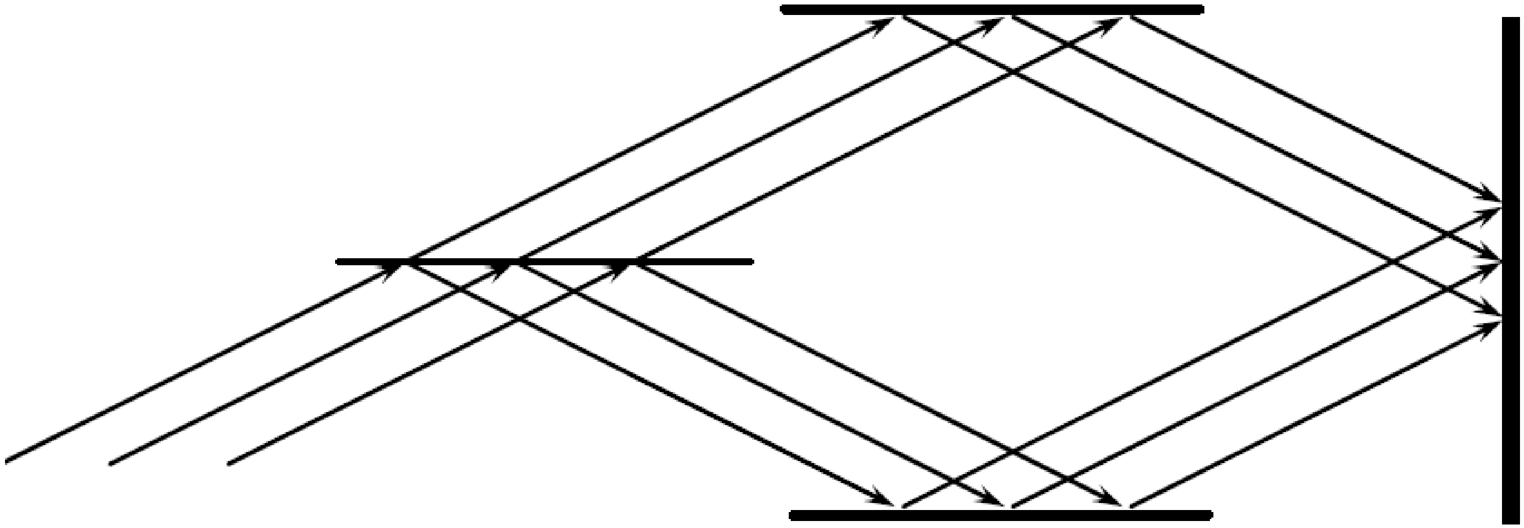
2.正弦光栅

透过率是空间的正弦函数



$$\tilde{t}(x, y) = t_0 + t_1 \cos\left(q_x x + q_y y + \varphi_0\right)$$

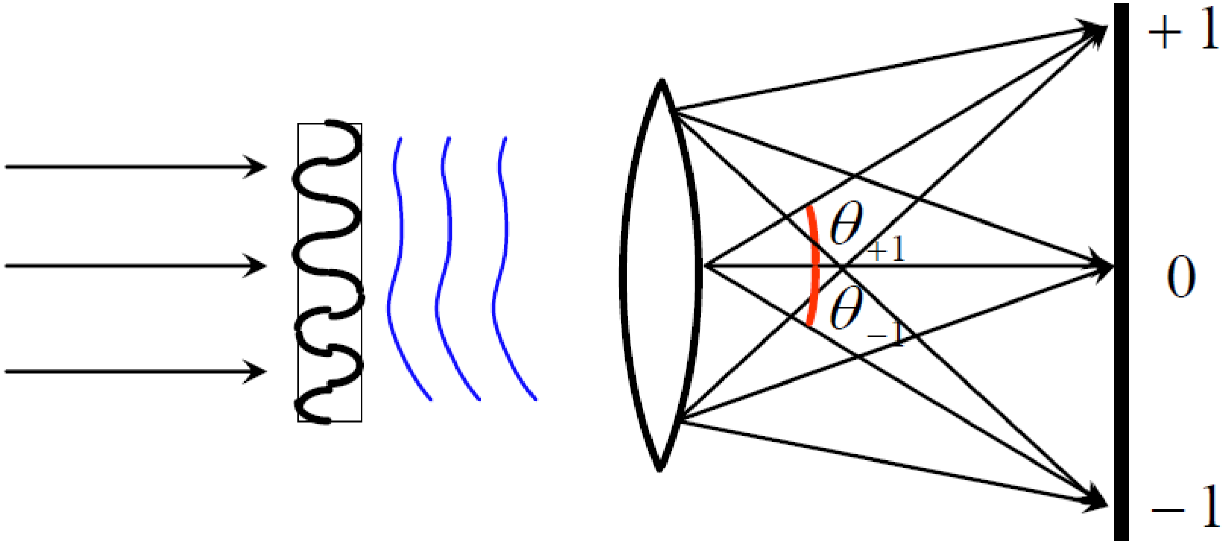
制备方法：照相底片记录两束平行光束的干涉场



$$I = I_0\left[1+\gamma\cos\left(q_x x+q_y y+\phi_0\right)\right]$$

线性冲洗即的正弦光栅

3.正弦光栅的衍射图样



屏函数 $$\tilde{t}(x) = t_0 + t_1\cos(2\pi fx + \varphi_0)$$

平行光正入射 $$\tilde{U}_1(x) = A_1$$

透射波 $$\tilde{U}_2(x) = \tilde{U}_1(x)\tilde{t}(x)$$

$$= A_1[t_0 + t_1\cos(2\pi fx + \varphi_0)]$$

