

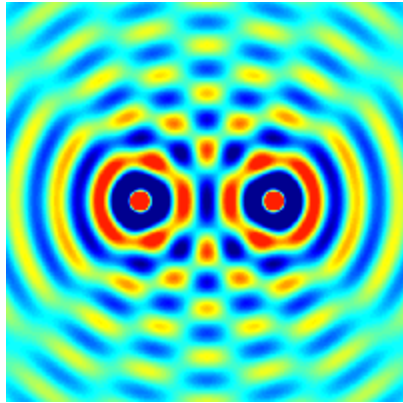
光 学

Optics

第二章 光的干涉

注：本课件应用了众多其他老师和网络上的图表，恕不能一一做引用说明，请见谅并在此表示感谢！

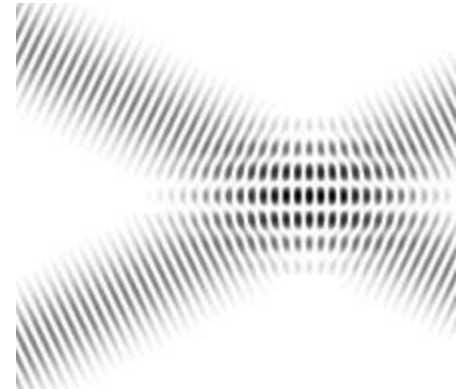
[http://en.wikipedia.org/wiki/Interference_\(wave_propagation\)](http://en.wikipedia.org/wiki/Interference_(wave_propagation))



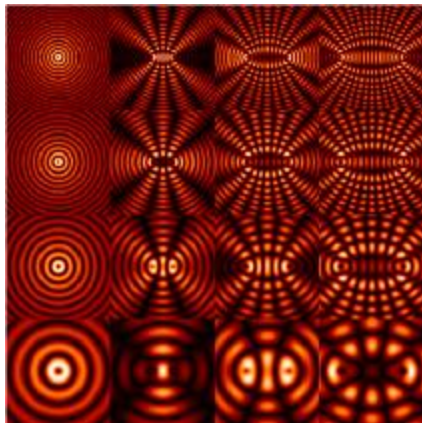
Interference of waves from two point sources.



Interference caused by the reflection on a CD



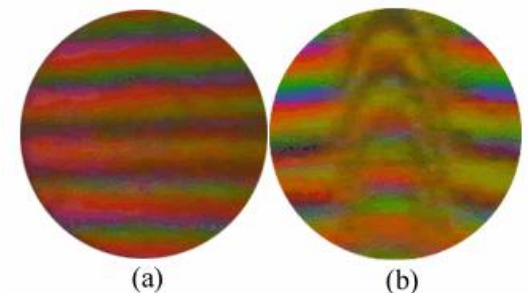
Interference fringes in overlapping plane waves



Optical interference between two point sources for different wavelengths and source separations



White light interference in a soap bubble

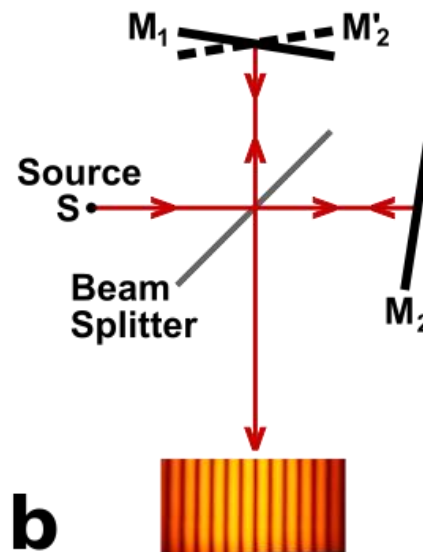
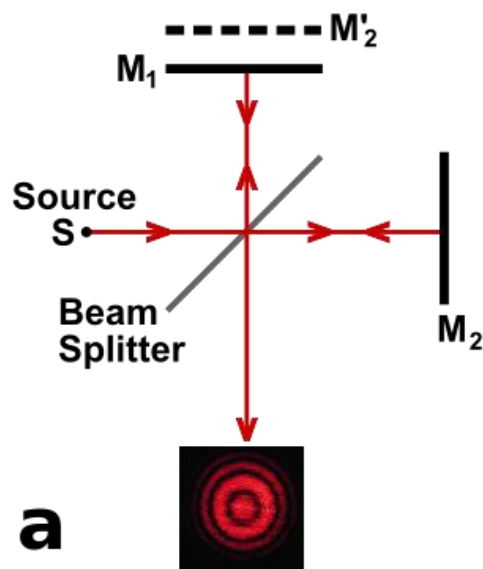


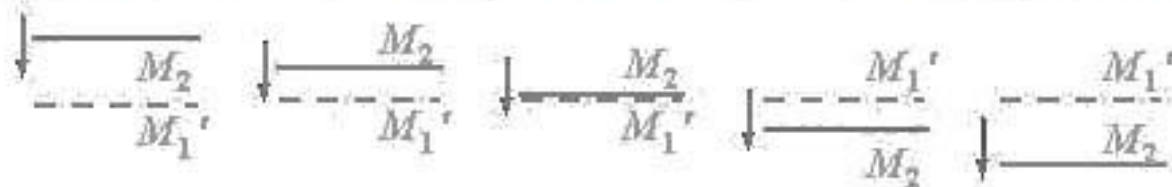
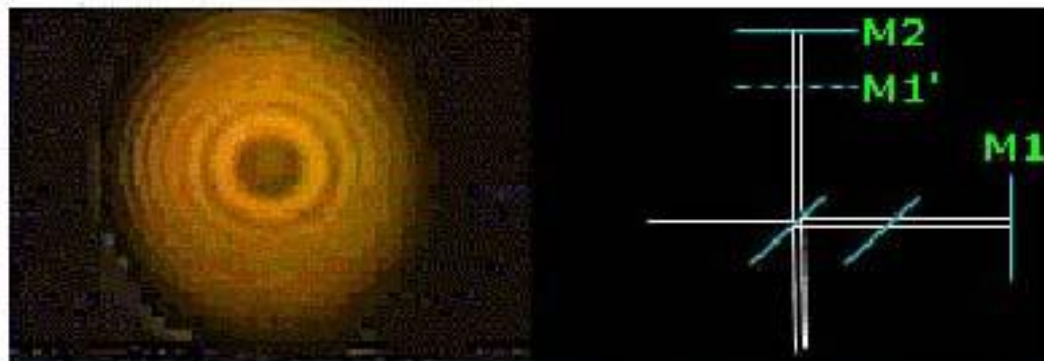
迈克尔逊干涉仪

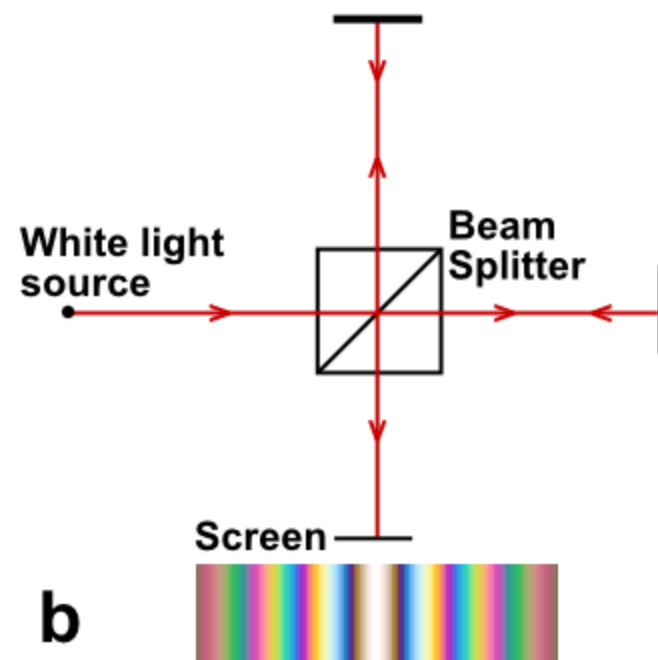
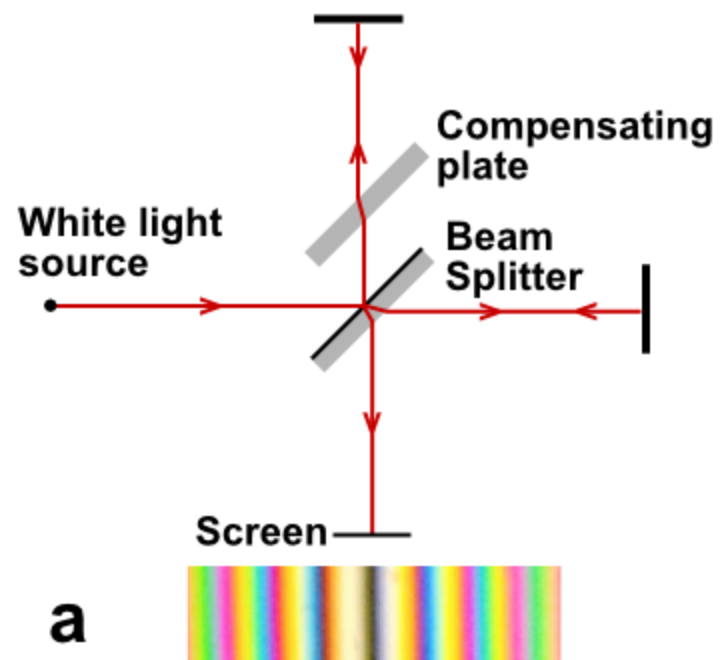
$S'_2 \bullet$

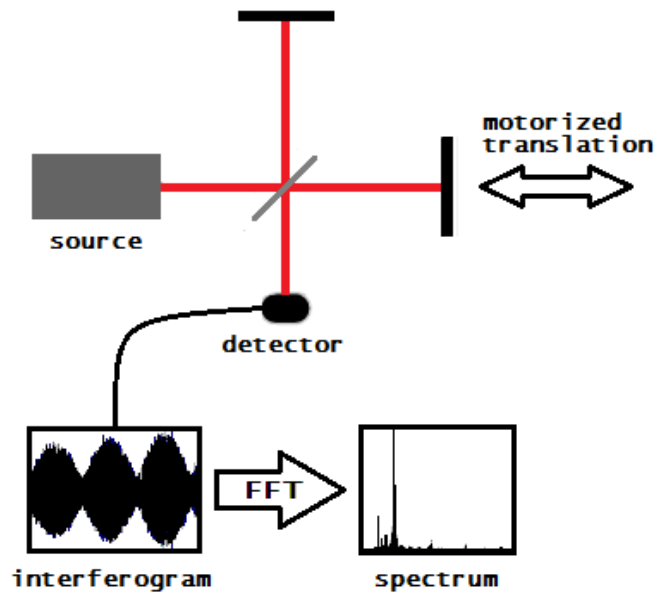
$S'_1 \bullet$

S'_2 S'_1

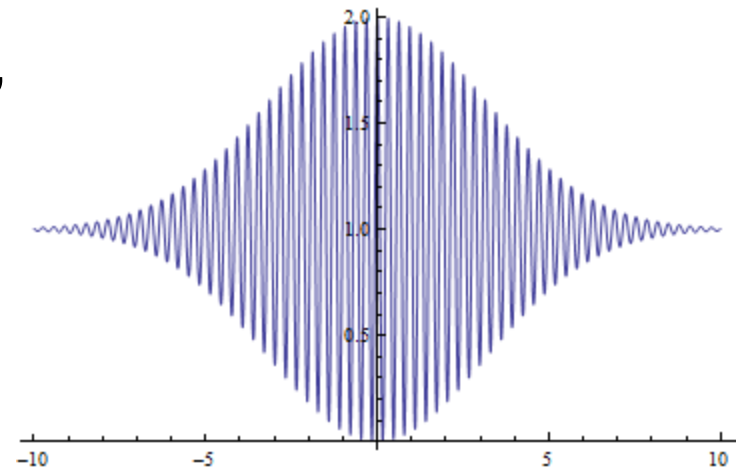




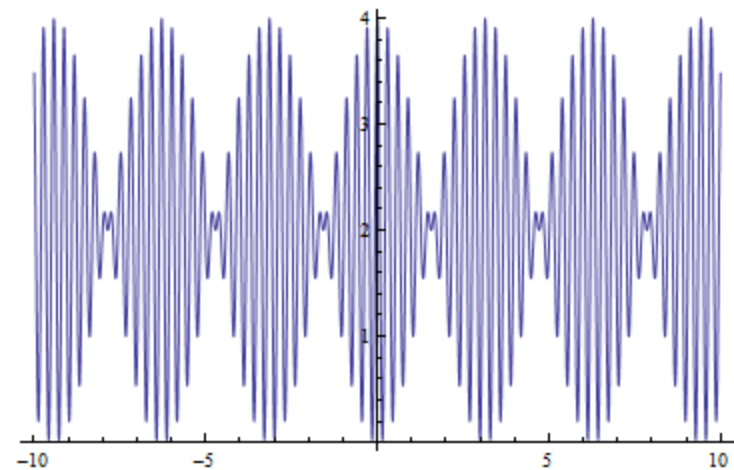




$$\Delta\lambda$$

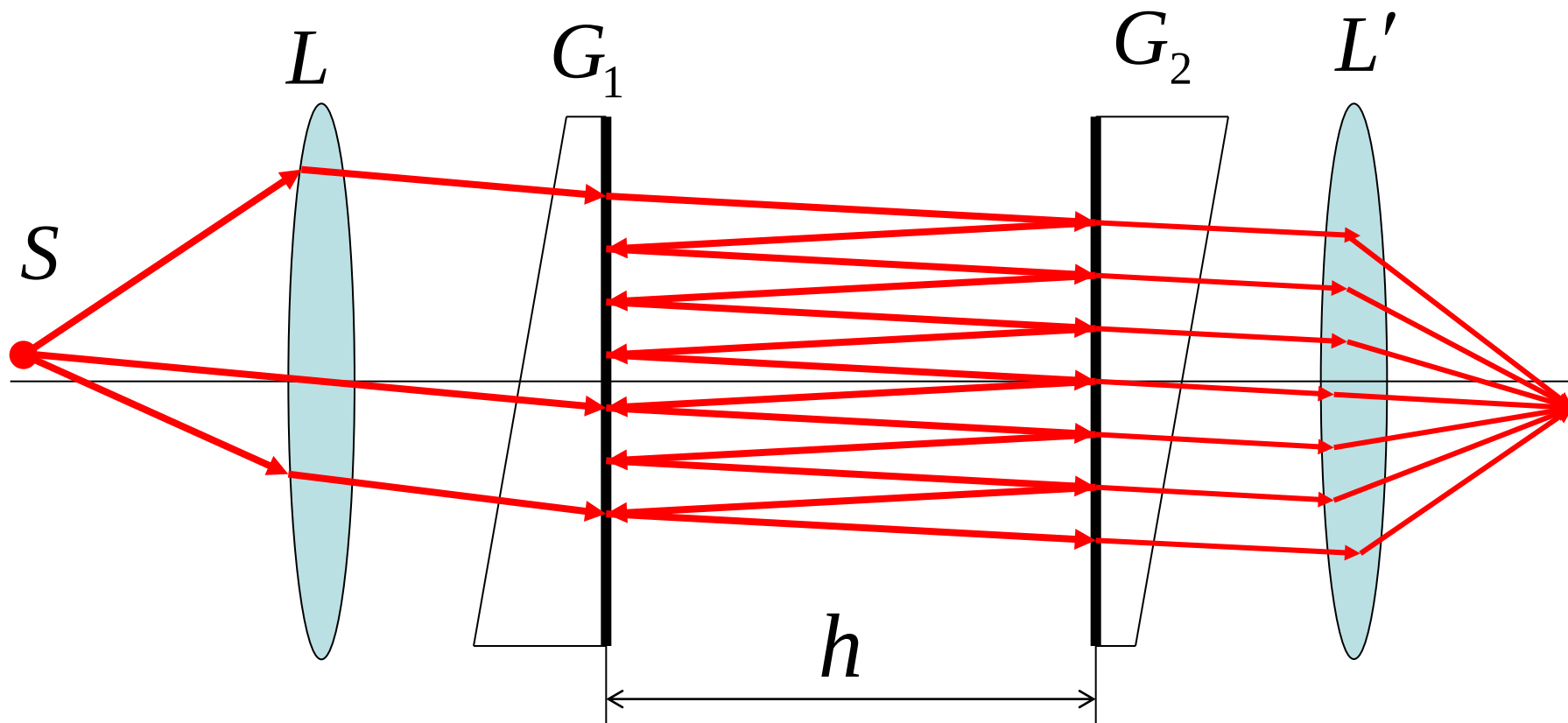


$$\lambda_1 + \lambda_2$$



多光束干涉：Fabry-Perot干涉仪和标准具

- 在薄膜干涉中，如果膜的反射率足够大，则无论是反射光还是透射光，相邻光束的强度相差不大，是多光束的相干叠加。
- Fabry-Perot干涉仪和标准具

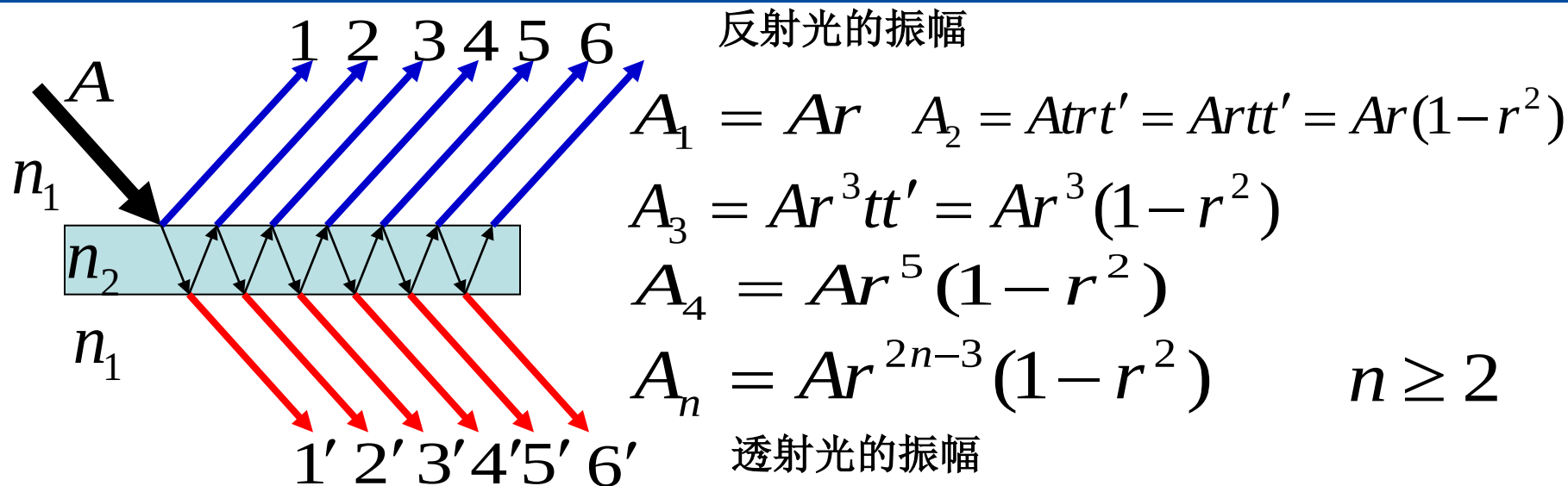


相对两面镀有半透半反膜。

如果 h 固定，为Fabry-Perot标准具。

如果 h 可调，为Fabry-Perot干涉仪。

条纹特性



$$A'_1 = Att' = A(1-r^2) \quad A'_2 = Ar^2(1-r^2)$$

$$A'_3 = Ar^4(1-r^2) \quad A'_n = Ar^{2(n-1)}(1-r^2)$$

相邻两列波的位相差

$$\Delta\varphi = k\delta \quad \delta = 2n_2h \cos i_2 = 2h\sqrt{n_2^2 - n_1^2 \sin^2 i_1}$$

第一列反射波有半波损失，所以第二列反射波与其间的光程差为

$$\delta \pm \lambda/2$$

各列波的复振幅可以表示为

反射波

$$\tilde{U}_1 = A r e^{i(\varphi_0 + \pi)} = A \rho^{1/2} e^{i(\varphi_0 + \pi)} \quad \rho = |r|^2 = |r'|^2 \text{ 光强反射率}$$

$$\tilde{U}_n = A r^{2n-3} (1-r^2) e^{i[\varphi_0 + (n-1)\Delta\varphi]} = A \rho^{n-3/2} (1-\rho) e^{i[\varphi_0 + (n-1)\Delta\varphi]}$$

透射波

$$\tilde{U}'_n = A_0 r^{2n-2} e^{i[\varphi'_0 + (n-1)\Delta\varphi]} = A_0 \rho^{n-1} e^{i[\varphi'_0 + (n-1)\Delta\varphi]} \quad A_0 = A(1-r^2)$$

透射波的合振动

$$\begin{aligned} \tilde{U}_T &= \sum_{n=1}^N A_0 \rho^{n-1} e^{i[\varphi'_0 + (n-1)\Delta\varphi]} = A_0 e^{i\varphi'_0} \sum_{n=0}^{N-1} \rho^n e^{in\Delta\varphi} \\ &= A_0 e^{i\varphi'_0} \frac{1 - \rho^N e^{iN\Delta\varphi}}{1 - \rho e^{i\Delta\varphi}} \quad N \rightarrow \infty \quad \tilde{U}_T = \frac{A_0 e^{i\varphi'_0}}{1 - \rho e^{i\Delta\varphi}} \end{aligned}$$

$$\tilde{U}_T = \frac{A(1-r^2)e^{i\varphi'_0}}{1-\rho e^{i\Delta\varphi}}$$

$$I_T = \left| \frac{A(1-r^2)e^{i\varphi'_0}}{1-\rho e^{i\Delta\varphi}} \right|^2 = \frac{A^2(1-\rho)^2}{(1-\rho e^{i\Delta\varphi})(1-\rho e^{-i\Delta\varphi})}$$

$$(1-\rho e^{i\Delta\varphi})(1-\rho e^{-i\Delta\varphi})$$

$$= 1 - \rho(e^{i\Delta\varphi} + e^{-i\Delta\varphi}) + \rho^2 = 1 - 2\rho \cos(\Delta\varphi) + \rho^2$$