$$\tilde{U}_1(x) = A_1 \quad \tilde{U}_2(x) = A_1[t_0 + t_1 \cos(2\pi f x + \varphi_0)]$$

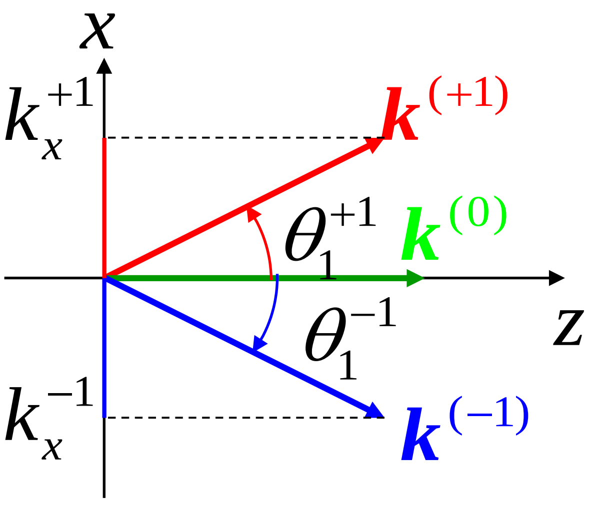
$$\cos(2\pi f x + \varphi_0) = \frac{1}{2}[\mathrm{e}^{i(2\pi f x + \varphi_0)} + \mathrm{e}^{-i(2\pi f x + \varphi_0)}]$$

$$\tilde{U}_2(x) = A_1 t_0 + \frac{1}{2} A_1 t_1 \mathrm{e}^{i(2\pi f x + \varphi_0)} + \frac{1}{2} A_1 t_1 \mathrm{e}^{-i(2\pi f x + \varphi_0)}$$

$\tilde{U}_2(x) = \tilde{U}_0(x) + \tilde{U}_{+1}(x) + \tilde{U}_{-1}(x)$ **透射波实际上变为三列平面波**

三列透射波方向各不相同

$$\tilde{U}_{+1}(x) = \frac{1}{2} A_1 t_1 \mathrm{e}^{i(2\pi f x + \varphi_0)}$$

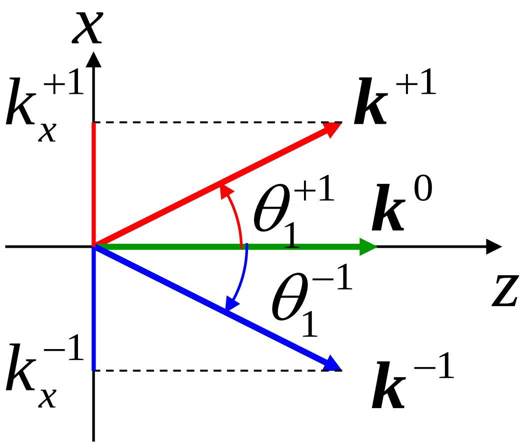


$$k_x^{(+1)} = 2\pi f \qquad \sin\theta_1^{(+1)} = \frac{k_x^{(+1)}}{k^{(+1)}} = \frac{2\pi f}{\frac{2\pi}{\lambda}} = f\lambda$$

正弦光栅的三级（三列）衍射光波

$$\tilde{U}_2(x) = A_1 t_0 + \frac{1}{2} A_1 t_1 \mathrm{e}^{i(2\pi fx+\varphi_0)} + \frac{1}{2} A_1 t_1 \mathrm{e}^{-i(2\pi fx+\varphi_0)}$$

$$= \tilde{U}_0(x) + \tilde{U}_{+1}(x) + \tilde{U}_{-1}(x)$$



$\tilde{U}_0(x) = A_1 t_0$ 　0级波，方向 　$\sin\theta_0 = 0$

$\tilde{U}_{+1}(x) = \frac{1}{2} A_1 t_1 \mathrm{e}^{i(2\pi fx+\varphi_0)}$ 　+1级波，方向 　$\sin\theta_1^{+1} = f\lambda$

$\tilde{U}_{-1}(x) = \frac{1}{2} A_1 t_1 \mathrm{e}^{-i(2\pi fx+\varphi_0)}$ 　-1级波，方向 　$\sin\theta_1^{-1} = -f\lambda$