$$= (1-\rho)^2 + 2\rho - 2\rho \cos \Delta\varphi$$

$$= (1-\rho)^2 + 2\rho \left(1 - 1 + 2 \sin^2 \frac{\Delta\varphi}{2} \right)$$

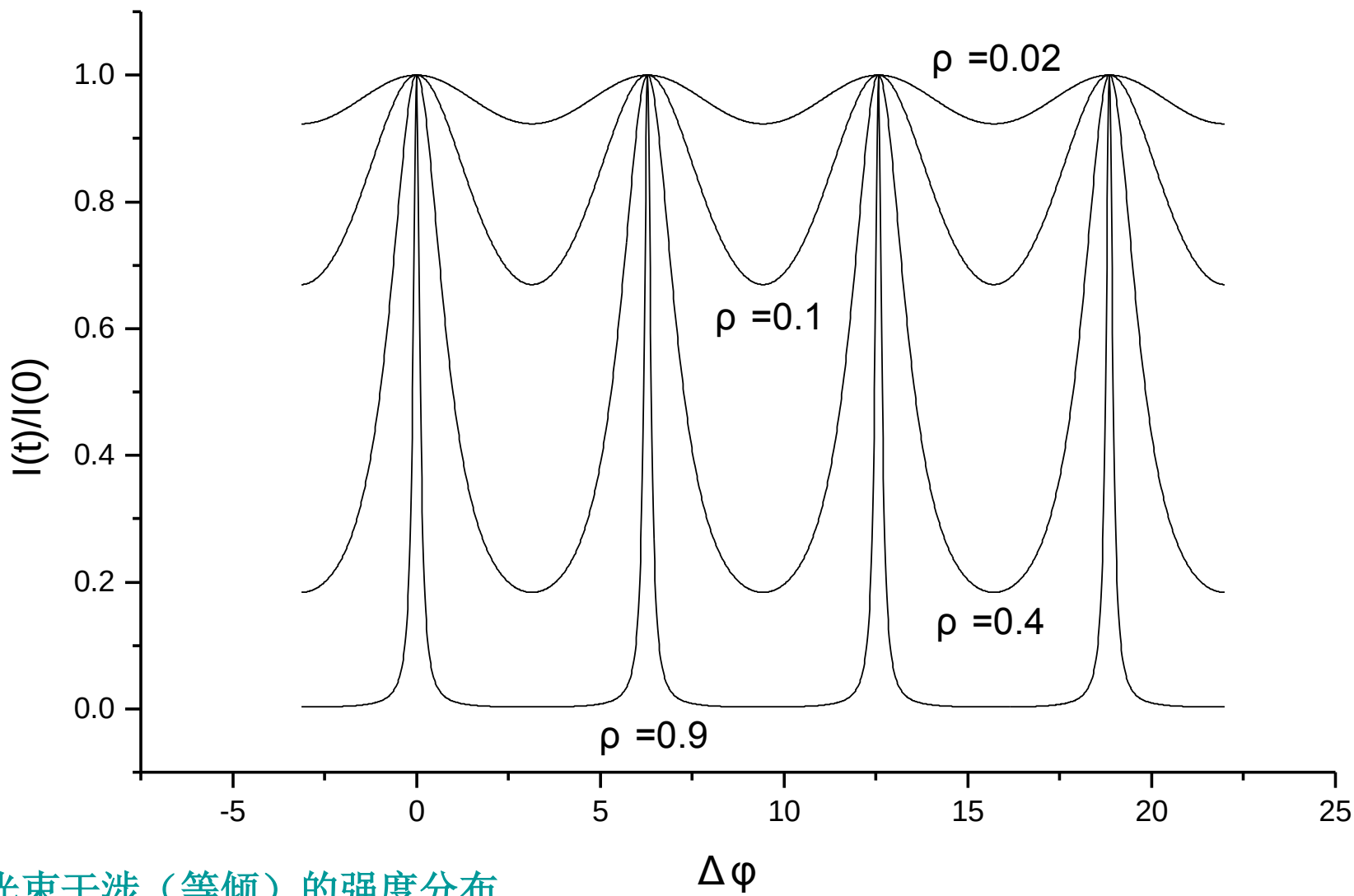
$$= (1-\rho)^2 + 4\rho \sin^2 \frac{\Delta\varphi}{2}$$

$$I_T = \frac{A^2(1-\rho)^2}{(1-\rho)^2 + 4\rho \sin^2 \frac{\Delta\varphi}{2}} = \frac{I}{1 + \frac{4\rho \sin^2 \frac{\Delta\varphi}{2}}{(1-\rho)^2}}$$

$I = A^2$ 入射光强

反射光的光强分布

$$I_R = I - I_T = I - \frac{I}{1 + \frac{4\rho \sin^2 \frac{\Delta\varphi}{2}}{(1-\rho)^2}} = \frac{I}{1 + \frac{(1-\rho)^2}{4\rho \sin^2 \frac{\Delta\varphi}{2}}}$$

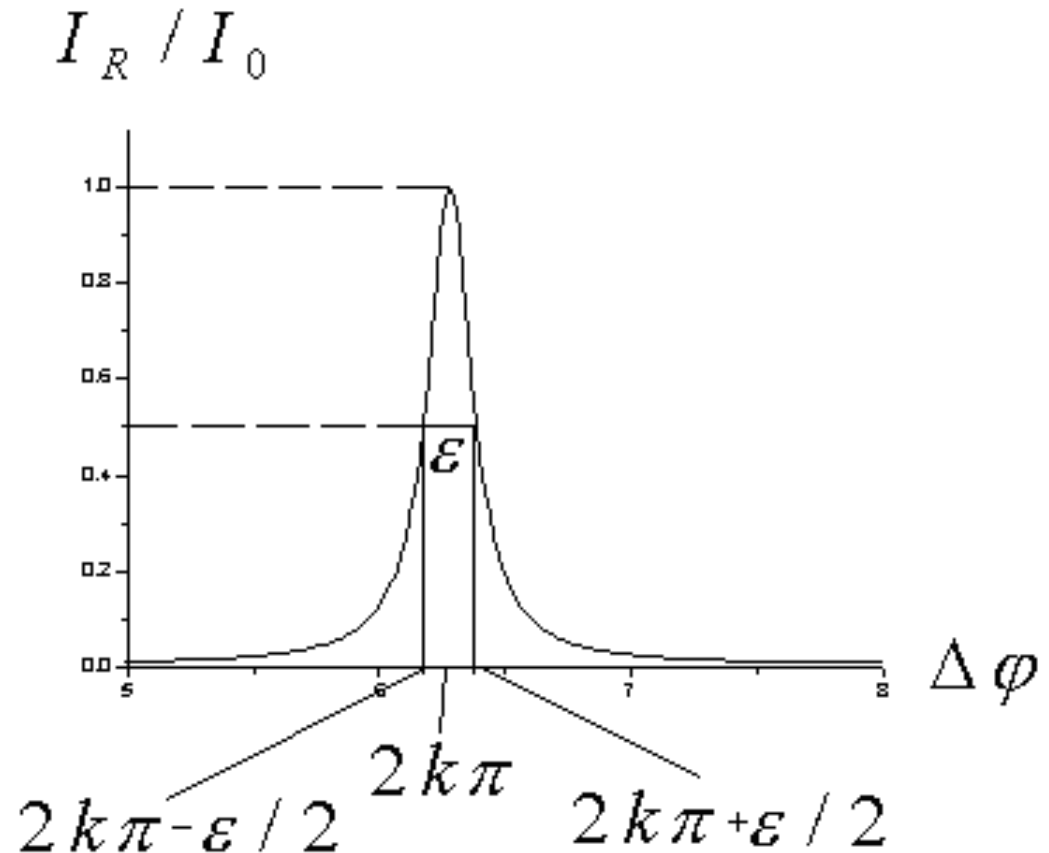


多光束干涉（等倾）的强度分布

条纹的角宽度

- 半值宽度：光强降为峰值一半时峰的宽度。
- 对于反射光和透射光，光强一半都有

$$\frac{4\rho \sin^2 \frac{\Delta\varphi}{2}}{(1-\rho)^2} = 1$$



$$\frac{4\rho \sin^2 \frac{\Delta\varphi}{2}}{(1-\rho)^2} = 1 \quad \Delta\varphi = \frac{2\pi}{\lambda} 2n_2 h \cos i_2$$

$\Delta\varphi$ 从 $2k\pi$ 变到 $2k\pi \pm \varepsilon/2$ 时，光强变为极大值的一半。

ε 为条纹的半值宽度位相值

$$\frac{4\rho \sin^2 \frac{\varepsilon}{4}}{(1-\rho)^2} \approx \frac{4\rho \left(\frac{\varepsilon}{4}\right)^2}{(1-\rho)^2} = \frac{\rho \varepsilon^2}{4(1-\rho)^2} = 1$$

$$\varepsilon = \frac{2(1-\rho)}{\sqrt{\rho}}$$

可以表示条纹的几何宽度，即角宽度。

$$\varepsilon = \frac{2(1-\rho)}{\sqrt{\rho}} = 2\left(\frac{1}{\sqrt{\rho}} - \sqrt{\rho}\right)$$

ρ 越大, ε 越小, 条纹越细锐。

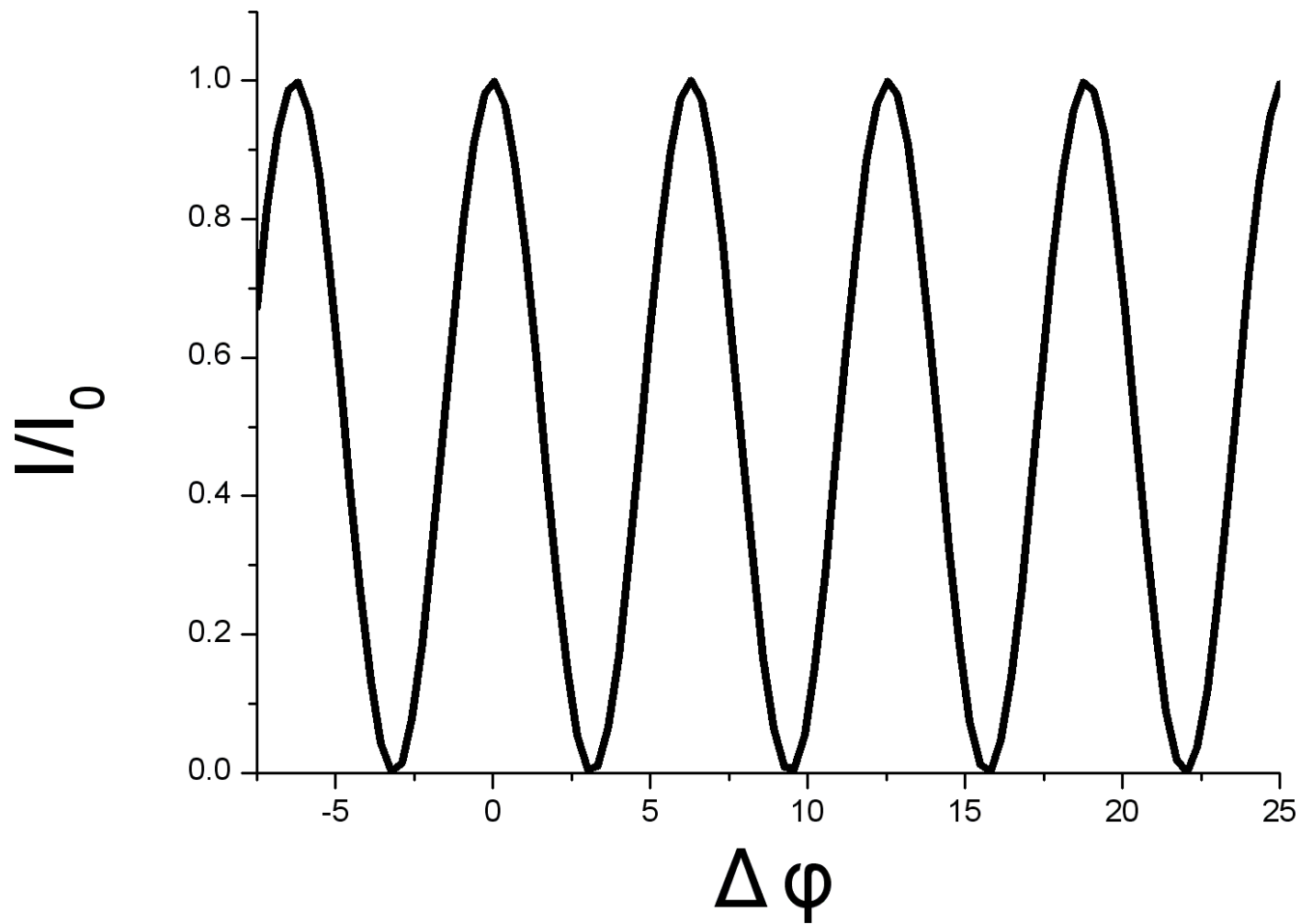
对于Michelson干涉仪

$$\delta = 2h \cos i \quad A_1 = A_2$$

$$\begin{aligned} I &= 2I_1 + 2I_1 \cos \Delta\varphi = 2I_1(1 + \cos \Delta\varphi) \\ &= 2I_1\left(1 + 2\cos^2 \frac{\Delta\varphi}{2} - 1\right) = I_0 \cos^2 \frac{\Delta\varphi}{2} \end{aligned}$$