4. 正弦光栅的组合

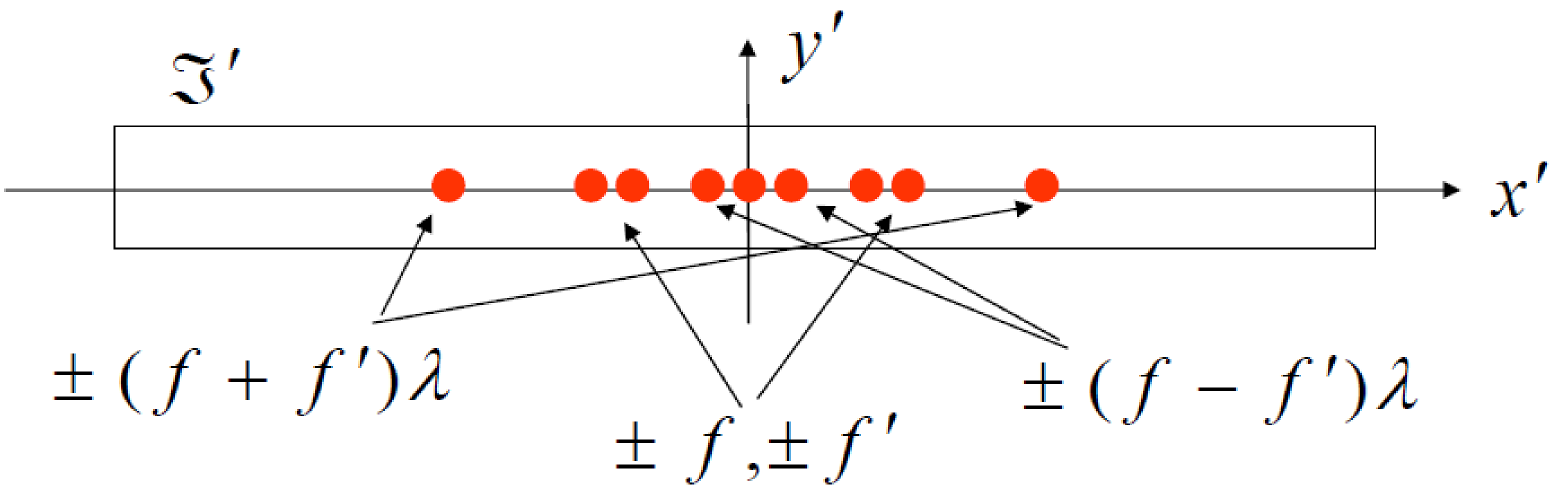
平行密接
正交密接
两套光栅

平行密接 $\begin{cases} G_1 : t(x) = t_0 + t_1 \cos 2\pi f x \\ G_1' : t'(x) = t_0' + t_1' \cos 2\pi f' x \end{cases}$

$$\begin{aligned}
\tilde{U}_2(x) &= \tilde{U}_1(x) t(x) t'(x) \\
&= A_1 \left(t_0 + t_1 \cos 2\pi f x\right)\left(t_0' + t_1' \cos 2\pi f' x\right) \\
&= A_1 \big[t_0 t_0' + t_0 t_1' \cos 2\pi f' x + t_0' t_1 \cos 2\pi f x \\
&+ t_1 t_1' \cos 2\pi f x \cos 2\pi f' x\big] \\
&= A_1 \Big[t_0 t_0' + t_0 t_1' \cos 2\pi f' x + t_0' t_1 \cos 2\pi f x \\
&+ \frac{1}{2} t_1 t_1' \cos 2\pi (f - f') x + \frac{1}{2} t_1 t_1' \cos 2\pi (f + f') x\Big]
\end{aligned}$$

平行密接

$$(\sin\theta_1, \sin\theta_2) = \begin{cases} 0 & (0\text{级}) \\ \pm f\lambda & (f\text{的} \pm 1\text{级}) \\ \pm f'\lambda & (f'\text{的} \pm 1\text{级}) \\ \pm (f - f')\lambda & (\text{差频的} \pm 1\text{级}) \\ \pm (f + f')\lambda & (\text{和频的} \pm 1\text{级}) \end{cases}$$



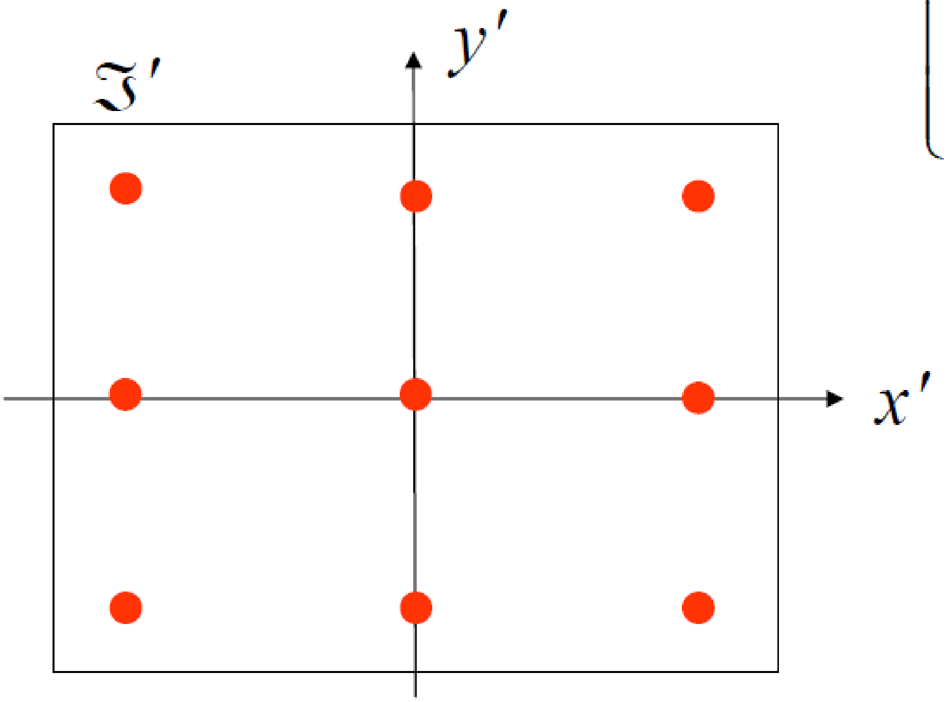
正交密接 $\begin{cases} G_1 : t(x) = t_0 + t_1 \cos 2\pi f x \\ G_1' : t'(y) = t_0' + t_1' \cos 2\pi f' y \end{cases}$