条纹要粗得多

Michelson干涉仪条纹



多光束干涉的特性

- 1、条纹角分布
- 2、频率（波长）分布
- 3、光谱的精细结构分析

第j级亮条纹

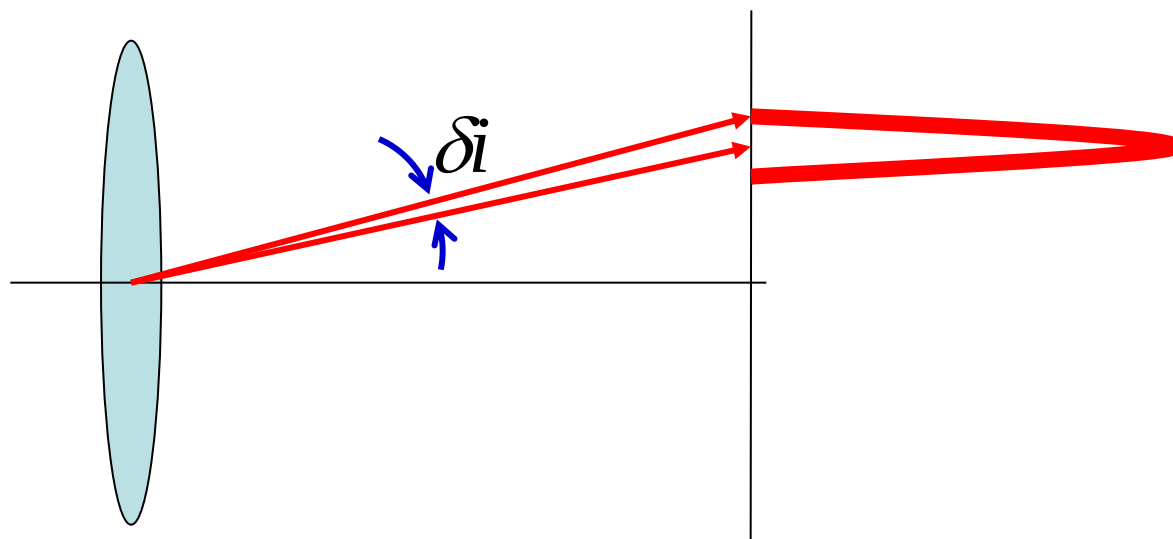
$$\Delta\varphi_j = \frac{2\pi}{\lambda} \delta = \frac{4\pi n_2 h \cos i_2}{\lambda} = 2j\pi$$

条纹角宽度

$$d(\Delta\varphi_j) = -\frac{4\pi n_2 h \sin i_2}{\lambda} di_2$$

$$d(\Delta\varphi_j) = \varepsilon \quad di_2 = \delta i_j \quad \text{条纹半角宽度}$$

$$\delta i_j = \frac{\lambda \varepsilon}{4\pi n_2 h \sin i_2} = \frac{\lambda}{2\pi n_2 h \sin i_2} \frac{1-\rho}{\sqrt{\rho}}$$



普通的薄膜干涉（Michelson干涉仪），即双光束干涉时

$$\delta i_2 = \frac{\lambda}{4n_2 h \sin i_2} = \frac{\lambda}{2\pi n_2 h \sin i_2} \frac{\pi}{2}$$

由于 $\rho \approx 1$ 所以 $\frac{1-\rho}{\sqrt{\rho}} \ll \frac{\pi}{2}$ 多光束干涉条纹要细锐的多

即出射的条纹发散角很小。保证了激光的平行性。

只有特殊的波长满足极大条件,

$$\Delta\varphi_j = 4\pi n_2 h \cos i_2 / \lambda = 2j\pi$$

j级亮条纹中, 极大值处的波长

$$\lambda_j = \frac{1}{j} 2n_2 h \cos i_2$$

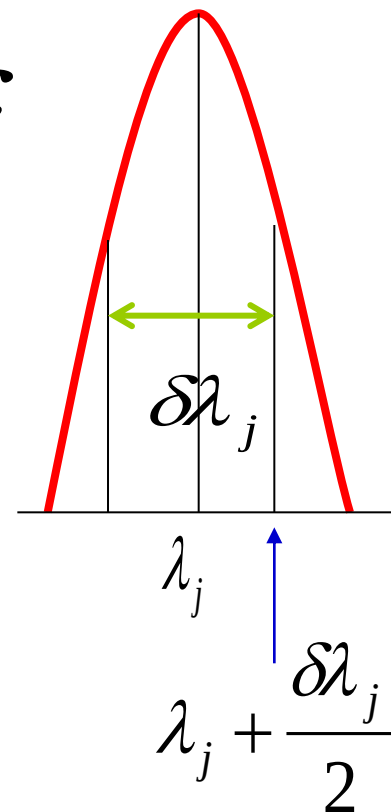
在其附近, 波长改变, 强度下降, 到达半值宽度时, 相应波长的改变量

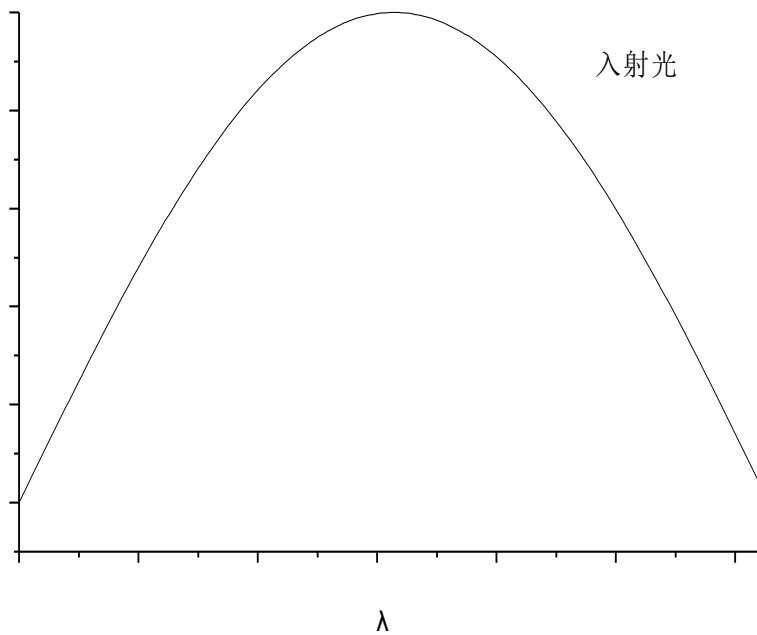
$$d(\Delta\varphi_j) = -(4\pi n_2 h \cos i_2 / \lambda_j^2) d\lambda_j = \varepsilon$$

$$\delta\lambda_j = \frac{\lambda_j^2}{2\pi n_2 h \cos i_2} \frac{1-\rho}{\sqrt{\rho}} = \frac{\lambda_j}{j\pi} \frac{1-\rho}{\sqrt{\rho}}$$

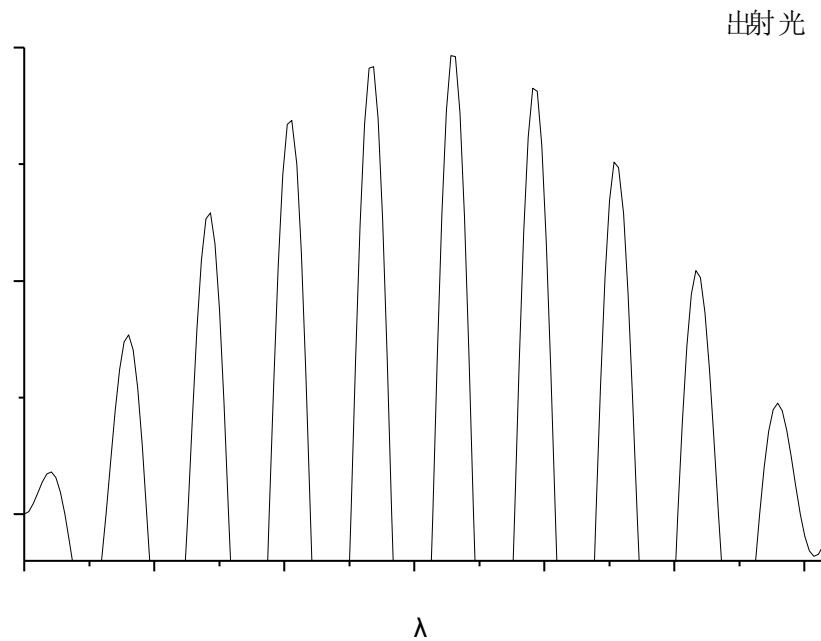
由于 $\frac{1-\rho}{\sqrt{\rho}} \ll 1$ 使得 $\delta\lambda_j \ll \lambda$

每一级出射的亮条纹都是很好的单色光波





白光入射



单色光出射

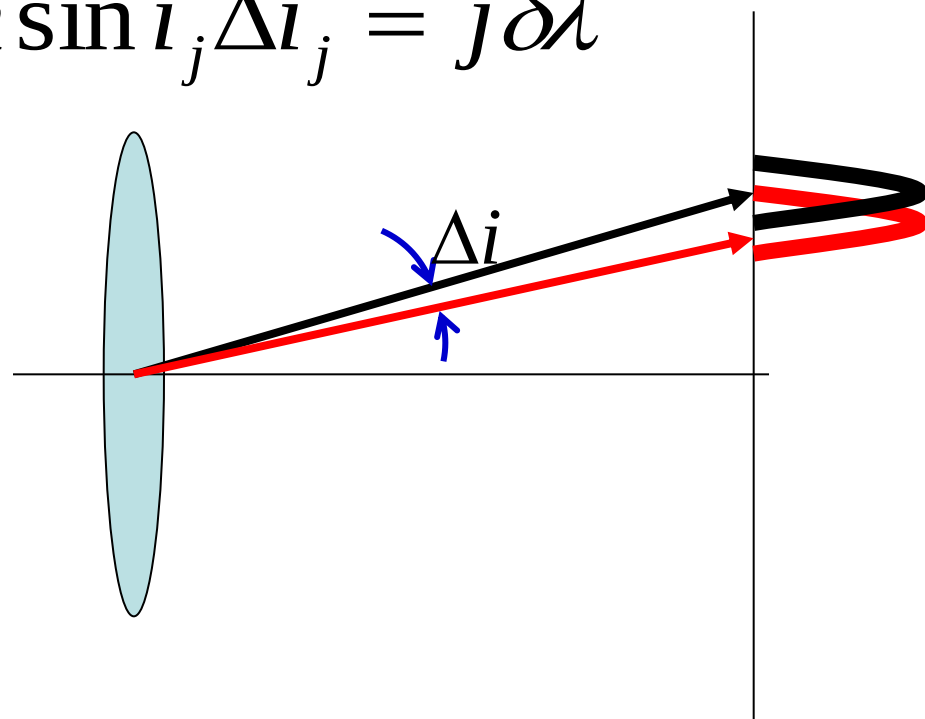
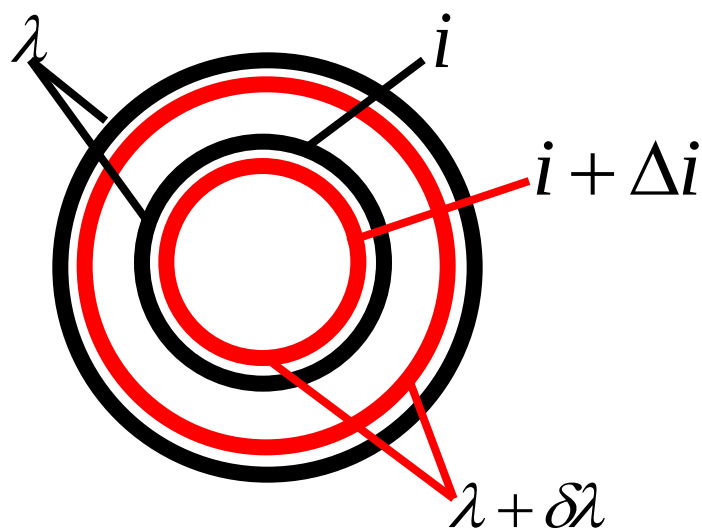
可用于选模。保证了激光的单色性。

j级亮条纹中心波长

$$2nh \cos i_j = j\lambda$$

$$2nh \cos(i_j + \delta i) = j(\lambda + \delta\lambda)$$

$$-2nh \sin i_j \Delta i_j = j\delta\lambda$$



$$\Delta i = \frac{j}{2nh \sin i_j} \delta\lambda$$

条纹间距

Rayleigh 判据或Taylor判据

$$\delta i = \Delta i$$

$$\delta i = \frac{\lambda}{2\pi n h \sin i_j} \frac{1-\rho}{\sqrt{\rho}}$$

$$\Delta i = \frac{j}{2n h \sin i_j} \delta \lambda$$

$$\frac{\lambda}{\pi} \frac{1-\rho}{\sqrt{\rho}} = j \delta \lambda$$

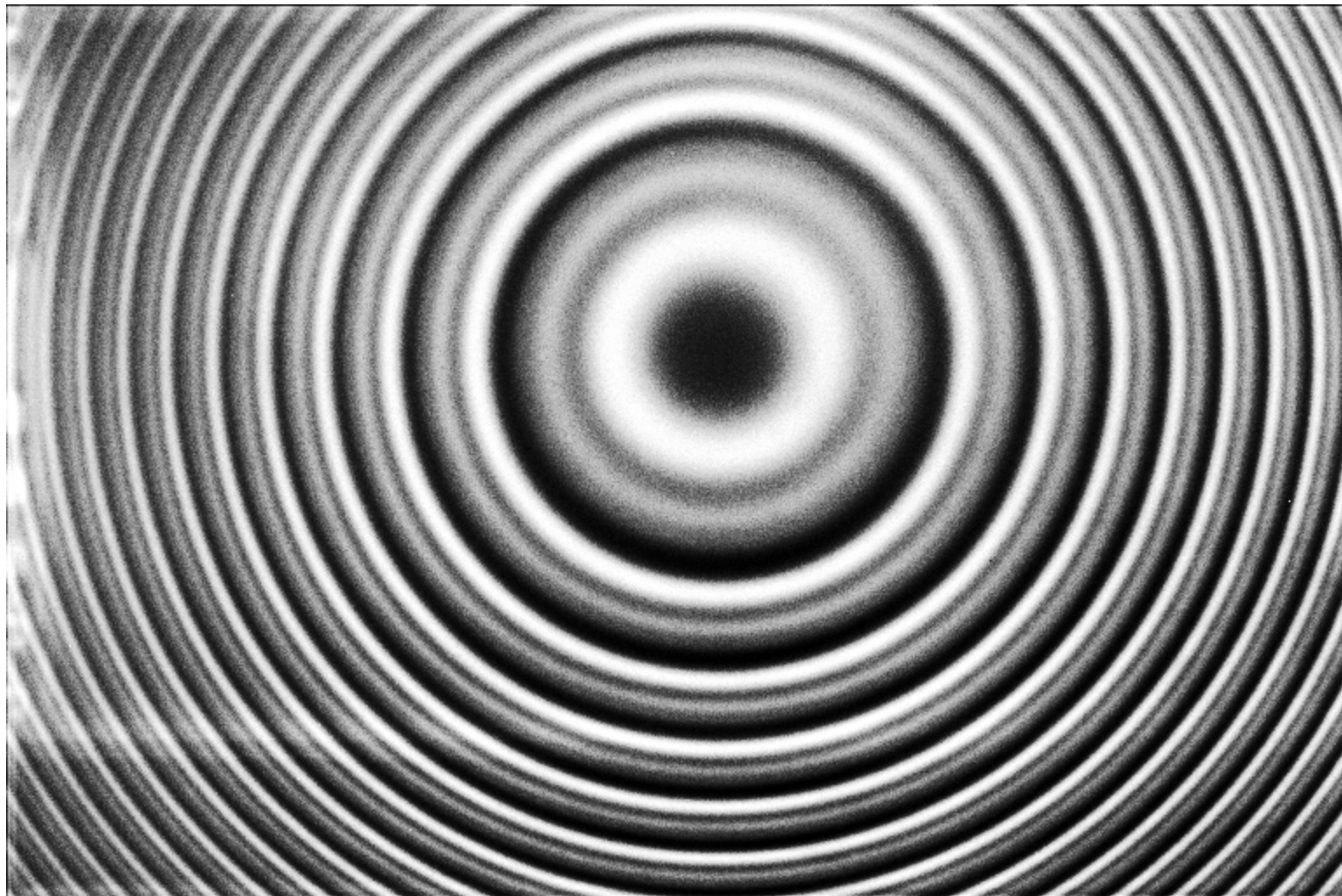
$$\delta \lambda = \frac{\lambda}{j\pi} \frac{1-\rho}{\sqrt{\rho}}$$

可分辨最小波长间隔

$$\frac{\lambda}{\delta \lambda} = \frac{\sqrt{\rho}}{1-\rho} j\pi$$

色（波长）分辨本领

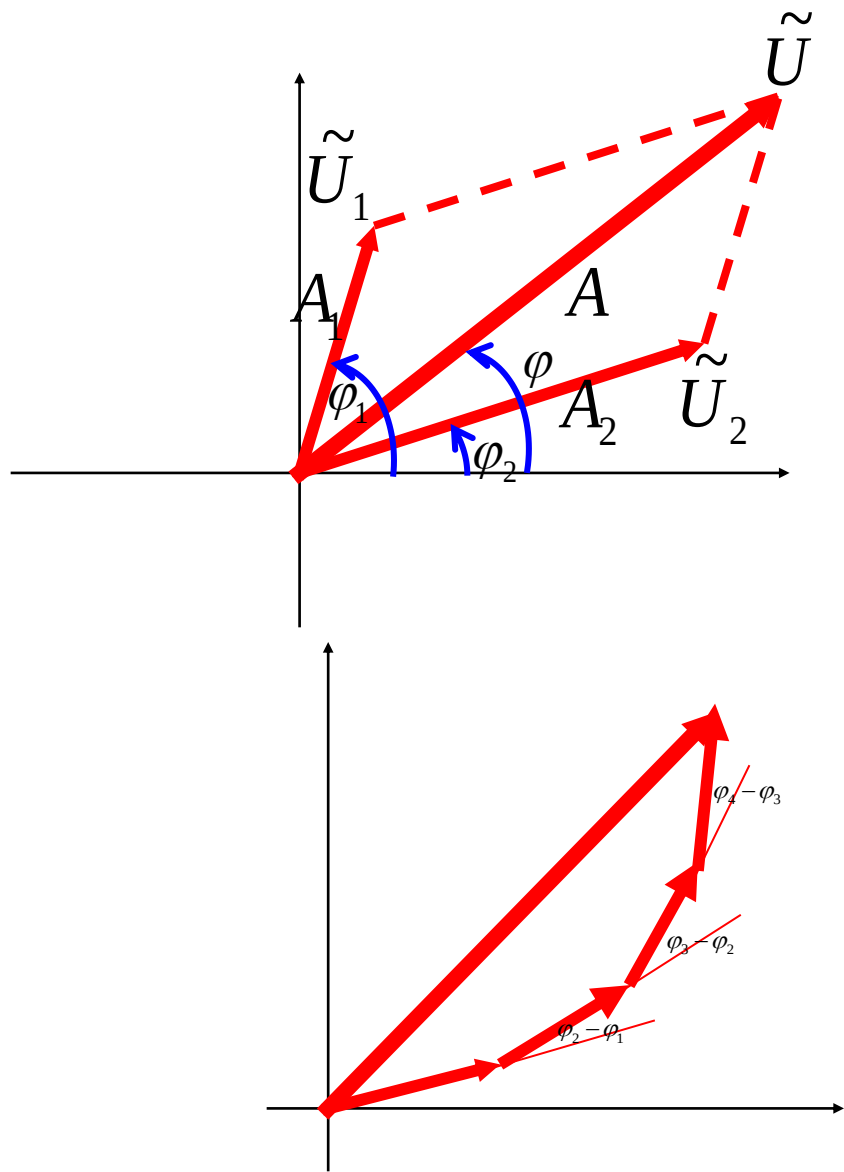
Fabry-Pérot 干涉条纹



光的干涉

波的叠加原理

- 传播介质为线性介质。
- 振动不十分强。在振动很强烈时，线性介质会变为非线性的。
- 注意要点：不是强度的叠加，也不是振幅的简单相加，而是振动矢量（瞬时值）的叠加。



干涉条件---存在不可区分性

(1)、 ω 相同

(2)、存在相互平行的振动分量

(3)、 $\Delta\varphi$ 稳定。

- 振动方向相同、传播方向相同，频率不同的两列波

$$\psi_1 = A_0 \cos(\omega_1 t - k_1 z) \quad \psi_2 = A_0 \cos(\omega_2 t - k_2 z)$$

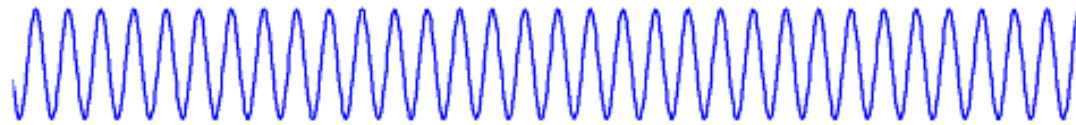
$$\psi = \psi_1 + \psi_2$$

$$= 2A_0 \cos \frac{(\omega_1 - \omega_2)t - (k_1 - k_2)z}{2} \cos \frac{(\omega_1 + \omega_2)t - (k_1 + k_2)z}{2}$$