$$\begin{aligned} \tilde{U}_2(x) &= \tilde{U}_1(x) t(x) t'(y) \\ &= A_1 (t_0 + t_1 \cos 2\pi f x)(t_0' + t_1' \cos 2\pi f' y) \\ &= A_1 [t_0 t_0' + t_0 t_1' \cos 2\pi f' y + t_0' t_1 \cos 2\pi f x \\ &+ t_1 t_1' \cos 2\pi f x \cos 2\pi f' y] \\ &= A_1 [t_0 t_0' + t_0 t_1' \cos 2\pi f' x + t_0' t_1 \cos 2\pi f y \\ &+ \frac{1}{2} t_1 t_1' \cos 2\pi (f x - f' y) + \frac{1}{2} t_1 t_1' \cos 2\pi (f x + f' y)] \end{aligned}$$

正交密接

$$(\sin\theta_1, \sin\theta_2) = \begin{cases} (0,0) & (0\text{级}) \\ (\pm f\lambda, 0) & (f\text{的}\pm 1\text{级}) \\ (0, \pm f'\lambda) & (f'\text{的}\pm 1\text{级}) \\ \pm(f\lambda, -f'\lambda) & \\ \pm(f\lambda, f'\lambda) & \end{cases}$$

$\pm(f\lambda, -f'\lambda)$、$\pm(f\lambda, f'\lambda)$：交叉项的±1级



透过函数： $t(x)=t_0+t_1\cos 2\pi fx+t_1'\cos 2\pi f'x$

两光栅之和

$$\tilde{U}_2(x)=\tilde{U}_1(x)t(x)$$
$$=A_1\left(t_0+t_1\cos 2\pi fx+t_1'\cos 2\pi f'x\right)$$

$$\begin{cases}\sin\theta=0 & (0\text{级}) \\ \sin\theta_{\pm1}=\pm f\lambda & (f\text{的}\pm1\text{级}) \\ \sin\theta_{\pm1}'=\pm f'\lambda & (f'\text{的}\pm1\text{级})\end{cases}$$

→ 展开

5.任意光栅的屏函数及其傅里叶级数展开

空间周期函数：$\tilde{t}(x+d)=\tilde{t}(x)$

正余弦展开：

$$t(x)=t_0+\sum_{n=1}^{\infty}a_n\cos(2\pi nfx)+\sum_{n=1}^{\infty}b_n\sin(2\pi nfx)$$

余弦展开：

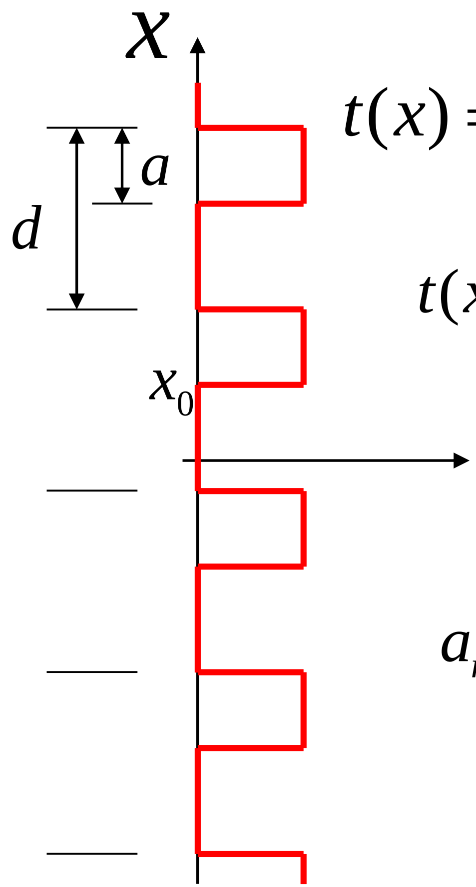
$$t(x)=t_0+\sum_{n=1}^{\infty}c_n\cos(2\pi nfx+\varphi_n)$$

Fourier级数展开：

$$\tilde{t}(x)=\sum_{n=-\infty}^{\infty}\tilde{t}_n e^{i2\pi nfx}$$

其中：$\tilde{t}_n=\dfrac{1}{d}\displaystyle\int_{-d/2}^{d/2}\tilde{t}(x)e^{-i2\pi nfx}dx$

一维周期性衍射屏的傅里叶展开



$$t(x)=t(x+d)=\begin{cases}1, & x_0+md<x<x_0+a+md\\ 0, & x_0+a+md<x<x_0+(m+1)d\end{cases}$$

$$t(x)=t_0+\sum_{n=1}^{\infty}a_n\cos 2\pi f_n x+\sum_{n=1}^{\infty}b_n\sin 2\pi f_n x$$

$$t_0=\frac{1}{d}\int_{-\frac{a}{2}}^{\frac{a}{2}}\mathrm{d}x=\frac{a}{d} \qquad b_n=\frac{2}{d}\int_{-\frac{a}{2}}^{\frac{a}{2}}\sin(2\pi f_n x)\mathrm{d}x=0$$

$$a_n=\frac{2}{d}\int_{-\frac{a}{2}}^{\frac{a}{2}}\cos(2\pi f_n x)\mathrm{d}x=\frac{1}{\pi f_n d}\sin(2\pi f_n x)\Big|_{-a/2}^{a/2}$$

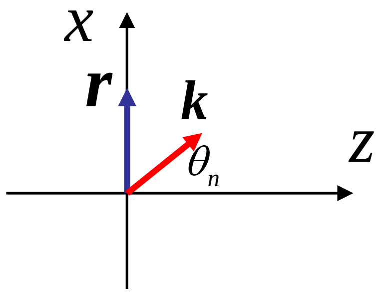
$$=\frac{2\sin(\pi f_n a)}{\pi f_n d}=\frac{a}{d}\frac{2\sin\frac{n\pi a}{d}}{\frac{n\pi a}{d}}$$

$$t(x)=t_0+\sum_{n=1}^{\infty}a_n\cos 2\pi f_n x+\sum_{n=1}^{\infty}b_n\sin 2\pi f_n x$$

$$=\frac{a}{d}+\frac{a}{d}\sum_{n=1}^{\infty}\frac{2\sin\frac{n\pi a}{d}}{\frac{n\pi a}{d}}\cos 2\pi f_n x=\frac{a}{d}+\frac{a}{d}\sum_{n=1}^{\infty}\frac{2\sin\frac{n\pi a}{d}}{\frac{n\pi a}{d}}\frac{e^{i2\pi f_n x}+e^{-i2\pi f_n x}}{2}$$

$$=\frac{a}{d}+\frac{a}{d}\sum_{n\neq 0}\frac{\sin\alpha_n}{\alpha_n}e^{i2\pi f_n x}$$