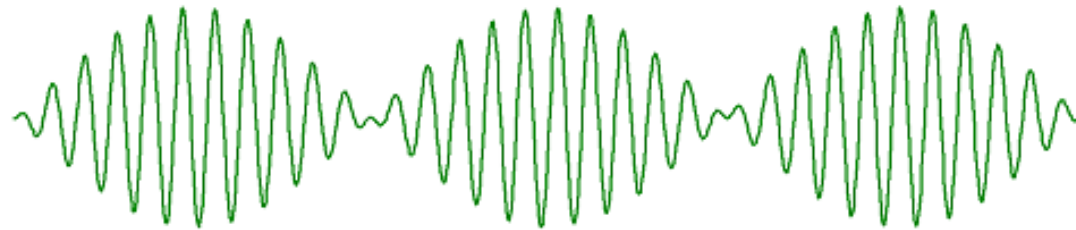
$$= 2A_0 \cos(\omega_m t - k_m z) \cos(\bar{\omega} t - \bar{k} z)$$

$$\Psi = 2A_0 \cos(\omega_m t - k_m z) \cos(\bar{\omega}t - \bar{k}z)$$

$$\cos(\bar{\omega}t - \bar{k}z)$$



$$2\cos(\omega_m t - k_m z)$$



$$2\cos(\omega_m t - k_m z)\cos(\bar{\omega}t - \bar{k}z)$$

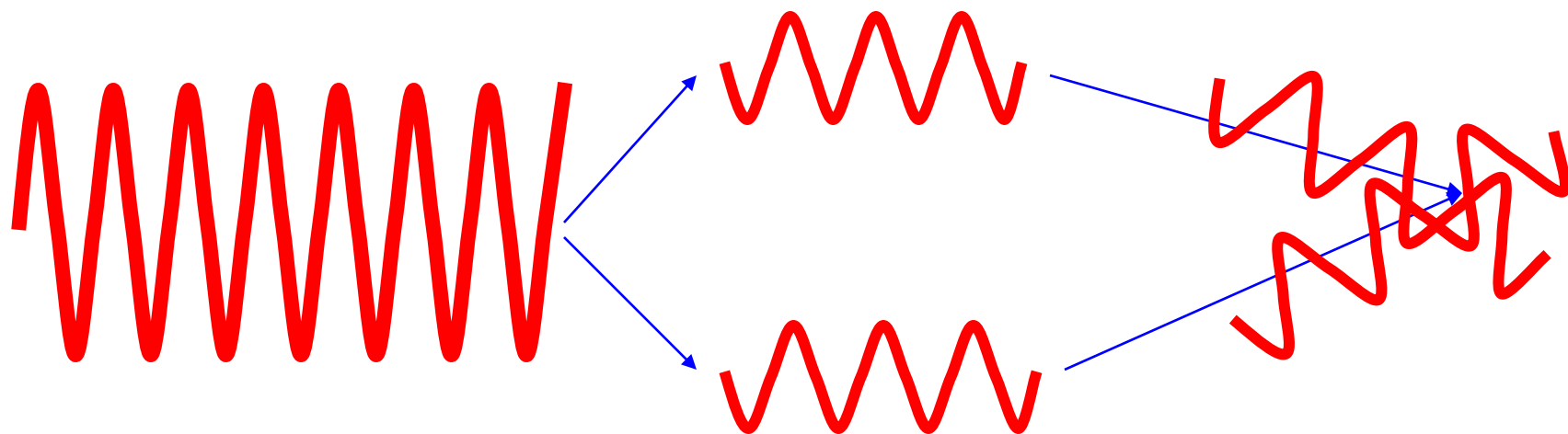
干涉条纹的反衬度（可见度）

- 反衬度的定义：在接收屏上一选定的区域中，取光强最大值和最小值，有

$$\gamma = \frac{I_M - I_m}{I_M + I_m} \quad I_M = (A_1 + A_2)^2, I_m = (A_1 - A_2)^2$$

$$\gamma = \frac{2A_1A_2}{A_1^2 + A_2^2} = \frac{2\frac{A_1}{A_2}}{1 + \left(\frac{A_1}{A_2}\right)^2}$$

- 当 $A_1=A_2$ 时， $\gamma=1$ ，反衬度最大
- 当 $A_1 \ll A_2$ 或 $A_1 \gg A_2$ 时，即 A_1 、 A_2 相差悬殊时， $\gamma=0$ ，反衬度最小

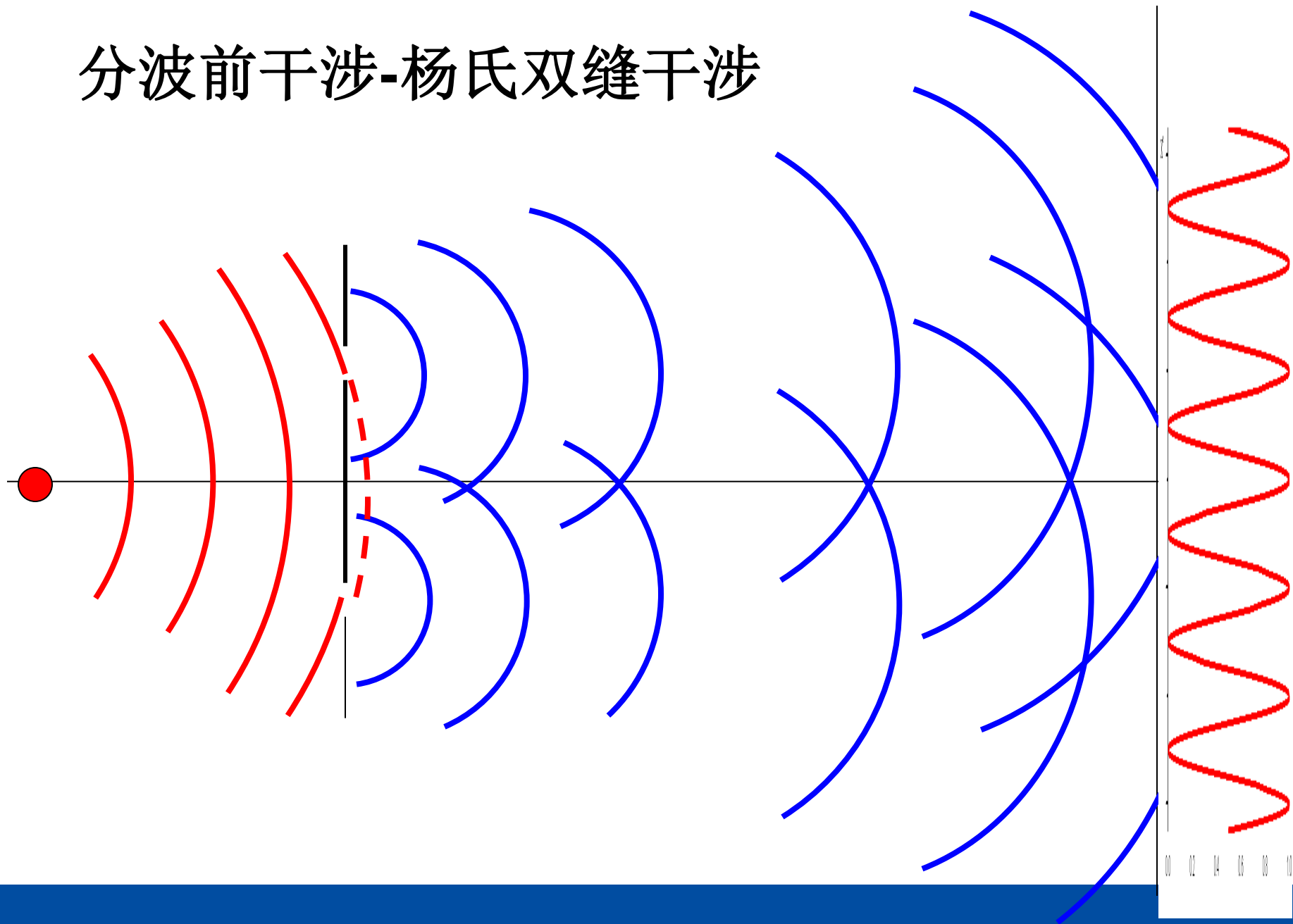


干涉相长 $\delta = r_2 - r_1 = j\lambda$

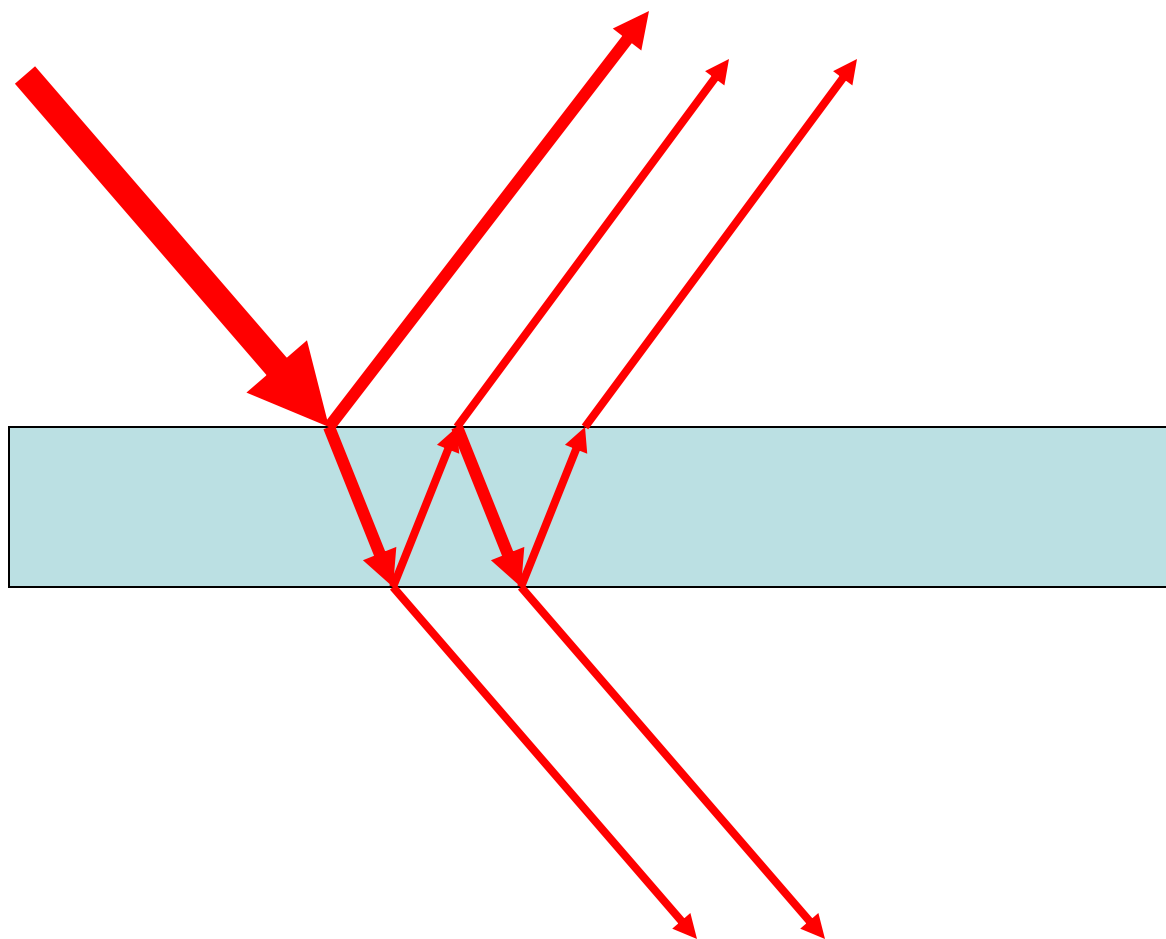
干涉相消 $\delta = r_2 - r_1 = (2j + 1)\frac{\lambda}{2}$