其中，$\alpha_n=\dfrac{n\pi a}{d}$



↑ 0级波　　↑ n级波

平面波，相位 $\boldsymbol{k}\cdot\boldsymbol{r}=\dfrac{2\pi}{\lambda}x\sin\theta=2\pi f_n x$

n级波的方向 $\sin\theta_n=\lambda f_n=\dfrac{n\lambda}{d}\Rightarrow d\sin\theta_n=n\lambda$ 光栅方程

$$\alpha_n=\frac{n\pi a}{d}=\frac{\pi a\sin\theta_n}{\lambda}=u_n\Rightarrow\frac{a}{d}\frac{\sin\alpha_n}{\alpha_n}=\frac{a}{d}\frac{\sin u_n}{u_n}=U(\theta_n)$$

单缝衍射因子

$$t(x)=\frac{a}{d}+\frac{a}{d}\sum_{n\neq 0}\frac{\sin\frac{n\pi a}{d}}{\frac{n\pi a}{d}}\mathrm{e}^{i\frac{2\pi nx}{d}}$$

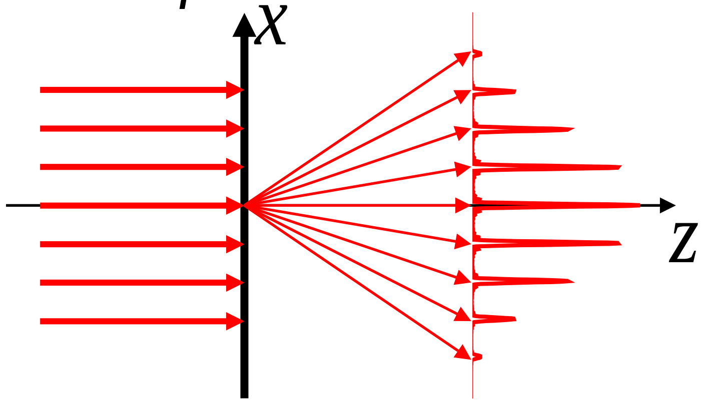
如果 $d=2a$ 平面波入射

$$\tilde{U}_2(x)=\tilde{U}_1(x)t(x)=A_1\frac{a}{d}\left(1+\sum_{n\neq 0}\frac{\sin n\pi fa}{n\pi fa}\mathrm{e}^{i2\pi nfx}\right)=\frac{A_1}{2}\left(1+\sum_{n\neq 0}\frac{\sin\frac{n\pi}{2}}{\frac{n\pi}{2}}\mathrm{e}^{i2\pi nfx}\right)$$

$$=\frac{A_1}{2}+\frac{A_1}{\pi}(\mathrm{e}^{i2\pi fx}+\mathrm{e}^{-i2\pi fx})-\frac{A_1}{3\pi}(\mathrm{e}^{i6\pi fx}+\mathrm{e}^{-i6\pi fx})+\frac{A_1}{5\pi}(\mathrm{e}^{i10\pi fx}+\mathrm{e}^{-i10\pi fx})-\cdots$$

$$=\frac{A_1}{2}+\frac{2A_1}{\pi}\cos 2\pi fx-\frac{2A_1}{3\pi}\cos 6\pi fx+\frac{2A_1}{5\pi}\cos 10\pi fx-\cdots$$

$$\sin\theta_n=\lambda f_n=\frac{n\lambda}{d}$$



例

黑白光栅屏函数的Fourier级数展开：

光栅常数*d*，宽度*a*

$$t(x)=\begin{cases}1, & |x|<a/2\\ 0, & |x|>a/2\end{cases}$$

$$t_0=\frac{1}{d}\int_{-a/2}^{a/2}dx=\frac{a}{d}$$

$$\tilde{t}_n=\frac{1}{d}\int_{-a/2}^{a/2}e^{-i2\pi nfx}dx=\frac{a}{d}\frac{\sin\pi f_n a}{\pi f_n a}$$

$$=\frac{a}{d}\frac{\sin n\pi fa}{n\pi fa}=\frac{a}{d}\frac{\sin(n\pi a/d)}{n\pi a/d}$$

黑白光栅的Fraunhofer衍射

平行光正入射：$\tilde{U}_1(x) = A_1$

$$\tilde{U}_2(x) = \tilde{U}_1(x)\tilde{t}(x) = A_1\tilde{t}(x)$$
$$= A_1 t_0 + A_1 \sum_{n\neq 0} \tilde{t}_n e^{i2\pi nfx}$$