$j=0, 1, 2, 3, 4, \dots$, 干涉级数

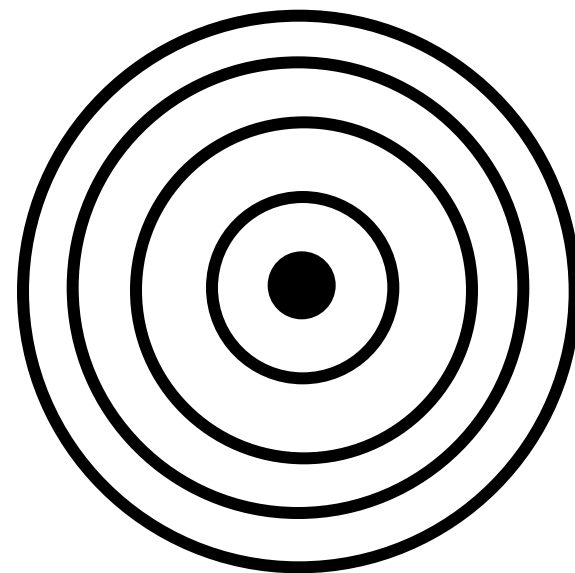
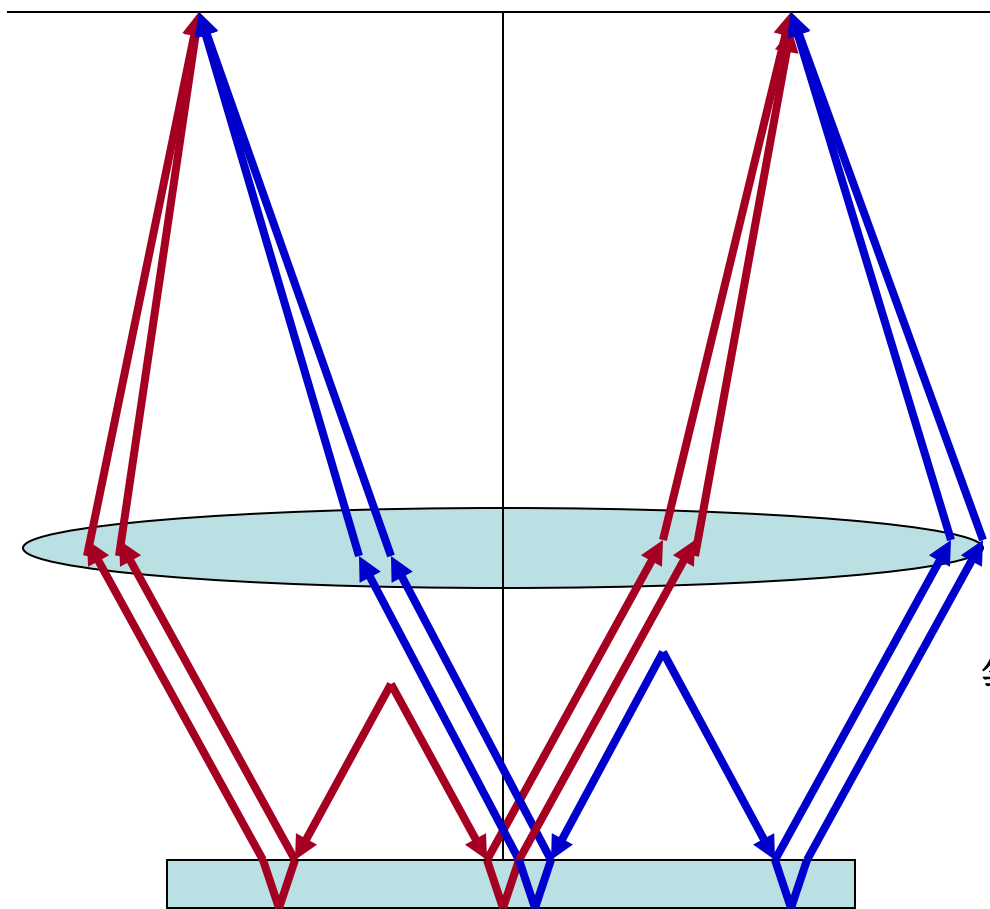
分波前干涉-杨氏双缝干涉



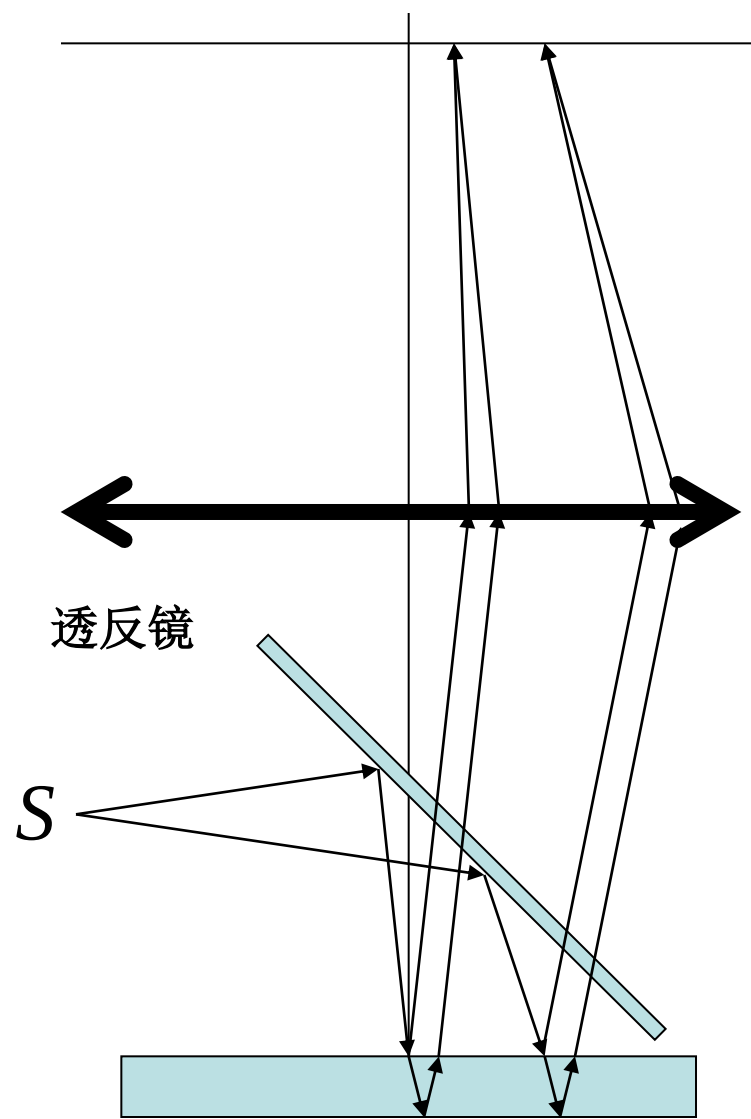
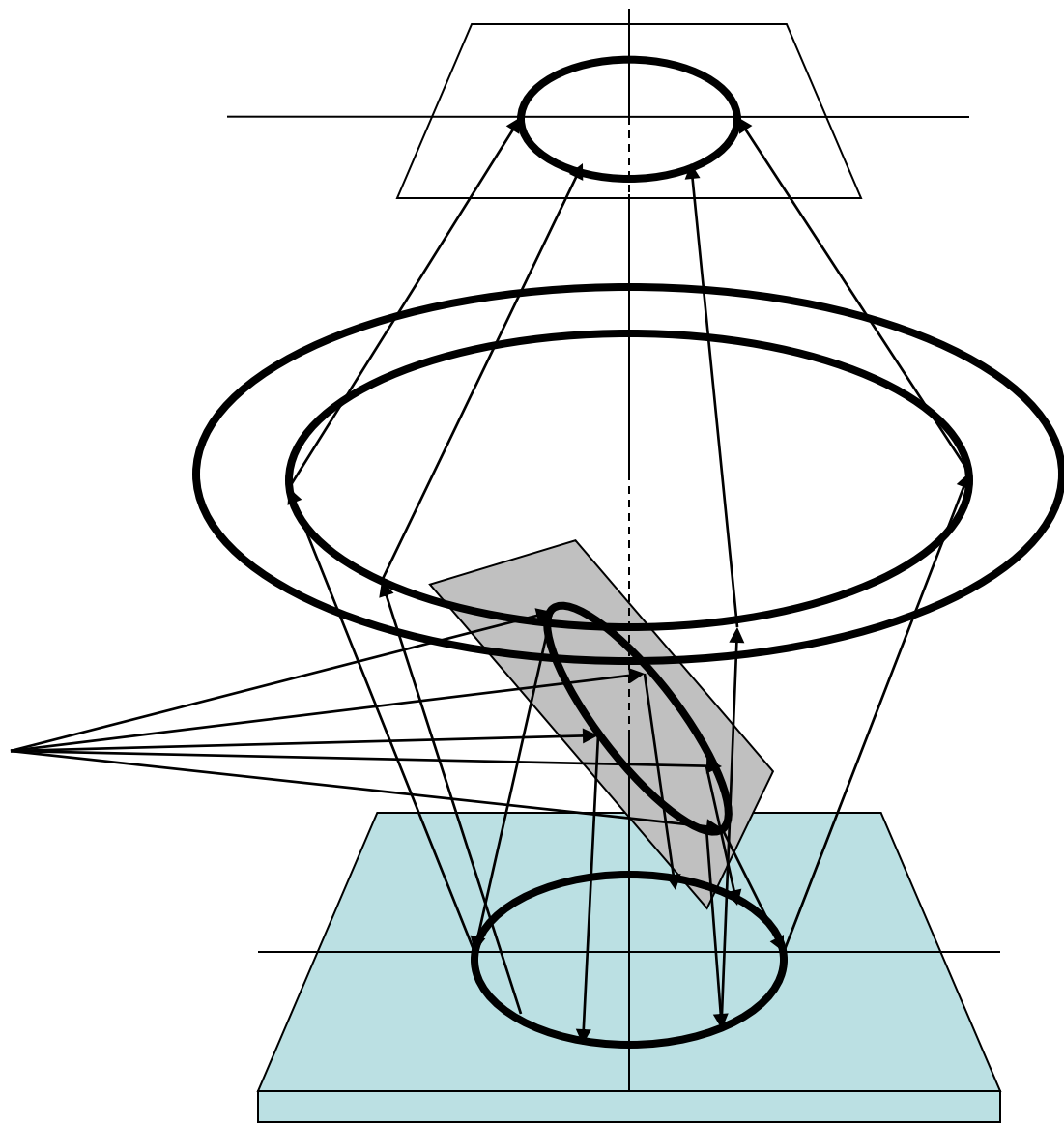
分振幅干涉-薄膜干涉



等倾干涉

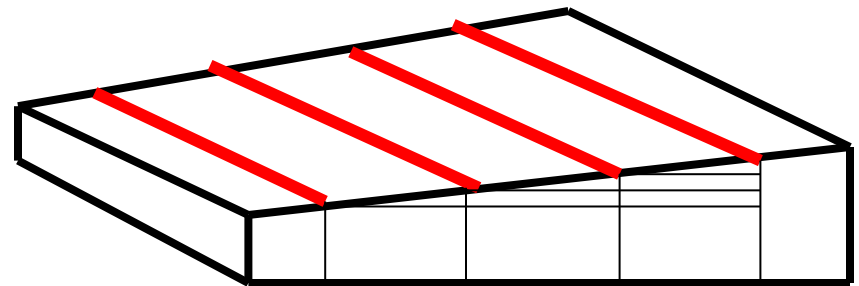
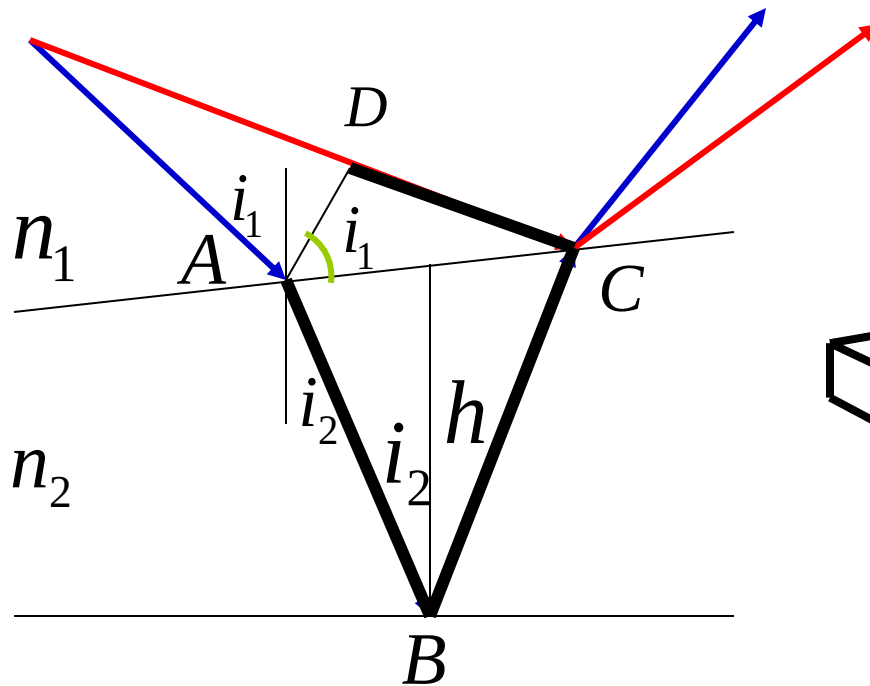


等倾干涉的条纹是同心圆环

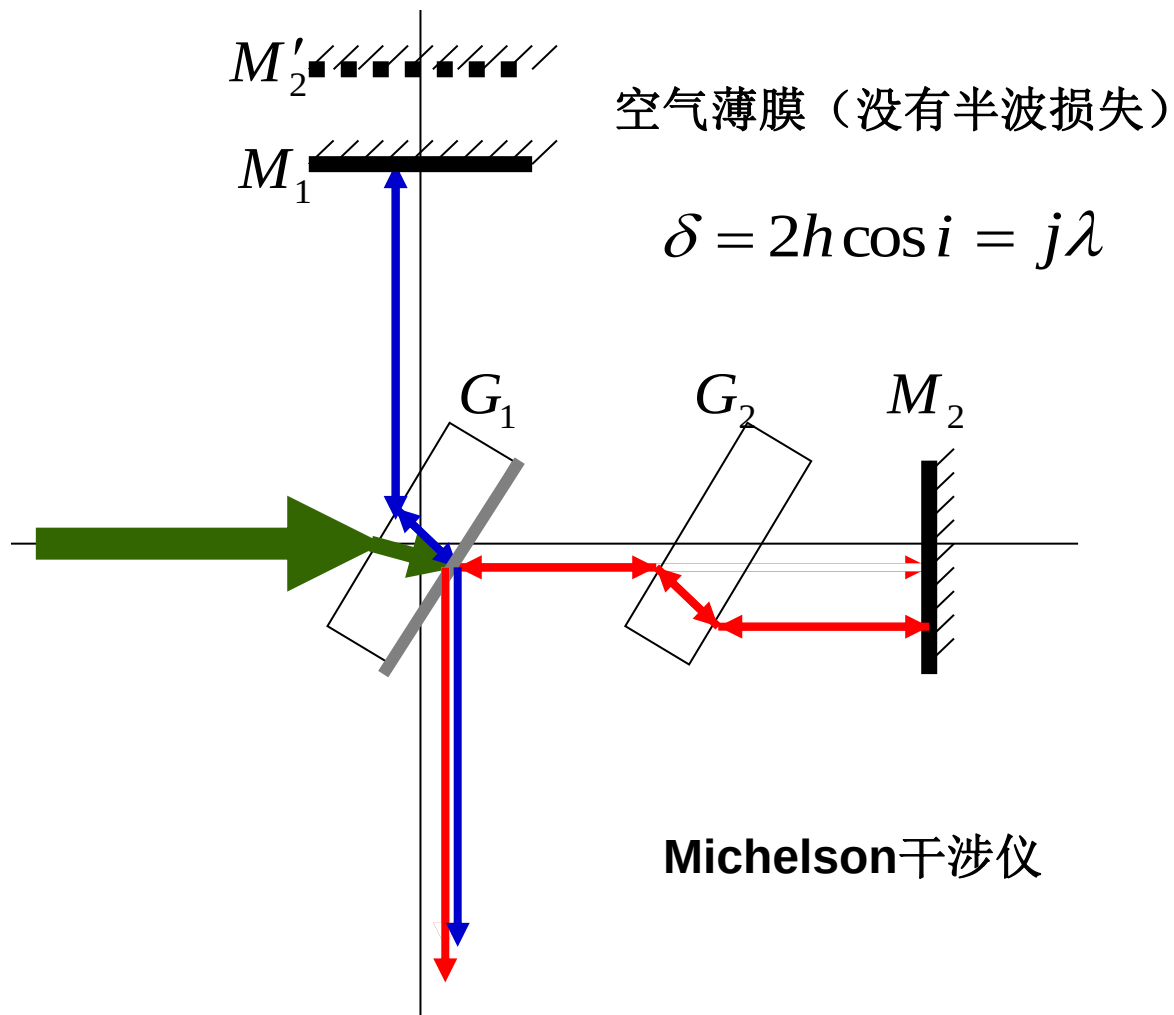


等倾干涉的观察装置

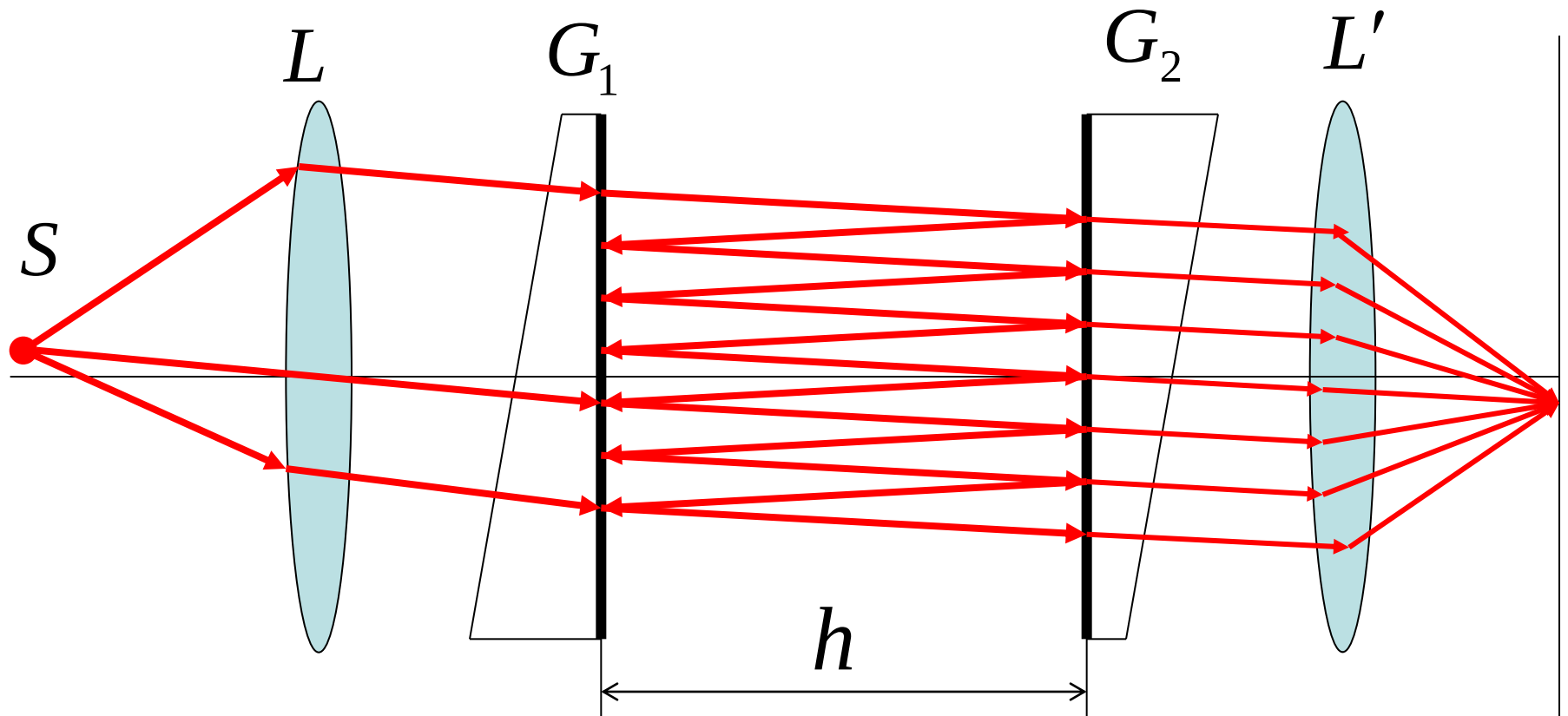
等厚干涉

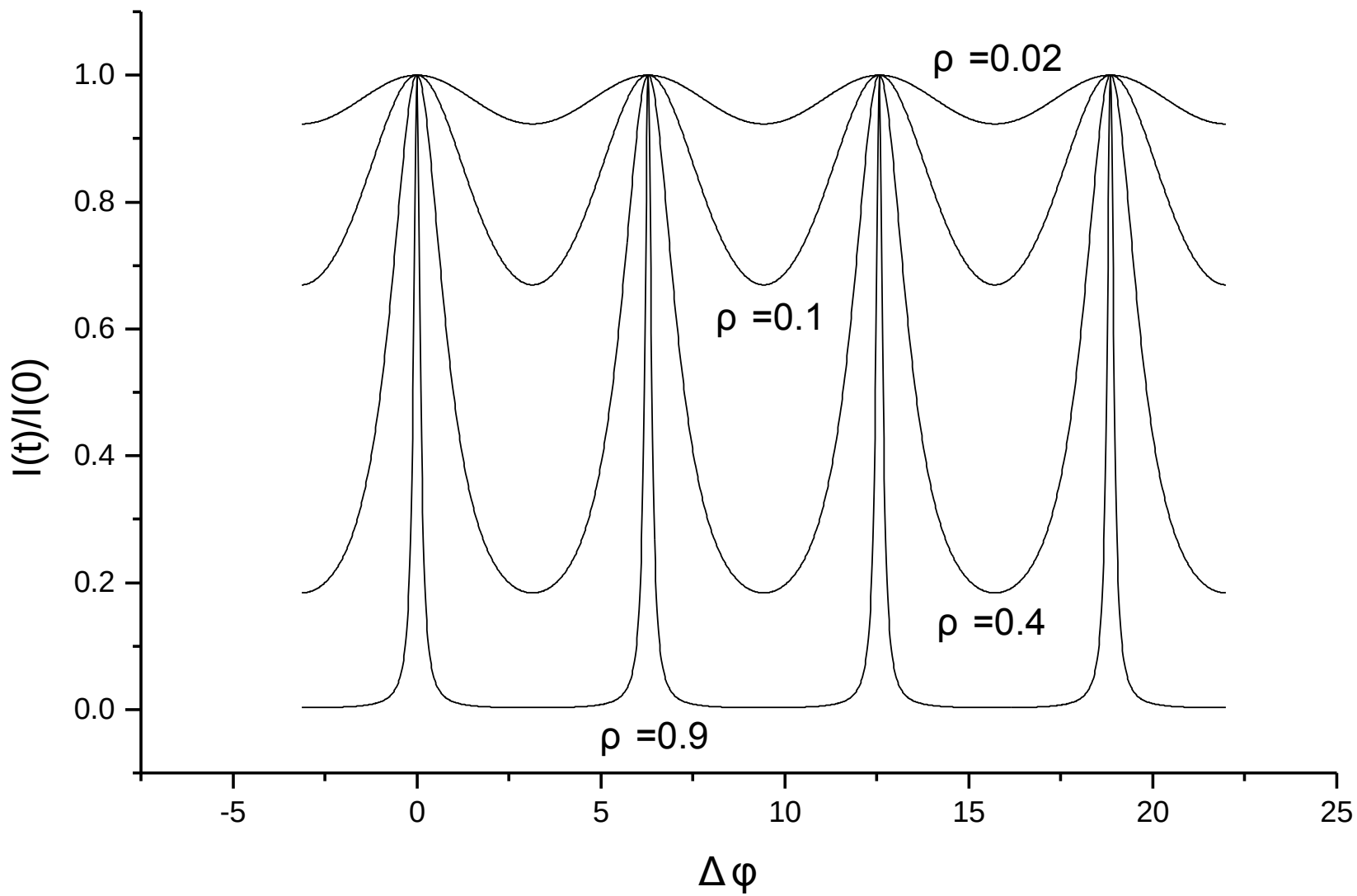


迈克尔逊干涉仪



多光束干涉：Fabry-Perot干涉仪和标准具





高反膜