***n*级平面波衍射方向：**

$$\sin\theta_n = nf\lambda = \frac{n\lambda}{d}$$

$$I_n \propto \left|\tilde{t}_n\right|^2 = \left(\frac{a}{d}\right)^2 \left(\frac{\sin a_n}{a_n}\right)^2$$

$$a_n = \pi a \sin\theta_n / \lambda$$

正是单缝衍射因子

6. 过高频信息产生衰逝波

当 $\sin\theta = f\lambda > 1$ 时

$$k_z = \sqrt{k^2 - k_x^2 - k_y^2} = i\kappa$$

其中：$\kappa = \sqrt{(f\lambda)^2 - 1}$

$$\tilde{U}(x, y, z) = Ae^{-\kappa z} e^{i2\pi f x}$$

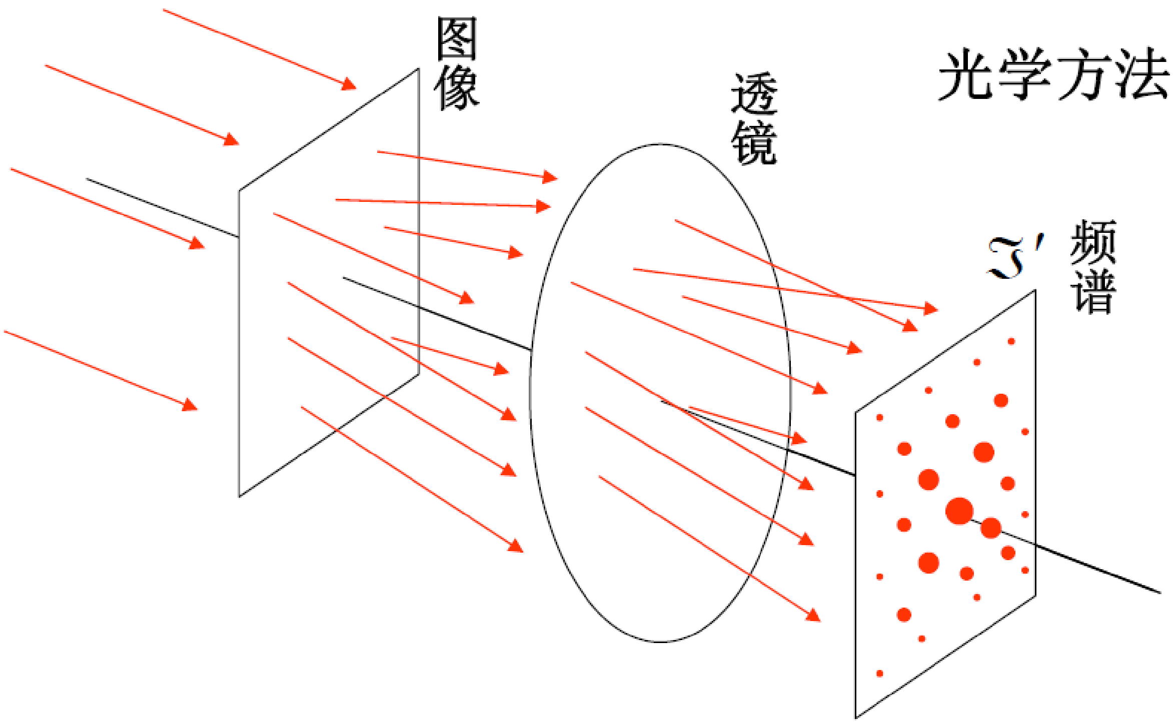
沿z方向衰减，衰失波

7. 对夫琅和费衍射的再认识

Fraunhofer衍射是衍射屏空间频率的频谱分析器！

光学方法实现Fourier变换

光学计算



作业：P67-69: 3, 4, 7, 9